Year 1 Pure   Chapter 13 INTEGRATION LIMITS   Exam Question ANSWERS

(1)

$$\int_1^4 2x + 3x^{1/2} \, dx$$

$$= \left[\frac{2x^2}{2} + \frac{3x^{3/2}}{3/2} \right]_1^4$$

$$= \left[x^2 + 2x^{3/2} \right]_1^4$$

$$= \left[x^2 + 2\sqrt{x}^3 \right]_1^4$$

$$= \left[4^2 + 2\sqrt{4}^3 \right] - \left[1^2 + 2\sqrt{1}^3 \right]$$

$$= \left[16 + 16 \right] - \left[1 + 2 \right]$$

$$= \left[32 \right] - \left[3 \right]$$

$$= 29$$

(2)

a) $f(x) = x^3 + 3x^2 + 5$

$f'(x) = 3x^2 + 6x$

$f''(x) = 6x + 6$

b) $$\int_1^2 x^3 + 3x^2 + 5 \, dx$$

$$= \left[\frac{x^4}{4} + \frac{3x^3}{3} + 5x \right]_1^2$$

$$= \left[\frac{x^4}{4} + x^3 + 5x \right]_1^2$$

$$= \left[4 + 8 + 10 \right] - \left[\tfrac{1}{4} + 1 + 5 \right]$$

$$= 22 - 6\tfrac{1}{4}$$

$$= 15\tfrac{3}{4}$$

(3)

$$\int_{-1}^{4} -x^2 + 3x + 4 \, dx$$

$$= \left[-\frac{x^3}{3} + \frac{3x^2}{2} + 4x \right]_{-1}^{4}$$

$$= \left[-\frac{64}{3} + 24 + 16 \right] - \left[\frac{1}{3} + \frac{3}{2} - 4 \right]$$

$$= \left[\frac{56}{3} \right] - \left[-\frac{13}{6} \right]$$

$$= 20\,{}^{5}\!/_{6}$$

4

$$y = x^3 - 6x^2 + 5x$$

$$\int x^3 - 6x^2 + 5x \quad dx$$

$$\text{Area} = \int_0^1 x^3 - 6x^2 + 5x \quad + \int_1^2 x^3 - 6x^2 + 5x$$

$$= \left[\frac{1}{4} - 2 + \frac{5}{2}\right] - \left[0\right] + \left[4 - 16 + 10\right] - \left[\frac{1}{4} - 2 + \frac{5}{2}\right]$$

$$= \frac{3}{4} + -2\tfrac{3}{4}$$

ignore the negative as just indicates the area is below the x axis.

$$= \frac{3}{4} + 2\tfrac{3}{4}$$

$$= 3\tfrac{1}{2}$$

5

a)

$$y = 6x - x^2$$

$$0 = 6x - x^2$$

$$0 = x(6-x)$$

$$\therefore x = 0 \text{ or } x = 6$$

b)

$$6x - x^2 = 2x$$

$$0 = x^2 - 4x$$

$$0 = x(x-4)$$

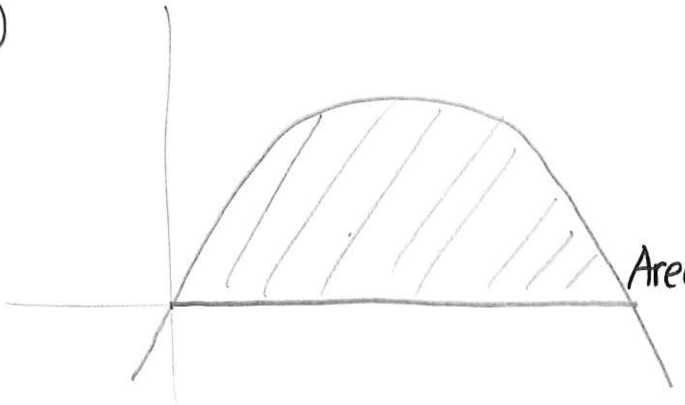
$$\therefore x = 0 \text{ or } x = 4$$

$y = 2x$ $\quad y = 0 \quad y = 8$

$(0,0) \quad (4,8)$

⑤

c)

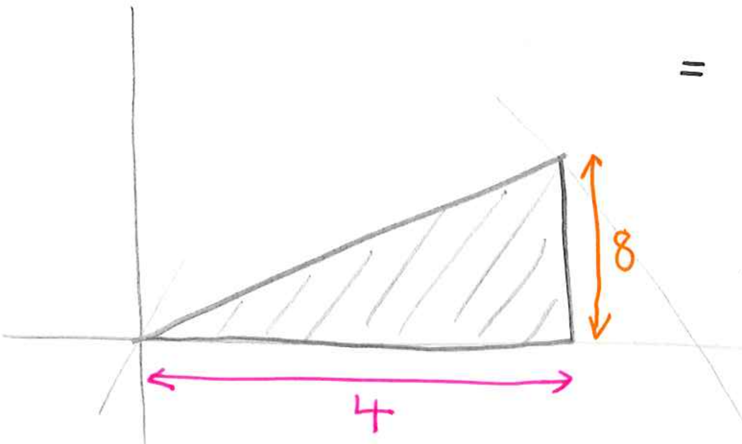


$$\int_0^4 6x - x^2$$

$$\text{Area} = \left[3x^2 - \frac{x^3}{3}\right]_0^4$$

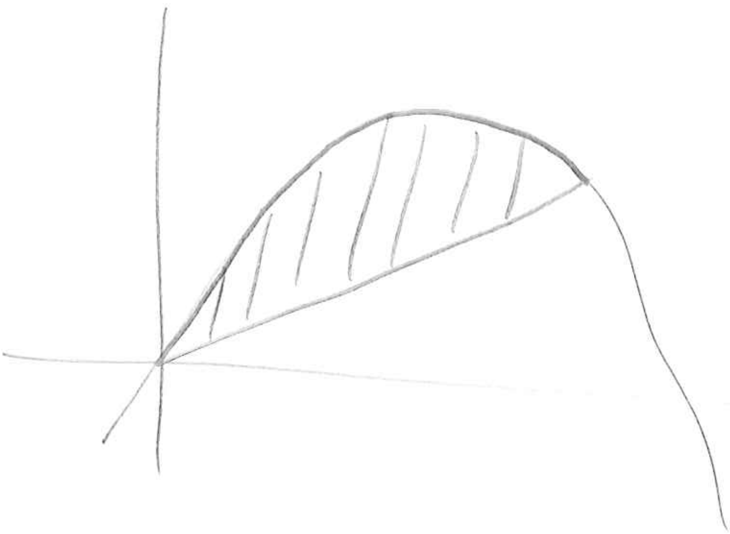
$$= \left[48 - \frac{64}{3}\right] - \left[0\right]$$

$$= 26\tfrac{2}{3}$$



$$\text{Area} = 4 \times 8 \times \tfrac{1}{2}$$

$$= 16$$



$$\text{Area} = 26\tfrac{2}{3} - 16$$

$$= \underline{\underline{10\tfrac{2}{3}}}$$

(6)

a) $$y = 10 + 8x - x^2 - x^3$$

$$\frac{dy}{dx} = 8 - 2x - 3x^2$$

$$\frac{dy}{dx} = (2-x)(4+3x)$$

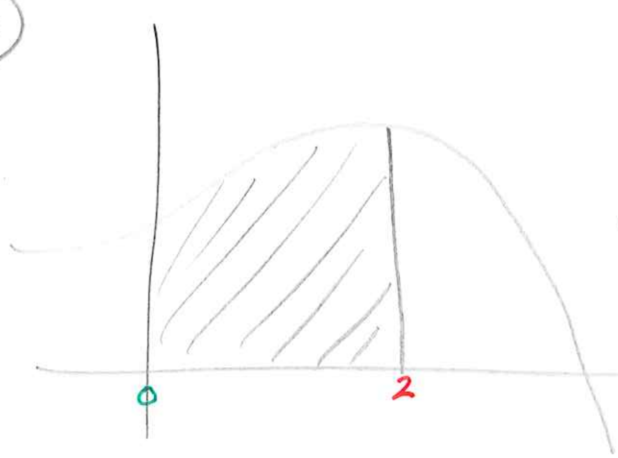
turning point $\therefore \frac{dy}{dx} = 0$

$$0 = (2-x)(4+3x)$$

$$\therefore \quad x = 2 \text{ or } x = \frac{-4}{3}$$

(6)

b)

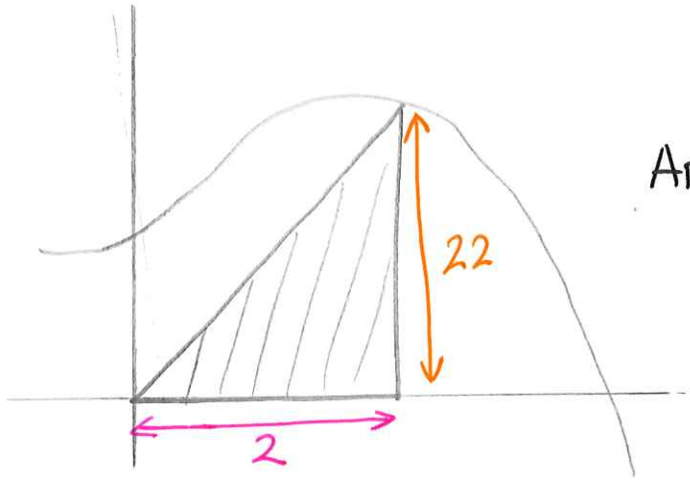


$$\int_0^2 10 + 8x + x^2 - x^3 \, dx$$

$$= \left[10x + 4x^2 + \frac{x^3}{3} - \frac{x^4}{4} \right]_0^2$$

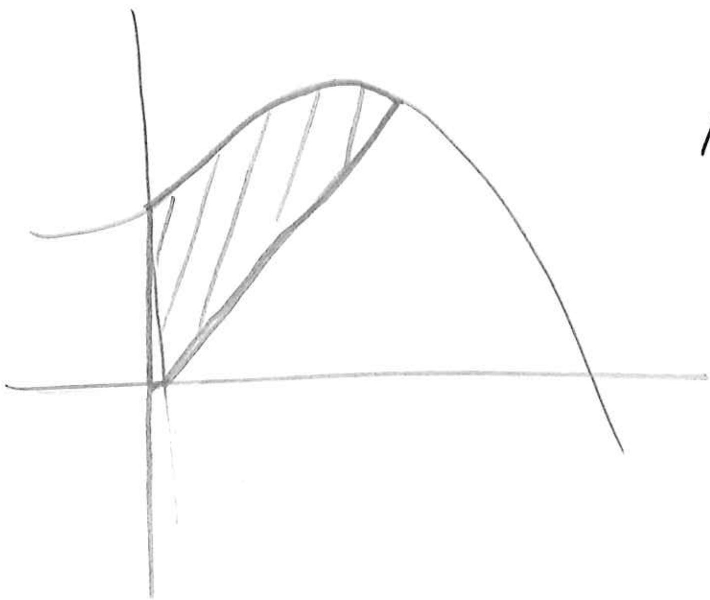
$$= \left[20 + 16 + \tfrac{8}{3} - 4 \right] - \left[0 \right]$$

$$= 34\tfrac{2}{3}$$



$$\text{Area} = 2 \times 22 \times \tfrac{1}{2}$$

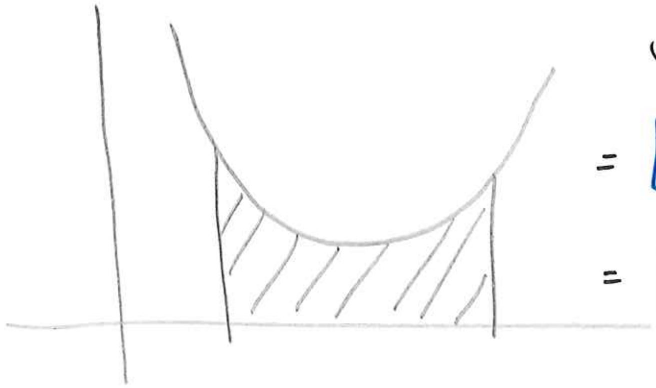
$$= 22$$



$$\text{Area} = 34\tfrac{2}{3} - 22$$

$$= \underline{\underline{12^{2}/3}}$$

(7)

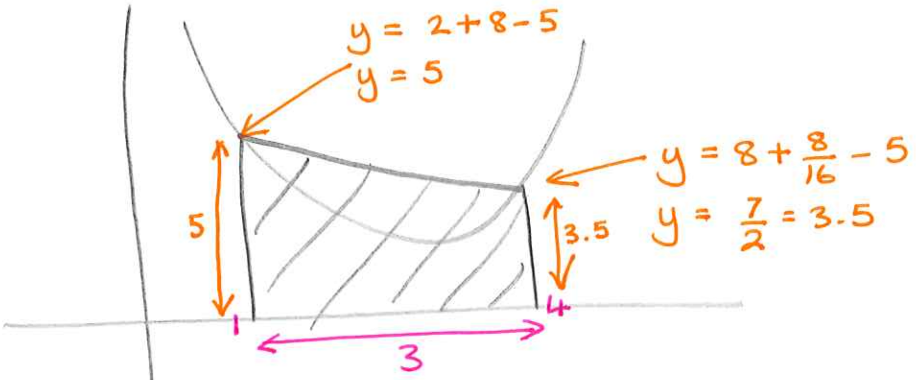


$$\int_1^4 2x + 8x^{-2} - 5$$

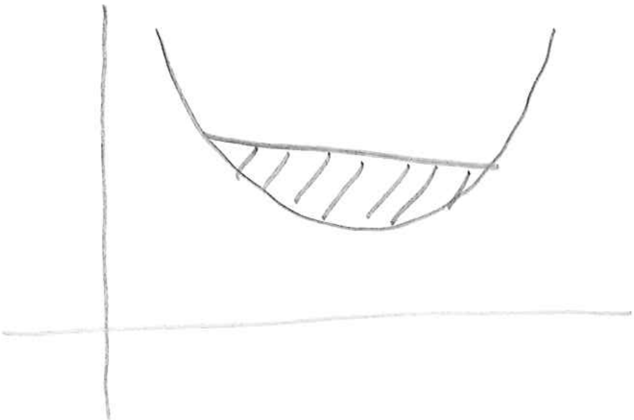
$$= \left[x^2 - \frac{8}{x} - 5x\right]_1^4$$

$$= [16-2-20] - [1-8-5]$$

$$= -6 - -12 = \underline{\underline{6}}$$



$$\frac{(5+3.5)}{2} \times 3 = 12.75$$



$$\text{Area} = 12.75 - 6$$

$$= \underline{\underline{6.75}}$$