FM1 – Mechanics Chapter 3
Exam Questions: Springs at an Angle

Q1.

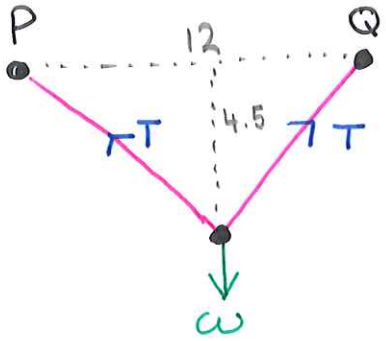
A light elastic string has natural length 8 m and modulus of elasticity 80 N.

The ends of the string are attached to fixed points *P* and *Q* which are on the same horizontal level and 12 m apart.

A particle is attached to the mid-point of the string and hangs in equilibrium at a point 4.5 m below *PQ*.

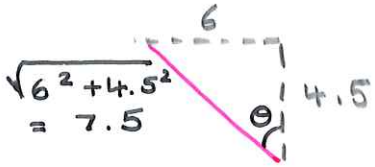
Calculate the weight of the particle.

(Total 6 marks)



$L = 8$

$\lambda = 80$

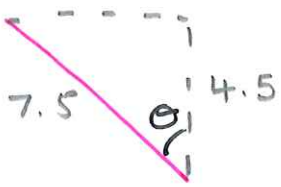


$x = 7.5 - 4$
$= 3.5$

$$T = \frac{\lambda x}{L}$$

$$T = \frac{80(3.5)}{4}$$

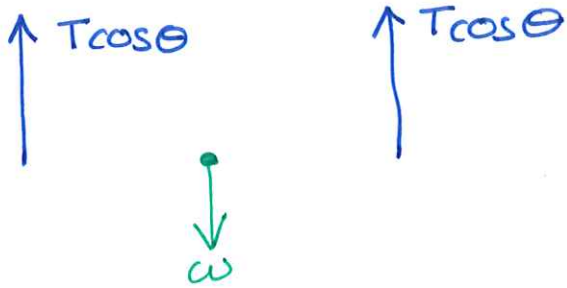
$$T = 70$$



$$\cos\theta = \frac{4.5}{7.5}$$

$$\cos\theta = \frac{3}{5}$$

(↕)

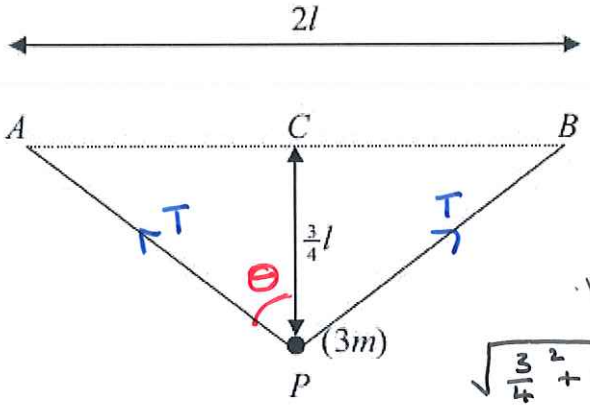


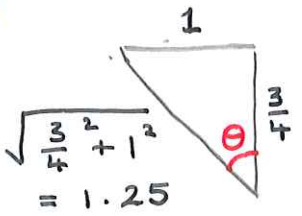
$$2T\cos\theta = \omega$$

$$2(70)\,\frac{3}{5} = \omega$$

$$\omega = 84 \text{ N}$$

Q2.





$\cos\theta = \frac{3/4}{1.25}$

$\cos\theta = 0.6$

Figure 4

A small ball of mass $3m$ is attached to the ends o two light elastic strings AP and BP, each of natural length l and modulus of elasticity kmg. The ends A and B of the strings are attached to fixed points on the same horizontal level, with $AB = 2l$. The mid-point of AB is C. The ball hangs in equilibrium at a distance $\frac{3}{4}l$ vertically below C as shown in Figure 4.

Show that $k = 10$

(Total 7 marks)

$\lambda = k mg$

$L = L$

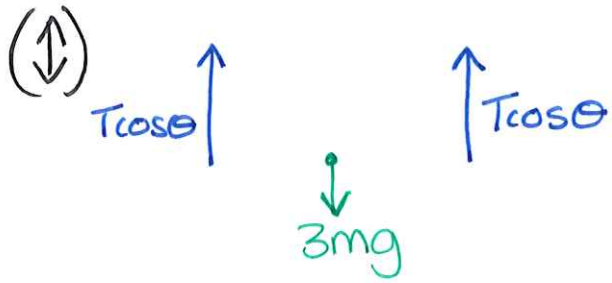
$x = 1.25L - 1L$

$= 0.25L$

$T = \frac{\lambda x}{L}$

$T = \frac{k mg (0.25L)}{L}$

$T = \frac{k mg}{4}$



$2T\cos\theta = 3mg$

$2\left(\frac{k mg}{4}\right)(0.6) = 3mg$

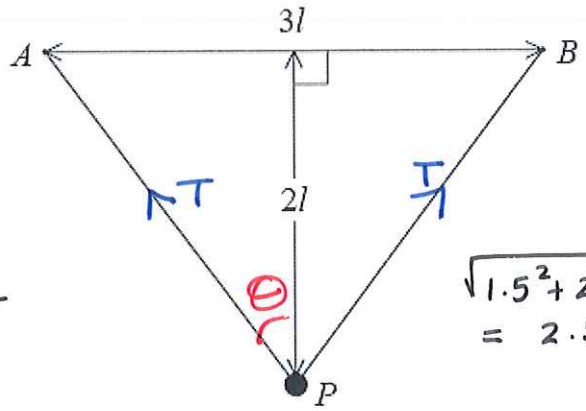
$0.3k mg = 3mg$

$0.3k = 3$

$k = 10$

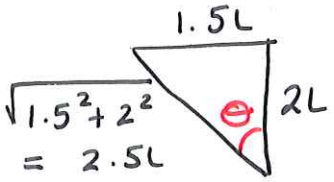
Q3.

Figure 1



$L = 1.5L$

$x = 2.5L - 1.5L$
$= 1L$



$\cos\theta = \frac{2}{2.5}$

$\cos\theta = \frac{4}{5}$

A light elastic string, of natural length $3l$ and modulus of elasticity λ, has its ends attached to two points A and B, where $AB = 3l$ and AB is horizontal.

A particle P of mass m is attached to the mid-point of the string. Given that P rests in equilibrium at a distance $2l$ below AB, as shown in Figure 1,

show that $\lambda = \frac{15mg}{16}$.

(Total 9 marks)

$T\cos\theta \uparrow$ $\uparrow T\cos\theta$

$\downarrow mg$

$2T\cos\theta = mg$

$2T\left(\frac{4}{5}\right) = mg$

$8T = 5mg$

$T = \frac{5mg}{8}$

$T = \frac{\lambda x}{L}$

$\frac{5mg}{8} = \frac{\lambda(1L)}{1.5L}$

$\frac{7.5mg}{8} = \lambda$

$\lambda = \frac{15mg}{16}$

Q4.

A particle P of mass 1.5 kg is attached to the mid-point of a light elastic string of natural length 0.30 m and modulus of elasticity λ newtons. The ends of the string are attached to two fixed points A and B, where AB is horizontal and $AB = 0.48$ m.
Initially P is held at rest at the mid-point, M, of the line AB and the tension in the string is 240 N.

(a) Show that $\lambda = 400$

(3)

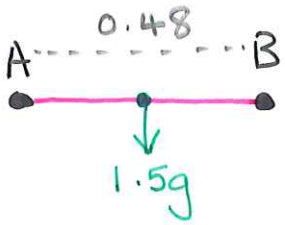
The particle is now held at rest at the point C, where C is 0.07 m vertically below M.

(b) Find the angle between the horizontal and the string.

(6)

(Total 15 marks)

(a)



$L = 0.3$

$x = 0.48 - 0.3 = 0.18$

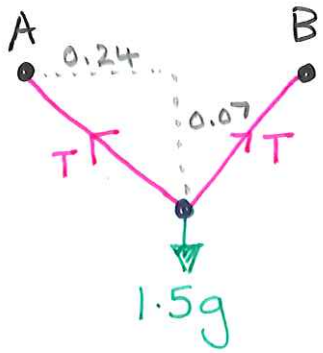
$T = 240$

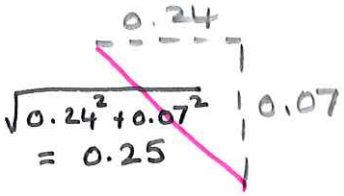
$$T = \frac{\lambda x}{L}$$

$$240 = \frac{\lambda(0.18)}{0.3}$$

$$\lambda = 400$$

(b)





$$\cos\theta = \frac{0.07}{0.25} = \frac{7}{25}$$

$L = 0.15$

$x = 0.25 - 0.15$
$= 0.1$

$$T = \frac{\lambda x}{L}$$

$$T = \frac{400(0.1)}{0.15}$$

$$T = \frac{800}{3}$$

$2T\cos\theta = 1.5g$

$$2\left(\frac{800}{3}\right)\cos\theta = 1.5g$$

$$\cos\theta = \frac{4.5g}{1600}$$

$$\theta = 88.4^\circ$$

Q5.

The ends of a light elastic string, of natural length 0.4 m and modulus of elasticity λ newtons,are attached to two fixed points A and B which are 0.6 m apart on a smooth horizontal table.

The tension in the string is 8 N.

(a) Show that $\lambda = 16$

(3)

A particle P is attached to the midpoint of the string. The particle P is now pulled **horizontally** in a direction perpendicular to AB to a point 0.4 m from the midpoint of AB. The particle is held at rest by a **horizontal** force of magnitude F newtons acting in a direction perpendicular to AB, as shown in Figure 5 below.

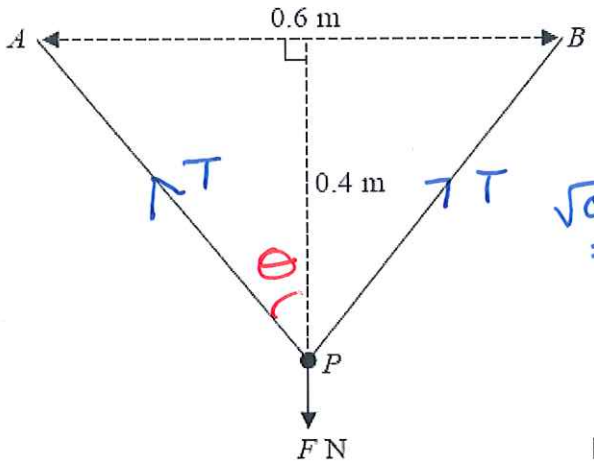


Figure 5

(b) Find the value of F.

(4)

(Total 7 marks)

(a) $L = 0.4$

$x = 0.6 - 0.4 = 0.2$

$T = 8$

$T = \frac{\lambda x}{L}$

$8 = \frac{\lambda (0.2)}{0.4}$ $\quad \lambda = 16$

(b) $T = \frac{\lambda x}{L}$ $\quad L = 0.4$, $x = 1 - 0.4$, $x = 0.6$

$T = \frac{16(0.6)}{0.4}$

$T = 24$ N

0.3, $\sqrt{0.3^2 + 0.4^2} = 0.5$, 0.4, θ

$\cos\theta = \frac{0.4}{0.5}$

$\cos\theta = \frac{4}{5}$

$T\cos\theta$ $\quad$ $T\cos\theta$

F

$2T\cos\theta = F$

$2T\left(\frac{4}{5}\right) = F$

$F = \frac{8 \times 24}{5}$

$F = 38.4$ N

Q6.

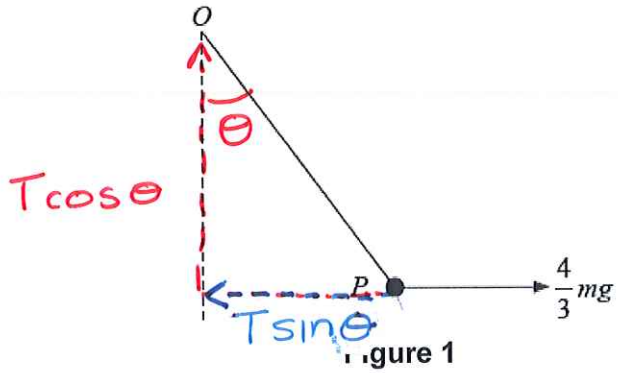


Figure 1

A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity 3mg. The other end of the string is attached to a fixed point O.

The particle P is held in equilibrium by a horizontal force of magnitude $\frac{4}{3}mg$ applied to P. This force acts in the vertical plane containing the string, as shown in Figure 1.

Find

(a) the tension in the string,

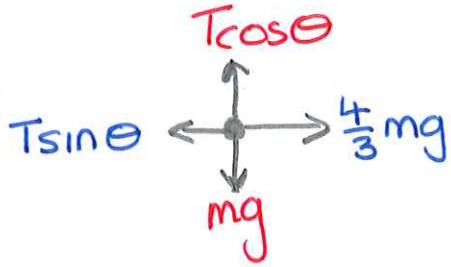
(5)

(b) the extension of the string.

(2)

(Total 7 marks)

(a)



① $T\cos\theta = mg$

② $T\sin\theta = \frac{4}{3}mg$

$①^2 + ②^2$

$$T^2\cos^2\theta + T^2\sin^2\theta = (mg)^2 + \left(\tfrac{4}{3}mg\right)^2$$

$$T^2(\cos^2\theta + \sin^2\theta) = (mg)^2 + \tfrac{16}{9}(mg)^2$$

$$T^2 = \tfrac{25}{9}(mg)^2$$

$$T = \tfrac{5}{3}mg$$

Q6.

(b) $\lambda = 3mg$

$l = a$

$T = \frac{5}{3}mg$

$$T = \frac{\lambda x}{l}$$

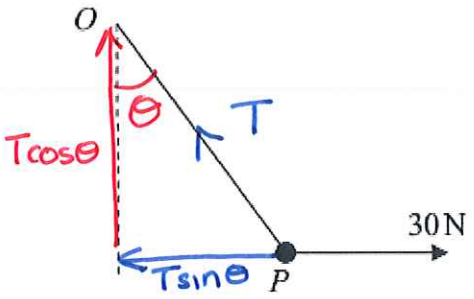
$$\frac{5}{3}mg = \frac{3mg\,x}{a}$$

$$5mga = 9mgx$$

$$5a = 9x$$

$$x = \frac{5a}{9}$$

Q7.



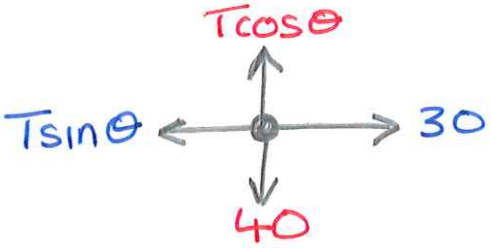
$L = 0.5$

$\lambda = 62.5$

Figure 3

A particle *P* of weight 40 N is attached to one end of a light elastic string of natural length 0.5 m. The other end of the string is attached to a fixed point *O*. A horizontal force of magnitude 30 N is applied to *P*, as shown in Figure 3. The particle *P* is in equilibrium and the modulus of elasticity is 62.5 N.
Calculate the length *OP*.

(Total 10 marks)



① $T\cos\theta = 40$

② $T\sin\theta = 30$

① ^{2} $T^2\cos^2\theta = 1600$

② ^{2} $T^2\sin^2\theta = 900$

① ^{2} + ② ^{2}

$T^2\cos^2\theta + T^2\sin^2\theta = 2500$

$T^2(\cos^2\theta + \sin^2\theta) = 2500$

$T^2 = 2500$

$T = 50$

$T = \frac{\lambda x}{L}$

$50 = \frac{62.5x}{0.5}$

$25 = 62.5x$

$x = 0.4$

$OP = 0.5 + 0.4$

$= 0.9$