Dreams Pack

Maths Crib Sheets

Pure Year 1

THE STUFF
THAT DREAMS
ARE
MADE OF

Year 1 Pure

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n[2a+(n-1)d]$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1 \times 2 \times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Useful fomrulae and things to know!

Discrimate $b^2 - 4ac$

$b^2-4ac = 0$ 1 root

$b^2-4ac > 0$ 2 roots

$b^2-4ac < 0$ no roots

Equation Straight Line

(x_1, y_1)

$y - y_1 = m(x - x_1)$

Perpendicular $m \times -\frac{1}{m} = -1$

TRANSFORMATIONS

$f(x)+a$ ↑ $f(x+a)$ ← $f(-x)$ y axis reflection $f(ax)$ $(\frac{x}{a}, y)$

$f(x)-a$ ↓ $f(x-a)$ → $-f(x)$ x axis reflection $af(x)$ (x, ay)

CIRCLES

$(x-a)^2 + (y-b)^2 = r^2$

centre (a,b)

TRIGONOMETRY

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc\cos A$

Area $= \frac{1}{2}ab\sin C$

BINOMIAL EXPANSION

$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots\ldots$

TRIGONOMETRY IDENTITIES

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

VECTORS

$2i + 3j$ gradient $= \frac{3}{2}$

$v = u + at$

$F = ma$

DIFFERENTIATION

multiply by original power

subtract one from the power

$f(x) = a^n$

$f'(x) = na^{n-1}$

INTEGRATION

add one to the power

divide by the new power

$f'(x) = a^n$

$f(x) = \frac{a^{n+1}}{n+1} + c$

LOG LAWS

$\log A + \log B = \log AB$

$\log A - \log B = \log \frac{A}{B}$

$A\log B = \log B^A$

Year 1 Pure Chapter 1 – Algebraic Expression

Index Laws

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m+n}$

$(a^m)^n = a^{mn}$

Fractional Powers

$a^{\frac{1}{m}} = \sqrt[m]{a}$

$a^{\frac{n}{m}} = \sqrt[m]{a}^{\,n}$

Negative Powers

$a^{-m} = \frac{1}{a^m}$

Watch out for:

$$\frac{1}{3x^2} = \frac{1}{3}\frac{1}{x^2} = \frac{1}{3}x^{-2}$$

Example

$$\left(\frac{8}{27}\right)^{-\frac{2}{3}} = \left(\frac{27}{8}\right)^{\frac{2}{3}}$$

$$= \frac{\sqrt[3]{27}^{\,-2}}{\sqrt[3]{8}^{\,-2}}$$

$$= \frac{9}{4}$$

Surds

Simplify

$\sqrt{5}\sqrt{3} = \sqrt{15}$

Simplify

$\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

Simplify

$\frac{\sqrt{15}}{\sqrt{3}} = \sqrt{5}$

Simplify

$\frac{\sqrt{12}}{\sqrt{8}} = \frac{2\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}}$

Expand and Simplify

$(3+\sqrt{2})(2-\sqrt{2})$

$= 6 - 3\sqrt{2} + 2\sqrt{2} - \sqrt{4}$

$= 6 - \sqrt{2} - 2$

$= 4 - \sqrt{2}$

Rationalise the Denominator

$\frac{2}{\sqrt{3}} =$

$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\frac{2}{2+\sqrt{3}} =$

$\frac{2}{2-\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} =$

$\frac{4-2\sqrt{3}}{2-4\sqrt{3}+3} =$

$\frac{4-2\sqrt{3}}{5-4\sqrt{3}}$

$\frac{2}{1-2\sqrt{3}} =$

$\frac{2}{1-2\sqrt{3}} \times \frac{1+2\sqrt{3}}{1+2\sqrt{3}} =$

$\frac{2+4\sqrt{3}}{1-4\sqrt{9}} =$

$\frac{2+4\sqrt{3}}{-11}$

Year 1 Pure Chapter 2 - Quadratics

Solving Quadratics By Factoring

$6x^2 + 13x - 5 = 0$

$ac = -30$

$b = 13$

15 and -2

$6x^2 + 15x - 2x - 5 = 0$

$3x(2x+5) - 1(2x+5) = 0$

$(3x-1)(2x+5) = 0$

$x = \frac{1}{3}$ or $x = -\frac{5}{2}$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$6x^2 + 13x - 5 = 0$

$a = 6$ $b = 13$ $c = -5$

$x = \frac{1}{3}$ or $x = -\frac{5}{2}$

Completing the Square

$x^2 + bx = (x + \frac{b}{2})^2 - (\frac{b}{2})^2$

$x^2 + 4x$

$= (x+2)^2 - 4$

$x^2 + 6x + 3$

$= (x+3)^2 - 9 + 3$

$= (x+2)^2 - 6$

$2x^2 + 8x$

$= 2(x^2 + 4)$

$= 2[(x^2+2) - 4]$

$= 2(x^2+2) - 8$

$2x^2 + 8x + 3$

$= 2(x^2+4) + 3$

$= 2[(x^2+2) - 4] + 3$

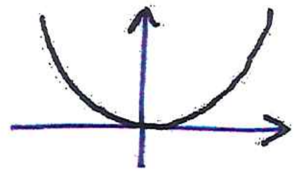
$= [2(x^2+2) - 8] + 3$

$= 2(x^2+2) - 16 + 3$

$= 2(x^2+2) - 13$

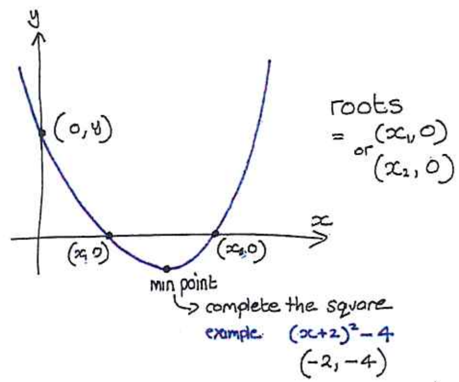
Quadratic Graphs

Positive Gradient



Negative Gradient

Roots and Turning Point



$y = (x+a)^2 + b$

Turning point

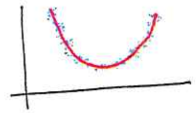
$(-a, b)$

Discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant $= b^2 - 4ac$

No roots

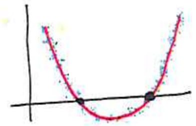


$b^2 - 4ac < 0$

One root

$b^2 - 4ac = 0$

Two roots



$b^2 - 4ac > 0$

Example

$x^2 + 4x + k = 0$

has two roots.

$a = 1, b = 4, c = k$

$b^2 - 4ac > 0$

$4^2 - 4(1)(k) > 0$

$16 - 4k > 0$

$k < 4$

Year 1 Pure Chapter 3 - Equations + Inequalities

Quadratic Simultaneous Equations

$$x + 2y = 3$$
$$x^2 + 3xy = 10$$

$$(3 - 2y)^2 + 3(3 - 2y)y = 10$$
$$9 - 12y + 4y^2 + 9y - 6y^2 = 10$$
$$2y^2 + 3y + 1 = 0$$

$$y = -\frac{1}{2} \text{ or } y = -\frac{1}{2}$$

Don't forget to find the x values.

$$x = 4 \text{ or } x = 5$$

Linear Inequalities

$$-2x - 3 < 5$$
$$-2x < 8$$
$$x > -4$$

If you multiply both sides of the equation by -1, then you need to change the direction of the inequality.

Shading Regions

$>$ and $\geq$

Shade above the line

$<$ and $\leq$

Shade below the line

$\leq$ and $\geq$

Solid line

$<$ and $>$

Dashed line

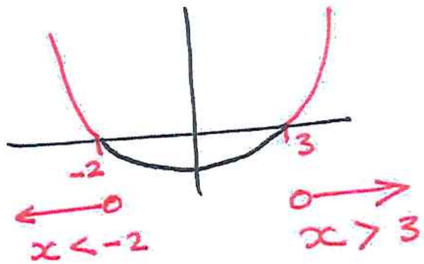
Quadratic Inequalities

Quadratic inequality > 0

Part of the graph that is above the x axis.

$$x^2 + 5x + 6 > 0$$

$$(x + 2)(x + 3) > 0$$

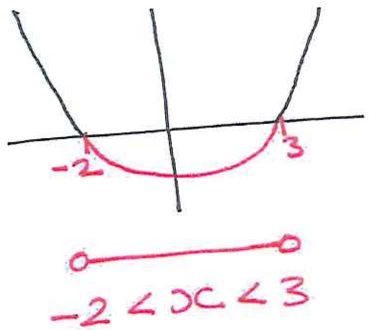


Quadratic inequality < 0

Part of the graph that is below the x axis.

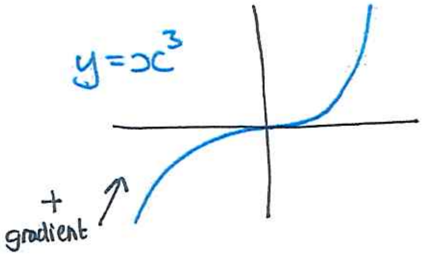
$$x^2 + 5x + 6 < 0$$

$$(x + 2)(x + 3) < 0$$

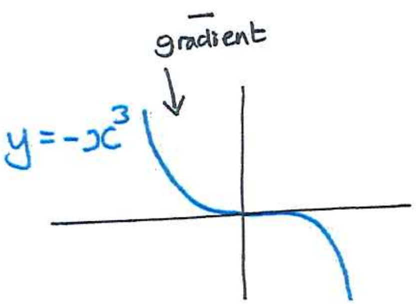


Cubic Graphs

Positve Graident



Negative Graident



y Intercept

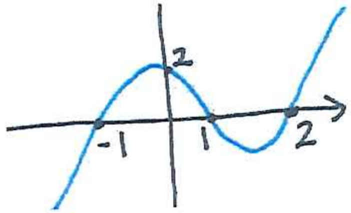
$y = ax^2 + bx + c$

y Intercept = constant c

$y = (x+3)^2(x-1)$
$y = (x+3)(x+3)(x-1)$
Constant = 3 x 3 x −1 = −9
y Intercept = $(0, -9)$

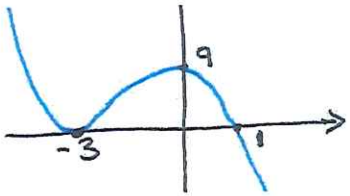
3 Roots

$y = (x-2)(x+1)(x-1)$
Roots **2, −1, 1**



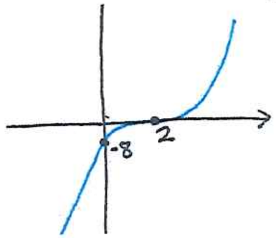
2 Roots

$y = (x+3)^2(1-x)$
$y = (x+3)(x+3)(1-x)$
Roots **−3, −3, 1**
Double root = bounce



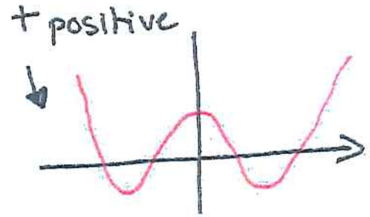
1 Root

$y = (x-2)^3$
$y = (x-2)(x-2)(x-2)$
Roots **2, 2, 2**
Treble root = inflection point

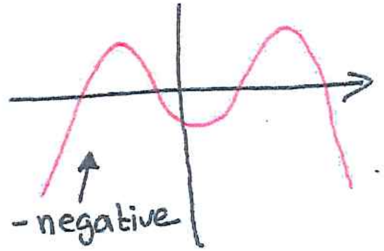


Quartic Graphs

Positve Graident



Negative Graident



y Intercept

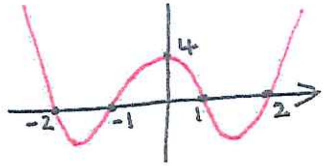
$y = ax^3 + bx^2 + cx + d$

y Intercept = constant d

$y = (x+3)^2(x-1)\ (x-2)$
Constant = 3 x 3 x −1 x −2 = 18
y Intercept = $(0, 18)$

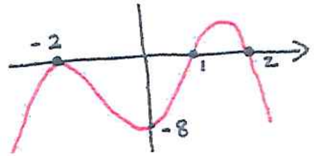
4 Roots

$y = (x-2)(1-x)(x+1)(x+2)$
Roots 2, 1, −1, −2



3 Roots

$y = (x+3)^2(1-x)(x-2)$
Roots −2, −2, 1, 2
Double root = bounce

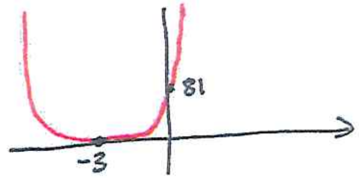


2 Root

$y = (x-2)^2(x+3)^2$
Roots 2, 2, −3, −3

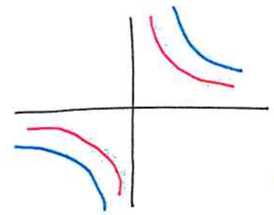
1 Root

$y = (x+3)^4$
Roots −3, −3, −3, −3
Quadruple root = inflection point



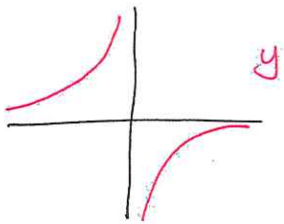
Year 1 Pure Chapter 4 – Graphs + Transformations

Reciprocal Graphs

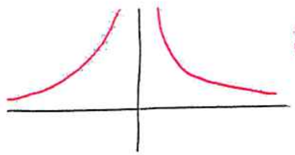


$y = \frac{1}{x}$

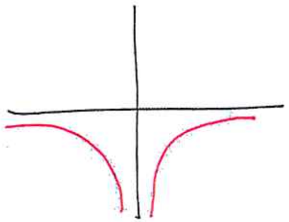
$y = \frac{4}{x}$



$y = -\frac{1}{x}$



$y = \frac{1}{x^2}$



$y = \frac{-1}{x^2}$

Points of Intersection

$g(x)$ intersects $f(x)$
when
$g(x) = f(x)$

Example

$$g(x) = 2x^2$$
$$f(x) = 4x$$

$$2x^2 = 4x$$

$$2x^2 - 4x = 0$$

$$2x(x - 2) = 0$$

$$x = 0 \ or \ x = 2$$

Don't forget to find the y value

$$f(x) = 4(0) = 0$$
$$f(x) = 4(2) = 8$$

Points of intersection
$(0, 0)$ and $(2, 8)$

Transformations

$y = f(x)$ starting position

Shift up
$y = f(x) + 3 \quad (x, y + 3)$
Shift down
$y = f(x) - 3 \quad (x, y - 3)$

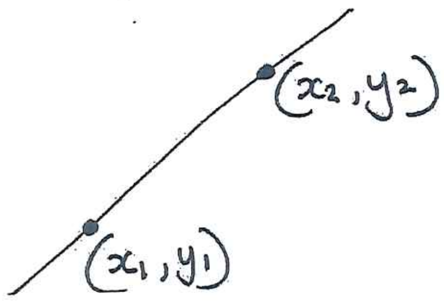
Shift left
$y = f(x + 2) \quad (x - 2, y)$
Shift right
$y = f(x - 2) \quad (x + 2, y)$

Horizontial Stretch
$y = f(4x) \quad (\frac{x}{4}, y)$
Vertical Stretch
$y = 4f(x) \quad (x, 4y)$

Reflection in the x axis
$y = f(-x) \quad (x, -y)$
Reflection in the y axis
$y = -f(x) \quad (-x, y)$

Year 1 Pure Chapter 5 - Straight Line Graphs

Straight Line Equation



$$y = mx + c$$

or

$$ax + by + c = 0$$

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = m(x - x_1)$$

Parallel:

same gradients

$$y = 3x - 2$$

$$y = 3x + 4$$

Perpendicular:

negative reciprocal

$$y = -\frac{2}{3}x - 2$$

$$y = \frac{3}{2}x + 4$$

Length

$$\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$$

If asked for the exact value, give your answer in surd form if necessary.

Example

Find the equation and length.

$(2, 3)$

$(-3, 5)$

$$\text{Gradient} = \frac{5 - 3}{-3 - 2}$$

$$= \frac{2}{-5}$$

$$y - 3 = \frac{2}{-5}(x - 2)$$

$$-5y + 15 = 2x - 4$$

$$-5y = 2x - 19$$

$$y = -\frac{2}{5}x + \frac{19}{5}$$

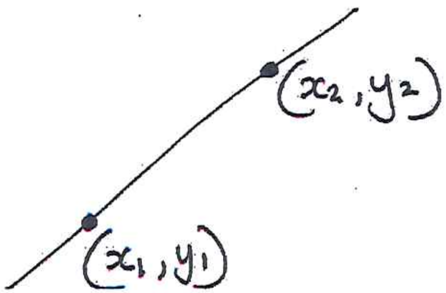
$$\sqrt{(-3 - 2)^2 - (5 - 3)^2}$$

$$= \sqrt{(-5)^2 - (2)^2}$$

$$= \sqrt{21}$$

Year 1 Pure Chapter 6 - Circles

Mid Points



$$\left(\left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)\right)$$

Equation of a Circle

$$(x - a)^2 + (y - b)^2 = r^2$$

Centre $= (a, b)$

Radius $= r$

Example

Find the centre and radius.

$$x^2 + y^2 + 6x + 8y = 0$$

Rearrange

$$x^2 + 6x + y^2 + 8y = 0$$

Complete the Square

$$(x + 3)^2 - 9 + (y + 4)^2 - 16 = 0$$

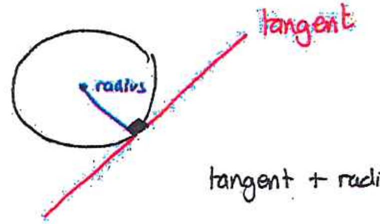
Equation Form

$$(x + 3)^2 + (y + 4)^2 = 25$$

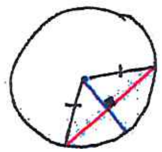
Centre $(\mathbf{-3, -4})$

Radius $= \sqrt{25} = \mathbf{5}$

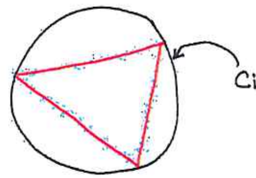
Circle Properties



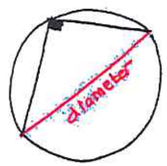
tangent + radius mee



radius/diameter bis



circumcircle



if diameter then meets circumference at 90°

Year 1 Pure Chapter 7 – Algebraic Methods

Algebraic Fractions

Golden rule:

factorise as soon as you can.

Long Division

$3x^2 - 2x + 4 \div x - 1$

$$
\begin{array}{r|l}
 & 3x^2 + 3x + 1 \\
\hline
x - 1 & 3x^3 + 0x^2 - 2x + 4 \\
 & \underline{3x^3 - 3x^2} \\
 & \quad 3x^2 - 2x \\
 & \quad \underline{3x^2 - 3x} \\
 & \qquad x + 4 \\
 & \qquad \underline{x - 1} \\
 & \qquad\quad 5
\end{array}
$$

The remainder is 5.

Factor Theorem

Is $x - 2$ a factor of $x^3 + x^2 - 4x - 4$?

$(2)^3 + (2)^2 - 4(2) - 4$

$= 0$

Remainder is 0

$\therefore x - 2$ is a factor.

Is $x + 3$ a factor $x^3 + x^2 - 4x - 4$?

$(-3)^3 + (-3)^2 - 4(-3) - 4$

$= -10$

Remainder is -10

$\therefore x + 3$ is a not a factor

Proofs by Deduction

Tips

- Show LHS ≡ RHS
- Complete square to show a max or min value
- Use properties of shapes

Proofs by Exhaustion

Tips

Break the statement down into smaller case and prove each smaller case.

Maybe break it down into odd and even.

Proofs by Counter-Example

Tips

You only need one counter-example to show that a statement is false.

Year 1 Pure Chapter 8 - Binomial Expansion

Binomial Expansion

$$(a+b)^0 = 1$$
$$(a+b)^1 = a + b$$
$$(a+b)^2 = a^2 + 2a^1b^1 + b^2$$
$$(a+b)^3 = a^3 + 3a^2b^1 + 3a^1b^2 + b^3$$
$$(a+b)^4 = a^4 + 4a^3b^1 + 6a^2b^2 + 4a^1b^3 + b^4$$
$$\dots\dots$$
$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots$$

where $\binom{n}{r} = {}^nC_r = \dfrac{n!}{r!(n-r)!}$

Identifying Values

$(1+2x)^{10}$ $a=1$ $b=2x$ $n=10$

$\left(10-\frac{1}{2}x\right)^6$ $a=10$ $b=-\frac{1}{2}x$ $n=6$

$\left(x-\frac{1}{x}\right)^5$ $a=x$ $b=-\frac{1}{x}$ $n=5$

Comparing Coefficients

$$1 + \underbrace{8q}_{\text{coefficient of } x}x + \underbrace{28q^2}_{\text{coefficient of } x^2}x^2$$

x^2 coefficient is double the coefficient of x

$$2(8q) = 28q^2$$
$$16q = 28q^2$$
$$16 = 28q$$
$$q = \frac{16}{28}$$

Binomial Expansion Estimation

Estimate 0.975^{10} to 4dp

Given

$$\left(1-\frac{x}{4}\right)^{10} = 1 - 2.5x + 2.8125x^2 - 1.875x^3$$

$$0.975 = 1 - \frac{x}{4}$$

$$x = 0.1$$

Now sub $x=0.1$ into the binomial expasion.

Year 1 Pure Chapter 9 - Trigonometric Ratios

Advanced Trigonometry

SIDE $a^2 = b^2 + c^2 - 2bc\cos A$

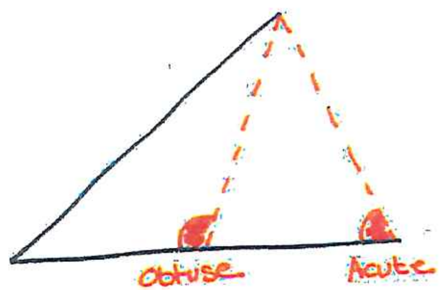
ANGLE $\cos A = \dfrac{a^2 - b^2 - c^2}{-2bc}$

SIDE $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

ANGLE $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

AREA OF TRIANGLE $= \frac{1}{2}ab\sin C$

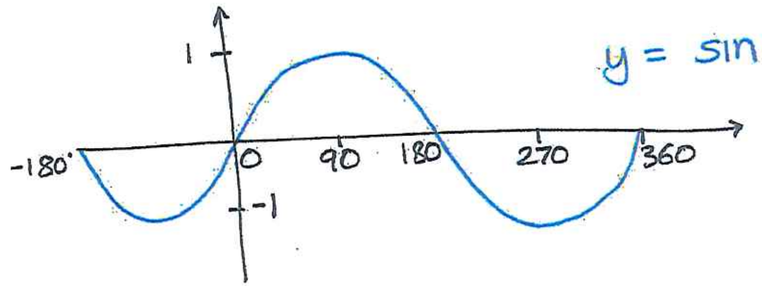
Obtuse and Acute Answers



obtuse = 180 - acute

Acute = 180 - obtuse

Transformation of Trigonometry Graphs



$y = \sin x$

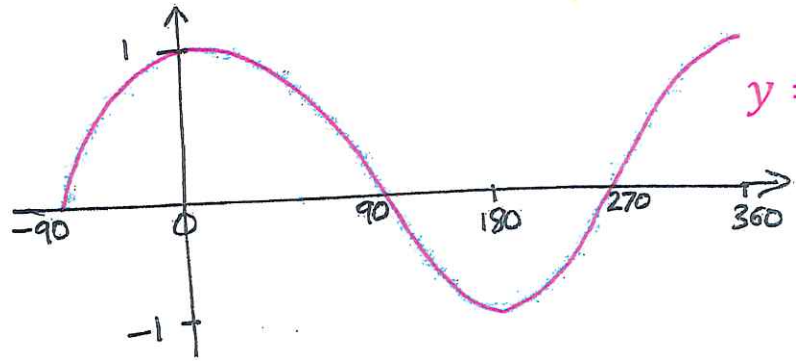
$y = 3 + \frac{1}{3}\sin(2\theta - 45°)$

up 3

max/min

number of cycles in 360°

right 45°



$y = \cos x$

$y = -2 + 2\cos(3\theta + 90°)$

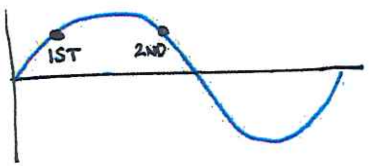
down 2

max/min

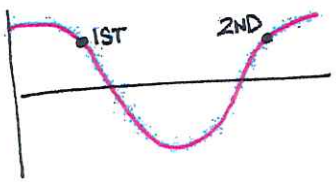
3 cycles in 360°

left 90°

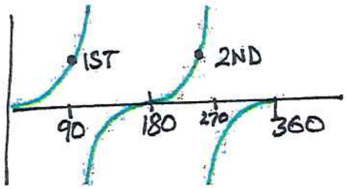
Finding Solutions



1ST SOLUTION = $\sin^{-1} x$
2ND SOLUTION = 180 – 1st solution
ALL OTHERS ± 360°



1ST SOLUTION = $\cos^{-1}$
2ND SOLUTION = 360 – 1ST solution
ALL OTHERS ± 360°



1ST SOLUTION = $\tan^{-1} x$
ALL OTHERS ±180°

Range of Answers

Watch out for the range of answers needed.

$\sin\theta = \frac{1}{2}$ $\quad 0 \le \theta \le 360$

$\sin 2\theta = \frac{1}{2}$ $\quad 0 \le \theta \le 360$

becomes

$0 \le 2\theta \le 720$

$\sin(2\theta + 45) = \frac{1}{2}$ $\quad 0 \le \theta \le 360$

becomes

$45 \le 2\theta + 45 \le 765$

Trig Identities

$$\sin^2\theta + \cos^2\theta = 1$$

Therefore $\quad \sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Therefore $\quad \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$

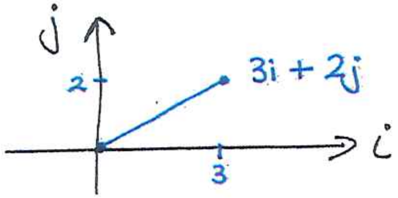
Year 1 Pure Chapter 11 – Vectors

Labels

$\overrightarrow{AB} = a$

$\overrightarrow{BA} = -a$

i + j Components



Parallel

Show they have the same gradient.

$2i + 4j$ gradient $= \frac{4}{2} = 2$

$6i + 12j$ gradient $= \frac{12}{6} = 2$

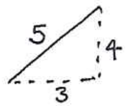
gradients are the same ∴ parallel

or

Show that one is a multiple of the other.

$2i + 4j = 2(a + 2b)$
$6i + 12j = 3(a + 2b)$

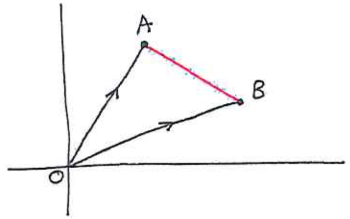
Magnitude and Length



$a = 3i + 4j$

$|a| = \sqrt{3^2 + 4^2} = 5$

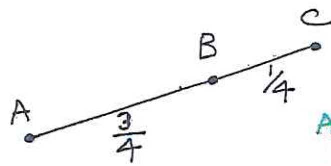
Position Vectors



$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

NOTE $\overrightarrow{AO} = -\overrightarrow{AO}$

Ratios



AB : BC
3 : 1

Velocity and Journeys

Current Position =
Velocity x time
+ starting Position

Velocity = 3i + 2j
Time = 5
Current Position = 5i + 5j

Find the starting position.

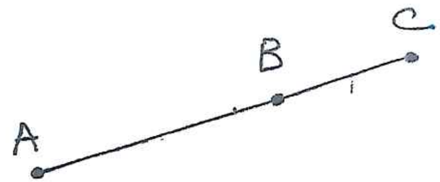
$5i + 9j = 5(3i + 2j) + Starting$

$5i + 9j = 15i + 10j + Starting$

$Starting = -10i - j$

Collinear

Collinear means lying in the same straight line.



To prove that three points are collinear you can show that AC is a multiple of AB.

AC = λ AB

λ is a multiple to be found.

Year 1 Pure Chapter 12 - Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x) = x^3$ then $f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

expand + simplify $= \lim_{h \to 0} 3x^2 + 3xh + h^2$

(as $h \to 0$ $3xh = 0$, $h^2 = 0$)

$f'(x) = 3x^2$

Notation

$f(x) \xrightarrow{\text{differentiate}} f'(x) \xrightarrow{\text{differentiate}} f''(x)$

$y \longrightarrow \frac{dy}{dx} \longrightarrow \frac{d^2y}{dx^2}$

$4x^3 \longrightarrow 12x^2 \longrightarrow 24x$

multiply by power then subtract 1 from power

multiply by power then subtract 1 from power

Differentiation

$y = \sqrt{x} = x^{1/2}$ $\quad \frac{dy}{dx} = \frac{1}{2}x^{-1/2}$

$y = \frac{1}{3x^3} = \frac{1}{3}x^{-3}$ $\quad \frac{dy}{dx} = -\frac{3}{3}x^{-4}$

$y = \frac{2x^3 + 3x}{x} = 2x^2 + 3$ $\quad \frac{dy}{dx} = 4x$

both terms get divided by x

Gradient of a Tangent

Find $\frac{dy}{dx}$ then sub in x

Max and Min Points

max point $\frac{d^2y}{dx^2} < 0$

min point $\frac{d^2y}{dx^2} > 0$

Year 1 Pure Chapter 13 - Integration

Integration

$$\frac{dy}{dx} = x^5 \xrightarrow{\text{Integrate}} y = \frac{x^6}{6} + c$$

add one to the power
divide by the new power

Find the Constant

To find the constant you need a co-ordinate that the curve passes through.

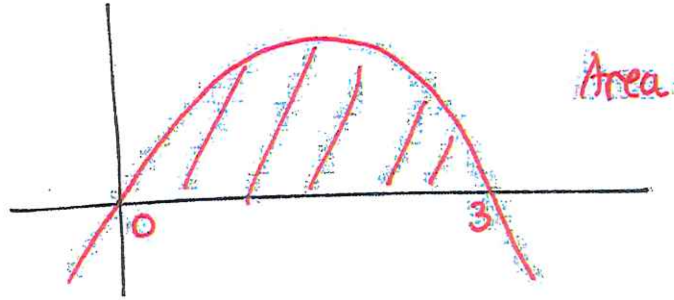
Notation

$$\int x^3 \, dx = \frac{x^4}{4} + c$$

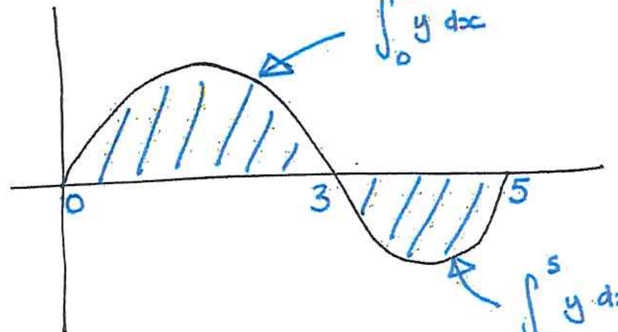
Limits

$$\int_1^2 3x^2 \, dx = [x^3]_1^2$$
$$= (2^3) - (1^3)$$
$$= 8 - 1$$
$$= 7$$

Area Under a Graph



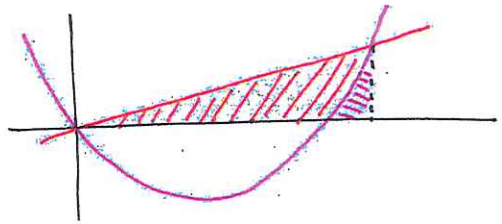
Area $= \int_0^3 y \, dx$

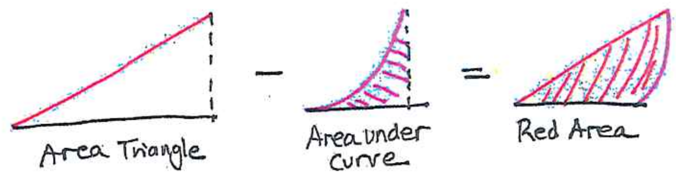


$\int_0^3 y \, dx$

$\int_3^5 y \, dx$

(Note: answer will be negative because below x axis. Just ignore the negative.)

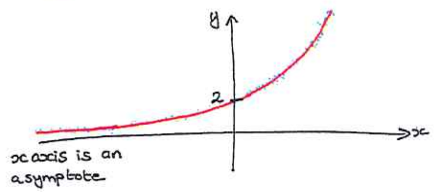




Year 1 Pure Chapter 14 - Exponential and Logs

Exponentials

$y = 2^x$ Index is a variable



Special Exponential

$y = e^x$ $e \approx 2.71828$

used for modelling

$f(x) = e^{kx}$

$f'(x) = ke^{kx}$

Logarithms

$3^2 = 9$

$\log_3 9 = 2$

Laws of Logarithms

multiplication law $\log x + \log y = \log xy$

division law $\log x - \log y = \log \frac{x}{y}$

power law $k \log x = \log x^k$

WATCH OUT FOR:

Only apply multiplication + division law after applying power law.

$5 \log 2 + \log 3$

$= \log 2^5 + \log 3$

$= \log 32 + \log 3 = \log 96$

Solving Equations Using Logs

$3^x = 2^{x+1}$

$\log 3^x = \log 2^{x+1}$

$x \log 3 = (x+1) \log 2$

$x \log 3 = x \log 2 + \log 2$

$x \log 3 - x \log 2 = \log 2$

$x(\log 3 - \log 2) = \log 2$

$x = \frac{\log 2}{\log 3 - \log 2}$

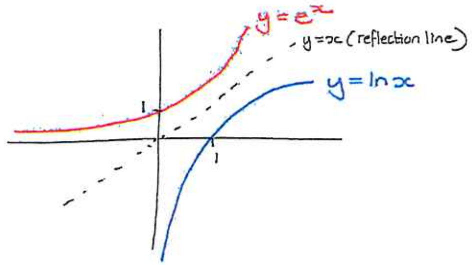
Quadratic Equations Using Logs

$2^{2x} - 6(2^x) + 5 = 0$

$y = 2^x$ $(2^x)^2 - 6(2^x) + 5 = 0$

$y^2 - 6y + 5 = 0$

Natural Logarithms



$e^x = 5$

$\ln e^x = \ln 5$

$x = \ln 5$

Natural Logarithms and Quadratics

$e^{2x} + 5e^x - 14 = 0$

$(e^x + 7)(e^x - 2) = 0$

$e^x = -7$ or $e^x = 2$

no solutions e^x is always positive

$\underline{\underline{x = \ln 2}}$

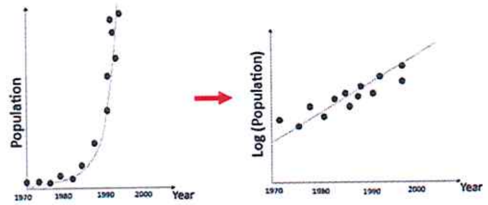
Year 1 Pure Chapter 14 – Exponential and Logs

Non-Linear Modelling

$y = ab^x$

y = Population
x = time

y = Log (Population)
x = time



Exponential Model to a Linear Model

$$y = ab^x$$

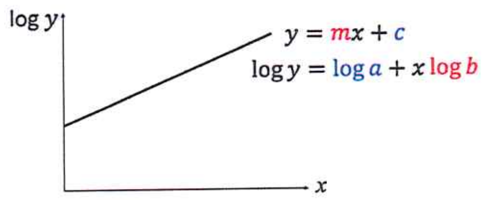
$$\log y = \log ab^x$$

$$\log y = \log a + \log b^x$$

$$\log y = \log a + x \log b$$

$$y = mx + c$$

If $y = ab^x$,
then the graph of $\log y$ against x
will be a straight line with
gradient $\log b$ and vertical intercept $\log a$.

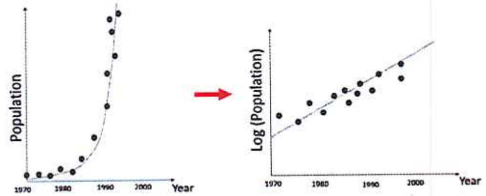


Non-Linear Modelling

$y = ax^n$

y = Population
x = time

y = Log (Population)
x = Log (time)



Exponential Model to a Linear Model

$$y = ax^n$$

$$\log y = \log ax^n$$

$$\log y = \log a + \log x^n$$

$$\log y = \log a + n \log x$$

$$y = mx + c$$

If $y = ax^n$,
then the graph of $\log y$ against $\log x$
will be a straight line with
gradient $\log b$ and vertical intercept $\log a$.