

$$\begin{aligned}
 1) \quad & x^2 - 4x + 7 \\
 & (x - 2)^2 - 4 + 7 \\
 & (x - 2)^2 + 3
 \end{aligned}$$

Min point (2, 3) $\therefore$ positive for all values of x

$$2) \quad n^2 - n + 3$$

when $n = 3$

$$(3)^2 - 3 + 3 = 9$$

9 is not prime $3 \times 3 = 9$ (disproof by counter example)

$$3) \quad 2n + 1 + 2n + 3$$

$$4n + 4$$

$$\underline{\underline{4(n + 1)}}$$

$$4) \quad \text{let } x = 1 \text{ and } y^2 = 2$$

$$(x + y)^2 = x^2 + y^2$$

$$(1 + 2)^2 = (1)^2 + (2)^2$$

$$3^2 = 1 + 4$$

$$9 = 1 + 4$$

$$9 = 5$$

disproof by counter example.

5a)

$n=1$	$(1)^2 + (1) + 11$	$= 13$	(prime)
$n=2$	$(2)^2 + (2) + 11$	$= 17$	(prime)
$n=3$	$(3)^2 + (3) + 11$	$= 23$	(prime)
$n=4$	$(4)^2 + (4) + 11$	$= 31$	(prime)
$n=5$	$(5)^2 + (5) + 11$	$= 41$	(prime)

proof by exhaustion.

b)

$n=10$	$(10)^2 + 10 + 11$	$= 121$	(not prime $11 \times 11 = 121$)
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6) ^{even} positive integers less than 10 2, 4, 6, 8

$2 + 4$	$= 6$	(even)
$2 + 6$	$= 8$	(even)
$2 + 8$	$= 10$	(even)
$4 + 6$	$= 10$	(even)
$4 + 8$	$= 12$	(even)
$6 + 8$	$= 14$	(even)

7) $2n + 5$

when $n = 10$	$2(10) + 5$	$= 25$	(True)
when $n = -10$	$2(-10) + 5$	$= -15$	(Not True)

The statement is sometimes true.

$$\begin{aligned}
 8) \quad & (2n+3)^2 - (2n-3)^2 \\
 & (2n+3)(2n+3) - (2n-3)(2n-3) \\
 & (4n^2 + 6n + 6n + 9) - (4n^2 - 6n - 6n + 9) \\
 & (4n^2 + 12n + 9) - (4n^2 - 12n + 9) \\
 & 4n^2 + 12n + 9 - 4n^2 + 12n - 9 \\
 & 24n \\
 & \underline{\underline{6(4n)}}
 \end{aligned}$$

$$\begin{aligned}
 9) \quad & (2n+1)^2 + (2n+3)^2 \\
 & (2n+1)(2n+1) + (2n+3)(2n+3) \\
 & 4n^2 + 2n + 2n + 1 + 4n^2 + 6n + 6n + 9 \\
 & 8n^2 + 16n + 10 \\
 & 8n^2 + 16n + 8 + 2 \\
 & \underline{\underline{8(n^2 + 2n + 1) + 2}}
 \end{aligned}$$

$$\begin{aligned}
 10) \quad & n^2 + 7n + 15 > n + 3 \\
 & n^2 + 6n + 12 > 0 \\
 & (n+3)^2 - 9 + 12 > 0 \\
 & (n+3)^2 + 3 > 0
 \end{aligned}$$

$$\begin{aligned}
 & (n+3)^2 + 3 \text{ has a minimum value when } n = -3 \\
 & \text{Min value} = 3 \\
 & 3 > 0
 \end{aligned}$$

$$\therefore n^2 + 7n + 15 > n + 3 \text{ for all values of } n$$