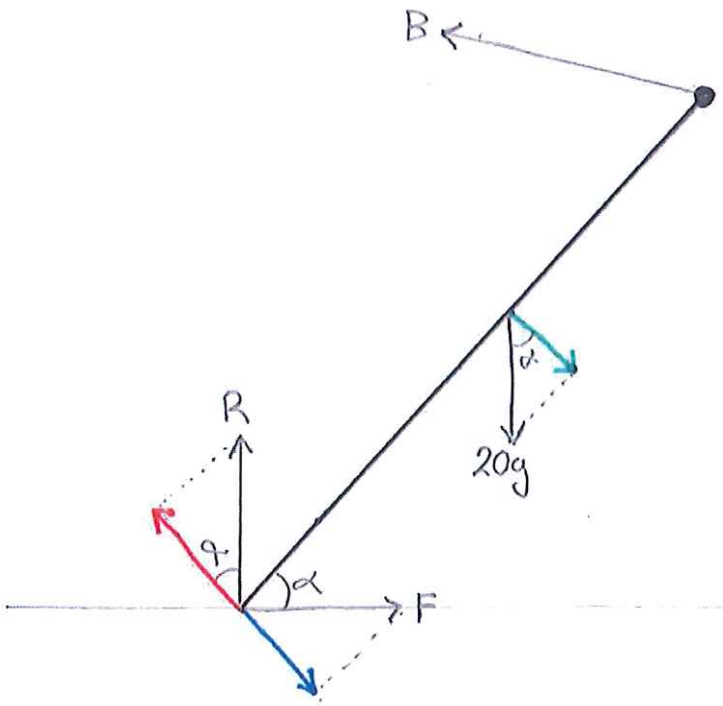
m2 static of rigid bodies

(1)



$\tan\alpha = \frac{3}{4}$

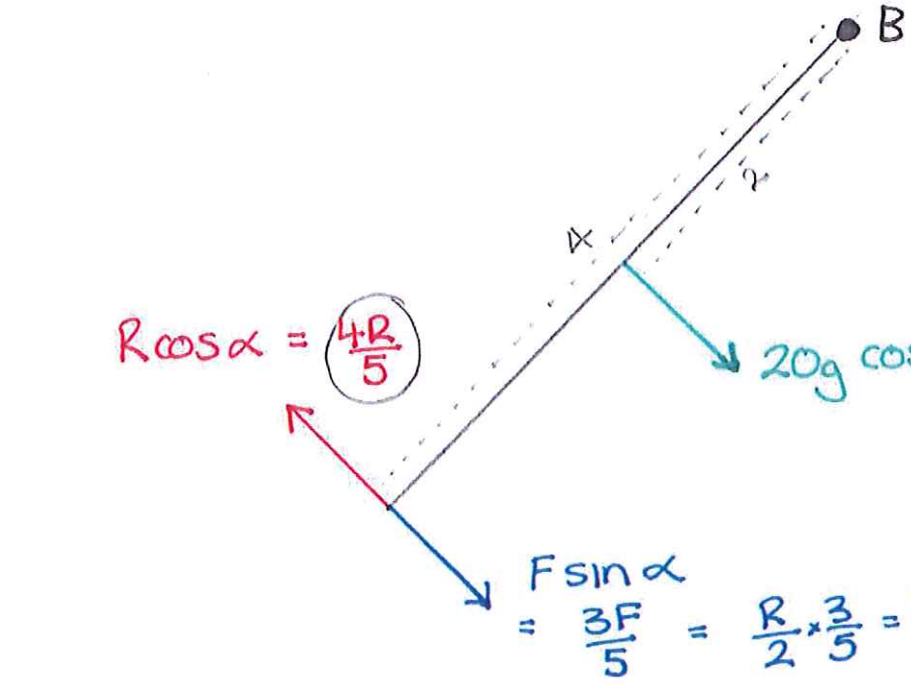
$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$\mu = \frac{1}{2}$

$F = \mu R$

$F = \frac{1}{2}R$



$R\cos\alpha = \frac{4R}{5}$

$20g\cos\alpha = 20g \times \frac{4}{5} = 16g$

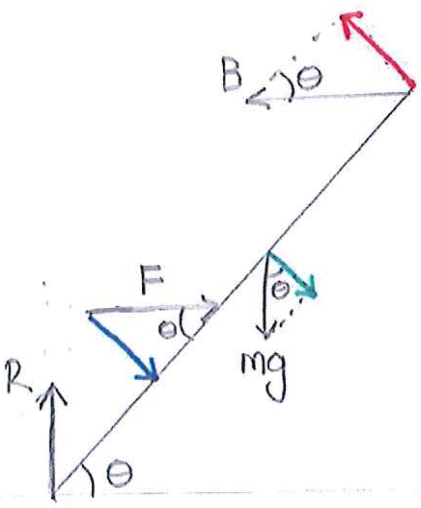
$F\sin\alpha = \frac{3F}{5} = \frac{R}{2} \times \frac{3}{5} = \frac{3R}{10}$

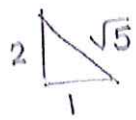
$M(B) \quad \frac{4R}{5} \times 4 = (16g \times 2) + (\frac{3R}{10} \times 4)$

$\frac{16R}{5} = 32g + \frac{12R}{10}$

$(\times 10) \quad 32R = 32g + 12R \quad \therefore R = 157\text{N}$

(2)





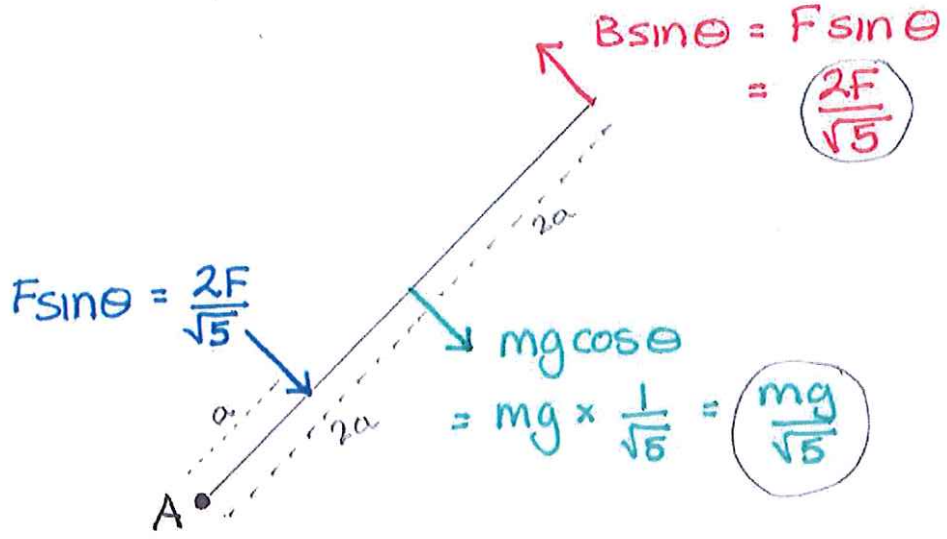
$\tan\theta = \frac{2}{1}$

$\sin\theta = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

(↑) $R = mg$

(→) $F = B$



$m(A)$ $\left(\frac{2F}{\sqrt{5}} \times a\right) + \left(\frac{mg}{\sqrt{5}} \times 2a\right) = \frac{2F}{\sqrt{5}} \times 4a$

$\frac{2Fa}{\sqrt{5}} + \frac{2mag}{\sqrt{5}} = \frac{8Fa}{\sqrt{5}}$

$2Fa + 2mga = 8Fa$

$2F + 2mg = 8F$

$2mg = 6F$

$F = \frac{2mg}{6} = \frac{mg}{3}$

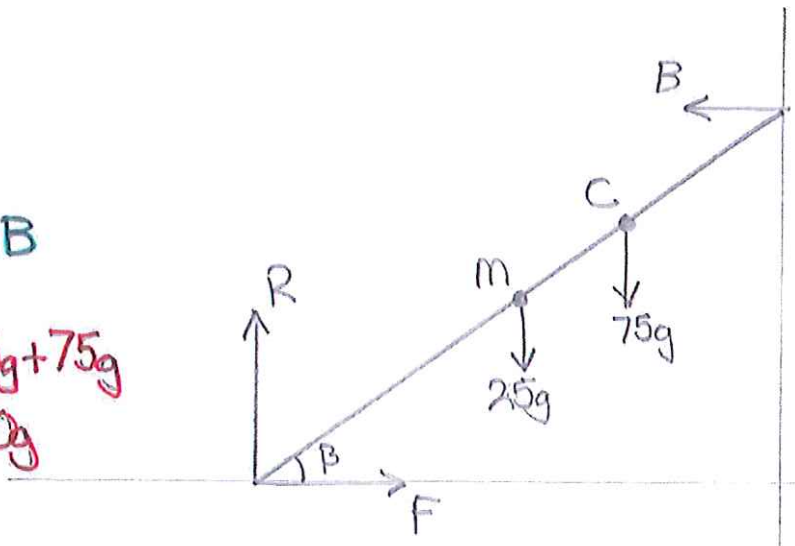
(3)

a)

(→) $F = B$

(↑) $R = 25g + 75g$

$R = 100g$



$\mu = \frac{11}{25}$

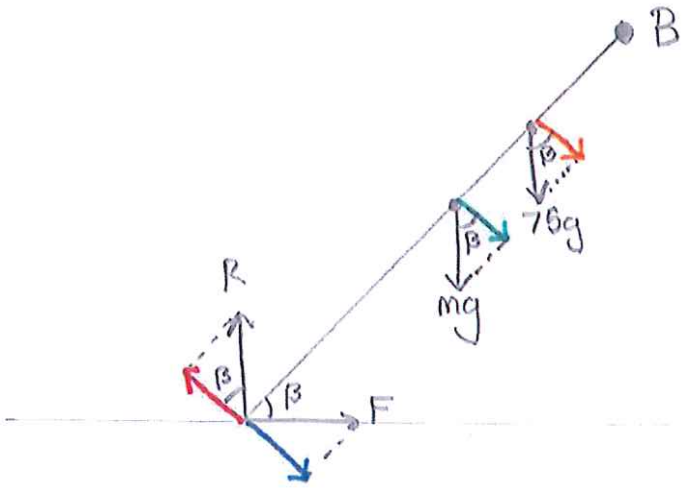
$F = \mu R$

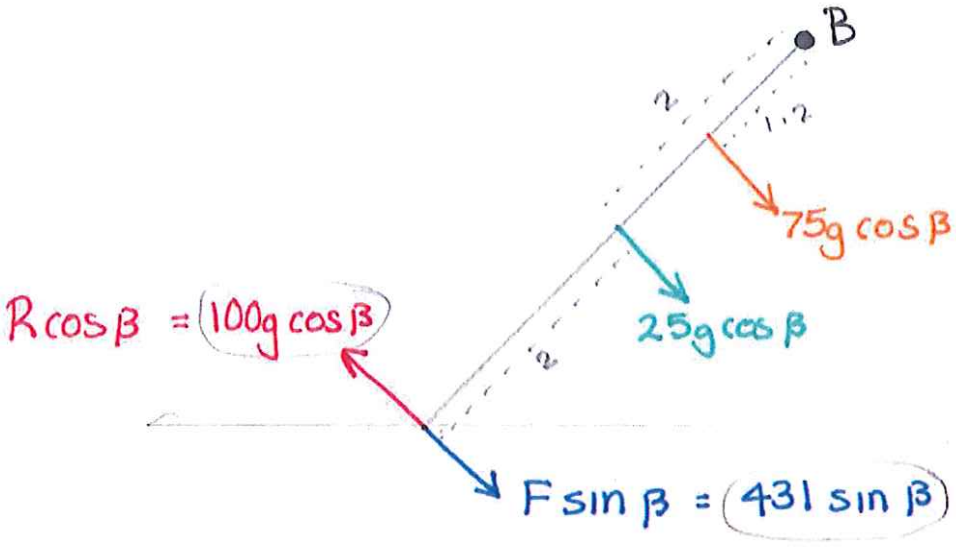
$F = \frac{11}{25} \times 100g$

$F = \underline{431N}$

(3)

b)





$$(100g\cos\beta \times 4) = (431\sin\beta \times 4) + (25g\cos\beta \times 2) + (75g\cos\beta \times 1.2)$$

$$400g\cos\beta = 1724\sin\beta + 50g\cos\beta + 140g\cos\beta$$

$$400g\cos\beta = 1724\sin\beta + 140g\cos\beta$$

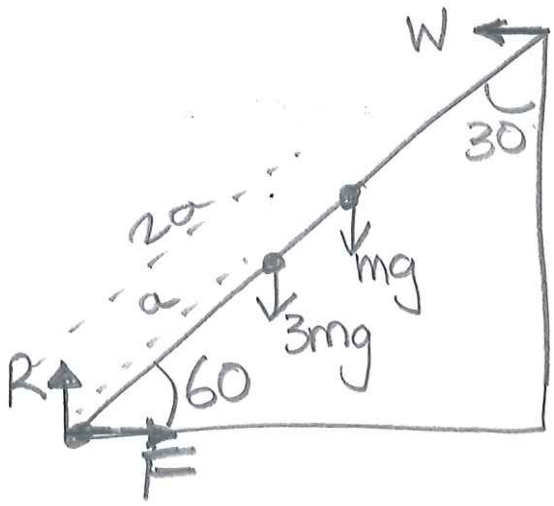
$$260g\cos\beta = 1724\sin\beta$$

$$\frac{260g}{1724} = \frac{\sin\beta}{\cos\beta}$$

$$\tan\beta = \frac{260g}{1724} \qquad \beta = 55.9°$$

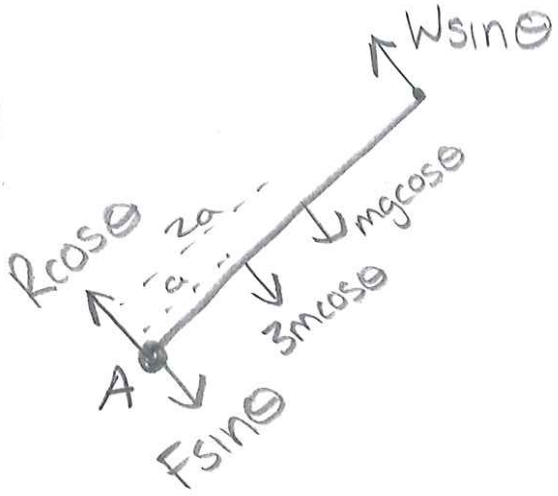
c) weight acts at a point

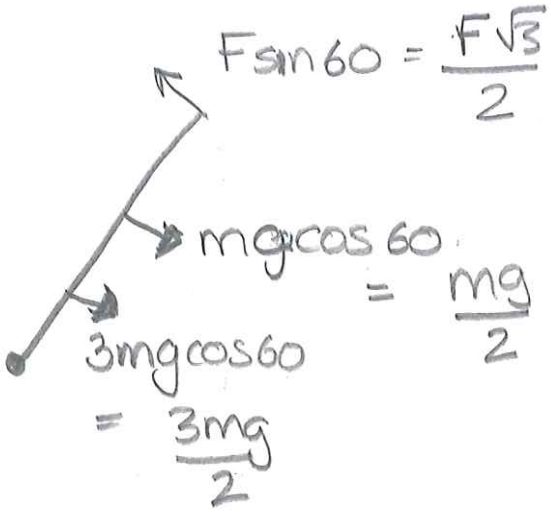
(4)



$(\updownarrow)\quad R = 4mg$

$(\leftrightarrow)\quad F = W$





$m(A)\quad 4a\,\frac{F\sqrt{3}}{2} = (2a)\frac{mg}{2} + \frac{3mg}{2}(a)$

$4a\,\frac{F\sqrt{3}}{2} = amg + \frac{3mga}{2}$

$\times \frac{2}{a}$

$4F\sqrt{3} = 2mg + 3mg$

$4F\sqrt{3} = 5mg$

$F = \frac{5mg}{4\sqrt{3}}$

$$\mu R = \frac{5mg}{4\sqrt{3}}$$

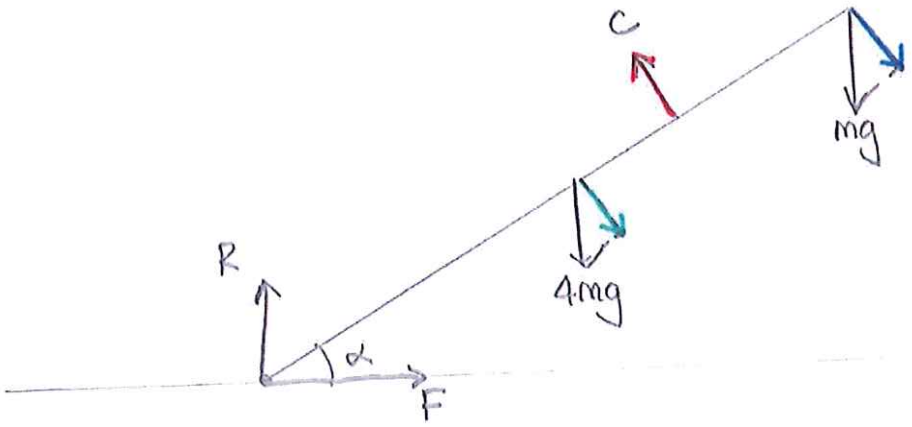
$$\mu(4mg) = \frac{5mg}{4\sqrt{3}}$$

$$\mu = \frac{5mg}{16\sqrt{3}\,mg}$$

$$\mu = \frac{5}{16\sqrt{3}}$$

(5)

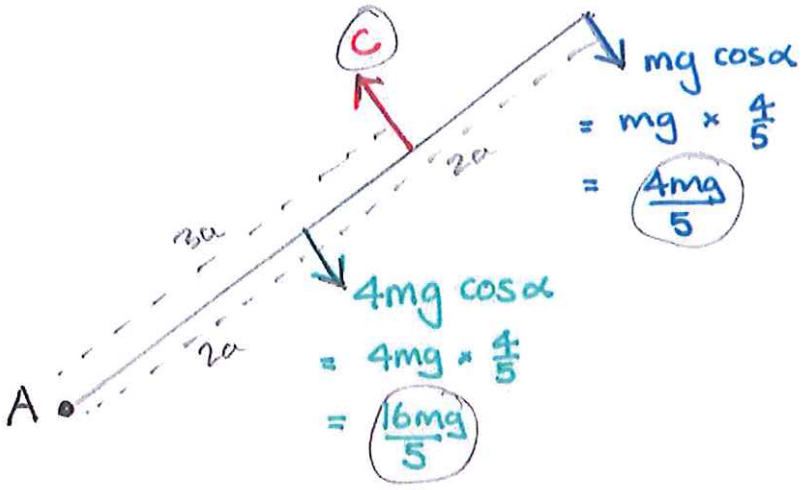
a)



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$



m(A) $\quad C \times 3a = \left(\frac{16mg}{5} \times 2a\right) + \left(\frac{4mg}{5} \times 4a\right)$

$$3aC = \frac{32mga}{5} + \frac{16mga}{5}$$

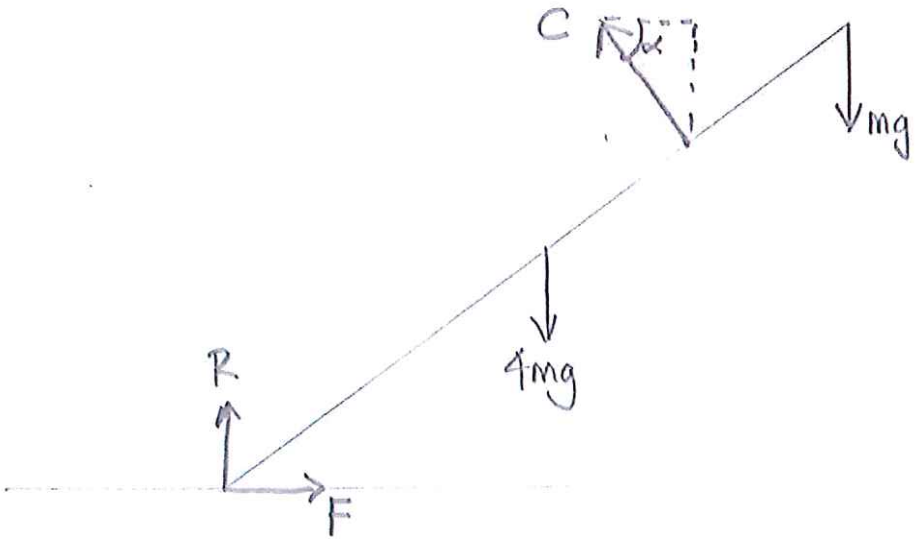
(×5)

$$15ac = 32amg + 16amg$$

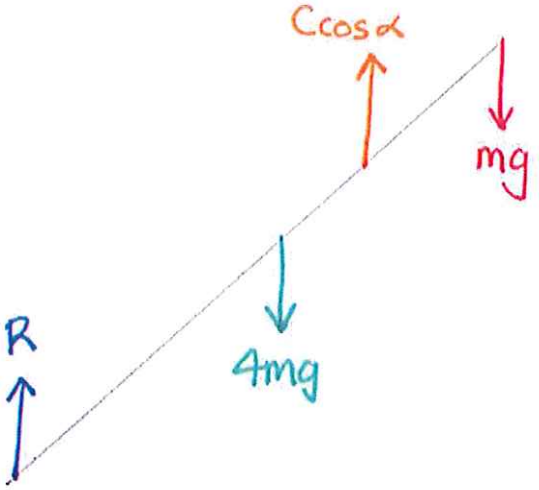
$$15c = 48mg$$

$$c = \frac{48mg}{15} = \frac{16mg}{5}$$

(5)
b)



(↑)



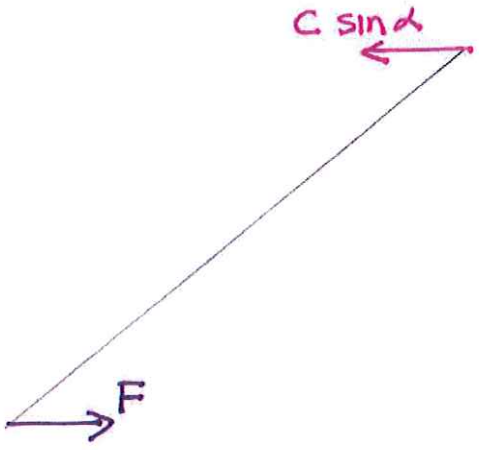
$$R + C\cos\alpha = 4mg + mg$$

$$R + \frac{4}{5} \times \frac{16mg}{5} = 5mg$$

$$R = \frac{64}{25}mg = 5mg$$

$$R = \frac{61}{25}mg$$

(→)



$$F = C\sin\alpha$$

$$F = \frac{16mg}{5} \times \frac{3}{5}$$

$$F = \frac{48mg}{25}$$

$$F_{max} = \mu R \quad \therefore \quad F \leq \mu R$$

$$\frac{48mg}{25} \leq \mu \frac{61}{25}mg$$

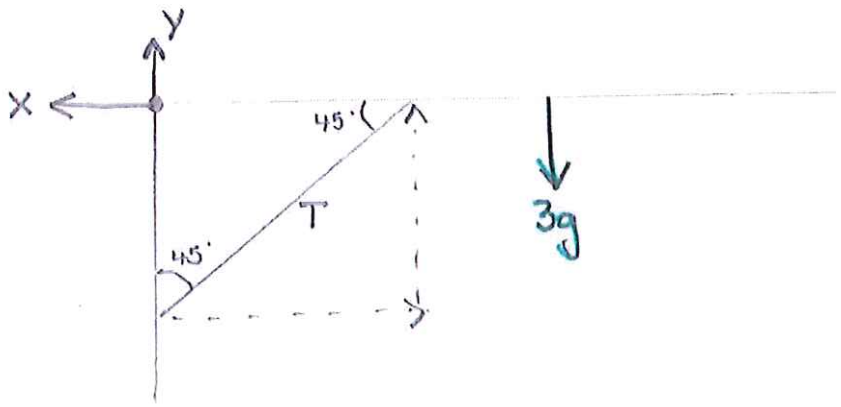
$$\mu \geq \frac{1200}{1525}$$

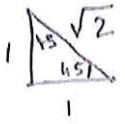
$$\mu \geq \frac{48}{61}$$

c)
No friction at the peg
or
C is perpendicular to plank

(6)

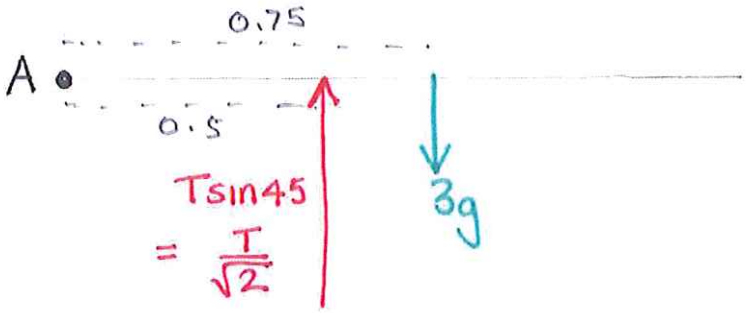
a)





$\sin 45 = \frac{1}{\sqrt{2}}$

$\cos 45 = \frac{1}{\sqrt{2}}$



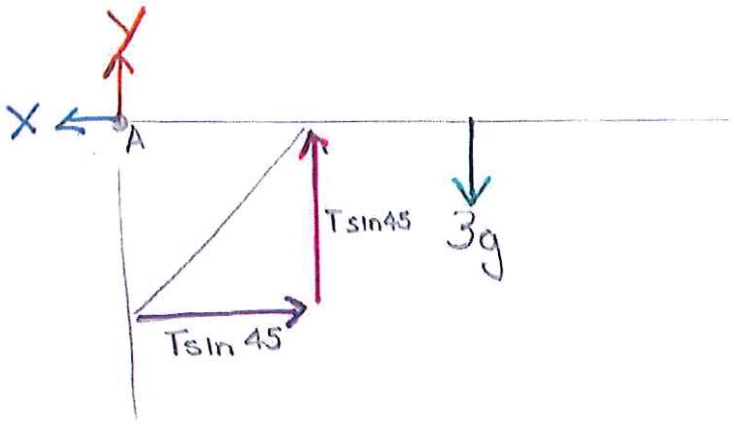
$m(A) \quad (3g \times 0.75) = \left(\frac{T}{\sqrt{2}} \times 0.5\right)$

$2.25g = \frac{T}{2\sqrt{2}}$

$T = \underline{62.4N}$

(6)

b)



$T = 62.4$

(↑) $Y + T\sin 45 = 3g$

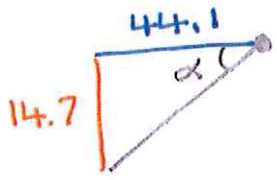
$Y = -14.7$

↑ direction: Y is working down

(→) $X = T\sin 45$

$X = 44.1$

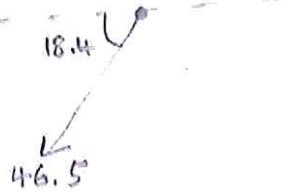
Therefore magnitude at A



$\sqrt{14.7^2 + 44.1^2} = \underline{46.5}$

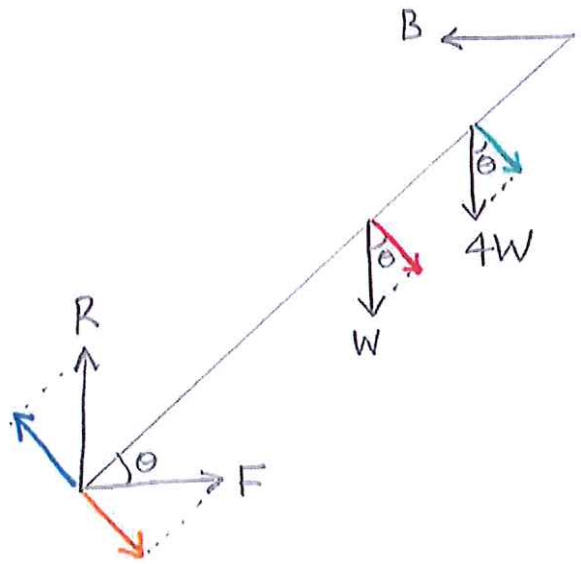
Angle

$\alpha = \tan^{-1}\left(\frac{14.7}{44.1}\right) = 18.4^\circ$



7

a)



$\tan\theta = \frac{2}{1}$

$\sin\theta = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

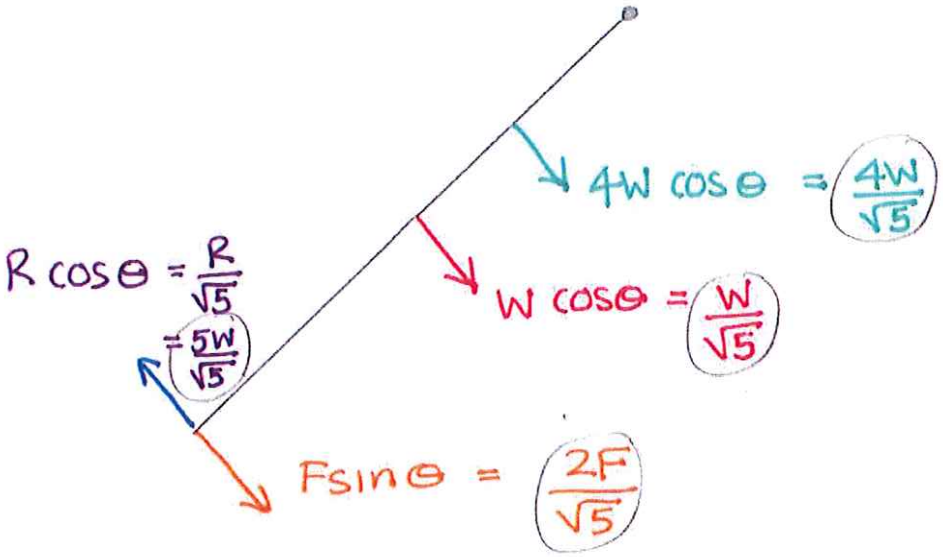
$(\uparrow)\quad R = 4W + W$

$R = 5W$

$(\rightarrow)\quad F = B$

Need to find F

take moments about B



$$\left(\frac{2F}{\sqrt{5}} \times 4a\right) + \left(\frac{W}{\sqrt{5}} \times 2a\right) + \left(\frac{4W}{\sqrt{5}} \times a\right) = \frac{5W}{\sqrt{5}} \times 4a$$

$$\frac{8aF}{\sqrt{5}} + \frac{2aW}{\sqrt{5}} + \frac{4aW}{\sqrt{5}} = \frac{20aW}{\sqrt{5}}$$

$\times \left(\frac{\sqrt{5}}{a}\right)$

$$8F + 2W + 4W = 20W$$

$$8F = 14W$$

$$F = 1.75W$$

(7)

a) continued

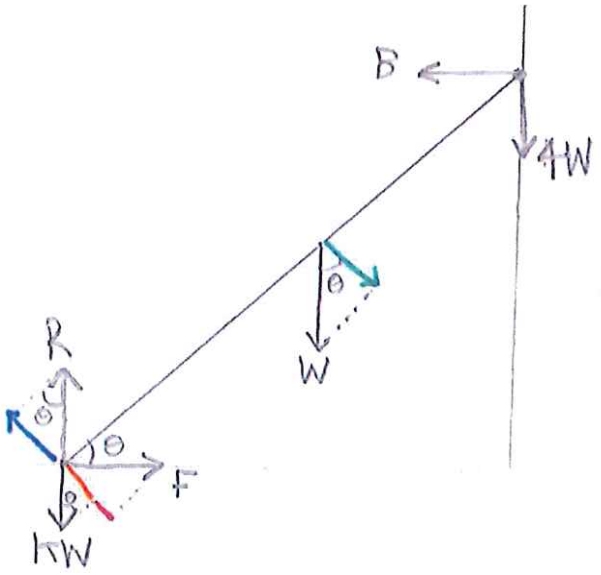
$$F = \mu R$$

$$1.75W = \mu \, 5W$$

$$\mu = \underline{0.35}$$

(7)

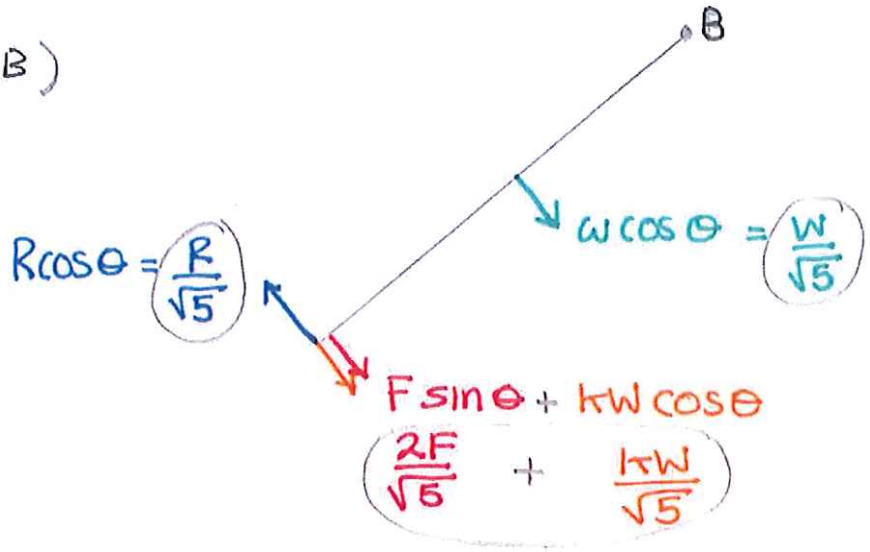
b)



(↑) $R = kW + W + 4W$

$R = kW + 5W$

m(B)



$$\left(\frac{W}{\sqrt{5}} \times 2a\right) + \left(\frac{2F}{\sqrt{5}} + \frac{kW}{\sqrt{5}}\right)4a = \frac{R}{\sqrt{5}} \times 4a$$

$$\frac{2aW}{\sqrt{5}} + \frac{8aF}{\sqrt{5}} + \frac{4akW}{\sqrt{5}} = \frac{4aR}{\sqrt{5}}$$

$$2W + 8F + 4kW = 4R$$

$$F = \mu R$$

$$F = 0.35(kW + 5W)$$

$$F = 0.35kW + 1.75W$$

Now sub into $2W + 8F + 4kW = 4R$

(7)

b) cont.

$$2W + 8F + 4kW = 4R$$

$$2W + 8(0.35kW + 1.75W) + 4kW = 4R$$

$$2W + 2.8kW + 14W + 4kW = 4(kW + 5W)$$

$$2W + 2.8kW + 14W + 4kW = 4kW + 20W$$

$$2.8kW = 4W$$

$$k = \frac{4}{2.8} = \frac{10}{7}$$

$$k \geq \frac{10}{7}$$