A level Mathematics Practice Paper – Differentiation (part 1) – Mark scheme

Question	Scheme	Marks
1(a)	$\frac{\mathrm{d}}{\mathrm{d}x(\ln(3x))} \to \frac{B}{x}$ for any constant B	M1
	Applying $vu' + uv'$ $\quad \ln(3x) \times 2x + x$	M1 A1 A1
		(4)
1(b)	Applying $\frac{vu' - uv'}{v^2}$ $\frac{x^3 \times 4\cos(4x) - \sin(4x) \times 3x^2}{x^6}$	M1 A1+A1 A1
	$= \frac{4x\cos(4x) - 3\sin(4x)}{x^4}$	A1
		(5)
		(9 marks)
2(a)	$\frac{1}{(x^2 + 3x + 5)} \times \ldots = \frac{2x + 3}{(x^2 + 3x + 5)}$	M1 A1
		(2)
2(b)	Applying $\frac{vu' - uv'}{v^2}$	M1
	$\frac{x^2 \times -\sin x - \cos x \times 2x}{(x^2)^2} = \frac{-x^2 \sin x - 2x\cos x}{x^4}$ $= \frac{-x\sin x - 2\cos x}{x^3}$ oe	A2, 1, 0
		(3)
		(5 marks)

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Question	Scheme		Marks
3(a)	$\dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)}$		
	$= \dfrac{(4x-1)(2x-1)-3}{2(x-1)(2x-1)}$	An attempts to form a single fraction	M1
	$= \dfrac{8x^2-6x-2}{2(x-1)(2x-1)}$	Simplifies to give a correct quadratic numerator over a correct quadratic denominator	A1 aef
	$= \dfrac{2(x-1)(4x+1)}{2(x-1)(2x-1)}$	An attempt to factorise a 3 term quadratic numerator	M1
	$= \dfrac{4x+1}{2x-1}$		A1
			(5)
3(b)	$\mathrm{f}(x) = \dfrac{4x-1}{2(x-1)} - \dfrac{3}{2(x-1)(2x-1)} - 2, x > 1$		
	$\mathrm{f}(x) = \dfrac{4x+1}{2x-1} - 2$		
	$= \dfrac{(4x+1)-2(2x-1)}{(2x-1)}$	An attempt to form a single fraction	M1
	$= \dfrac{4x+1-4x+2}{(2x-1)}$		
	$= \dfrac{3}{(2x-1)}$	Correct result	A1 *
			(2)
3(c)	$\mathrm{f}(x) = \dfrac{3}{(2x-1)} = 3(2x-1)^{-1}$		
	$\mathrm{f}'(x) = 3(-1)(2x-1)^{-2}(2)$		
		$\pm k(2x-1)^{-2}$	M1
			A1 aef
	$\mathrm{f}'(2) = \dfrac{-6}{9} = -\dfrac{2}{3}$	Either $\dfrac{-6}{9}$ or $-\dfrac{2}{3}$	A1
			(3)
			(9 amrks)

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Question	Scheme	Marks
4(a)	$\frac{\mathrm{d}x}{\mathrm{d}y} = 2\times 3\sec 3y \sec 3y \tan 3y = \left(6\sec^2 3y \tan 3y\right)$ $\left(\text{oe } \frac{6\sin 3y}{\cos^3 3y}\right)$	M1A1
		(2)
4(b)	Uses $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$ to obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6\sec^2 3y \tan 3y}$	M1
	$\tan^2 3y = \sec^2 3y - 1 = x - 1$	B1
	Uses $\sec^2 3y = x$ and $\tan^2 3y = \sec^2 3y - 1 = x - 1$ to get $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ in just x.	M1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$ CSO	A1*
		(4)
4(c)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{0 - [6(x-1)^{\frac{1}{2}} + 3x(x-1)^{-\frac{1}{2}}]}{36x^2(x-1)}$	M1A1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{6-9x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{2-3x}{12x^2(x-1)^{\frac{3}{2}}}$	dM1A1
		(4)
		(10 marks)
5(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{3}e^{x\sqrt{3}} \sin 3x + 3e^{x\sqrt{3}} \cos 3x$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $e^{x\sqrt{3}}(\sqrt{3}\sin 3x + 3\cos 3x) = 0$	M1
	$\tan 3x = -\sqrt{3}$	A1
	$3x = \frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{9}$	M1A1
		(6)
5(b)	At $x = 0$ $\frac{\mathrm{d}y}{\mathrm{d}x} = 3$	B1
	Equation of normal is $-\frac{1}{3} = \frac{y-0}{x-0}$ or any equivalent $y = -\frac{1}{3}x$	M1A1
		(3)
		(9 marks)

Question	Scheme	Marks
6	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \ \ldots$	B1
	$\ldots \quad = 2\ln x + 2x\left(\frac{1}{x}\right)$	M1 A1
	At $x=2$, $\quad \ln y = 2(2)\ln 2$	M1
	leading to $\quad y = 16 \quad$ Accept $y = \mathrm{e}^{4\ln 2}$	A1
	At $(2,16)$ $\quad \frac{1}{16}\frac{\mathrm{d}y}{\mathrm{d}x} = 2\ln 2 + 2$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 16(2 + 2\ln 2)$	A1
		(7 marks)
7(i)(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$	M1A1A1
	$= 3x^2 \ln 2x + x^2$	
		(3)
7(i)(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3(\boldsymbol{x} + \mathbf{sin2}\boldsymbol{x})^2 \times (1 + 2\cos 2x)$	B1M1A1
		(3)
7(ii)	$\frac{\mathrm{d}x}{\mathrm{d}y} = -\mathrm{cosec}^2 y$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\mathrm{cosec}^2 y}$	M1
	Uses $\mathrm{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ to get an expression in x	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\mathrm{cosec}^2 y} = -\frac{1}{1+\cot^2 y} = -\frac{1}{1+x^2} \quad$ cso	M1 A1*
		(5)
		(11 marks)

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8(a)(i)	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln(3x)) = \frac{3}{3x}$	M1
	$\frac{\mathrm{d}}{\mathrm{d}x}(x^{\frac{1}{2}}\ln(3x)) = \ln(3x)\times\frac{1}{2}x^{-\frac{1}{2}} + x^{\frac{1}{2}}\times\frac{3}{3x}$	M1A1
		(3)
8(a)(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(2x-1)^5\times -10-(1-10x)\times 5(2x-1)^4\times 2}{(2x-1)^{10}}$	M1A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{80x}{(2x-1)^6}$	A1
		(3)
8(b)	$x = 3\tan 2y \quad \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} = 6\sec^2 2y$	M1A1
	$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6\sec^2 2y}$	M1
	Uses $\sec^2 2y = 1+\tan^2 2y$ and uses $\tan 2y = \frac{x}{3}$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{6(1+(\frac{x}{3})^2)} = (\frac{3}{18+2x^2})$	M1A1
		(5)
		(11 marks)

Question	Scheme	Marks
9(a)	$\frac{2}{x+2}+\frac{4}{x^2+5}-\frac{18}{(x+2)(x^2+5)}=\frac{2(x^2+5)+4(x+2)-18}{(x+2)(x^2+5)}$	M1A1
	$=\frac{2x(x+2)}{(x+2)(x^2+5)}$	M1
	$=\frac{2x}{(x^2+5)}$	A1*
		(4)
9(b)	$\mathrm{h}'(x)=\frac{(x^2+5)\times 2-2x\times 2x}{(x^2+5)^2}$	M1 A1
	$\mathrm{h}'(x)=\frac{10-2x^2}{(x^2+5)^2}$ cso	A1
		(3)
9(c)	Maximum occurs when $\mathrm{h}'(x)=0\Rightarrow 10-2x^2=0\Rightarrow x=..$	M1
	$\Rightarrow x=\sqrt{5}$	A1
	When $x=\sqrt{5}\Rightarrow \mathrm{h}(x)=\frac{\sqrt{5}}{5}$	M1 A1
	Range of h(x) is $0\le h(x)\le\frac{\sqrt{5}}{5}$	A1ft
		(5)
		(12 marks)

Question	Scheme		Marks
10(a)	$y = \dfrac{3+\sin 2x}{2+\cos 2x}$		
	Apply quotient rule: $\begin{cases} u = 3+\sin 2x & v = 2+\cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{cases}$		
	$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2+\cos 2x) - -2\sin 2x(3+\sin 2x)}{(2+\cos 2x)^2}$	Applying $\dfrac{vu'-uv'}{v^2}$	M1
		Any one term correct on the numerator	A1
		Fully correct (unsimplified).	A1
	$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2+\cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2+\cos 2x)^2}$	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.	
	$= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2+\cos 2x)^2}$ (as required)		A1*
			(4)
10(b)	When $x = \frac{\pi}{2}$, $y = \dfrac{3+\sin\pi}{2+\cos\pi} = \dfrac{3}{1} = 3$	$y = 3$	B1
	At $\left(\frac{\pi}{2}, 3\right)$, $\mathrm{m}(\mathbf{T}) = \dfrac{6\sin\pi + 4\cos\pi + 2}{(2+\cos\pi)^2} = \dfrac{-4+2}{1^2} = -2$	$\mathrm{m}(\mathbf{T}) = -2$	B1
	Either **T**: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2\left(\frac{\pi}{2}\right) + c \Rightarrow c = 3 + \pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	T: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1
			(4)
			(8 marks)

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Question	Scheme		Marks
11(a)	$y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$		
	$\dfrac{dy}{dx} = -1(\cos x)^{-2}(-\sin x)$	Writes $\sec x$ as $(\cos x)^{-1}$ and gives $\dfrac{dy}{dx} = \pm\left((\cos x)^{-2}(\sin x)\right)$	M1
		$-1(\cos x)^{-2}(-\sin x)$ or $(\cos x)^{-2}(\sin x)$	A1
	$\dfrac{dy}{dx} = \left\{\dfrac{\sin x}{\cos^2 x}\right\} = \underline{\underline{\left(\dfrac{1}{\cos x}\right)\left(\dfrac{\sin x}{\cos x}\right)}} = \underline{\underline{\sec x \tan x}}$	Convincing proof. Must see both underlined steps.	A1 **AG**
			(3)
11(b)	$x = \sec 2y, \quad y \neq (2n+1)\frac{\pi}{4}, n \in \square$.		
	$\dfrac{dx}{dy} = 2\sec 2y \tan 2y$	$K \sec 2y \tan 2y$	M1
		$2\sec 2y \tan 2y$	A1
			(2)
11(c)	$\dfrac{dy}{dx} = \dfrac{1}{2\sec 2y \tan 2y}$	Applies $\dfrac{dy}{dx} = \dfrac{1}{\left(\frac{dx}{dy}\right)}$	M1
	$\dfrac{dy}{dx} = \dfrac{1}{2x \tan 2y}$	Substitutes x for $\sec 2y$.	M1
	$1 + \tan^2 A = \sec^2 A \Rightarrow \tan^2 2y = \sec^2 2y - 1$	Attempts to use the identity $1 + \tan^2 A = \sec^2 A$	M1
	So $\tan^2 2y = x^2 - 1$		
	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	$\dfrac{dy}{dx} = \dfrac{1}{2x\sqrt{(x^2-1)}}$	A1
			(4)
			(9 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	C3 Jan 2012	1	7.2, 7.4	Differentiation	1.1b
2	C3 2011	1	7.4	Differentiation	1.1b, 3.1a, 3.2a
3	C3 Jan 2011	2	2.6, 7.4	Partial fractions, Differentiation	1.1b, 2.1
4	C3 2013	5	7.2, 7.4, 5.4, 5.5	Differentiation and trigonometry	1.1b, 2.1, 3.1a
5	C3 2012	3	7.2, 7.3, 7.4, 5.7	Trigonometry, Differentiation	1.1b, 3.1a
6	C4 2011	5	7.5	Differentiation	1.1b, 3.1a, 3.4
7	C3 Jan 2013	5	7,1, 7.4, 5.5	Differentiation	1.1b, 2.1, 2.2a, 2.4, 3.1a, 3.4
8	C3 2012	7	7.2, 7.4, 5.5	Trigonometry, Differentiation	1.1b, 3.1a
9	C3 Jan 2013	7	2.6, 2.8, 7.3, 7.4	Algebra and functions, Differentiation	1.1b, 3.1a
10	C3 Jan 2011	7	7.3, 7.4, 5.5	Differentiation	1.1b
11	C3 Jan 2011	8	7.4, 5.4, 5.5	Differentiation, Trigonometry	1.1b, 2.1, 2.2a