AS and A level Mathematics Practice Paper – Parametric differentiation – Mark scheme

Question	Scheme	Marks
1(a)	$\frac{dx}{dt} = 4\sec^2 t,\ \frac{dy}{dt} = 10\sqrt{3}\cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3}\cos 2t}{4\sec^2 t}$ $\left\{= \frac{5}{2}\sqrt{3}\cos 2t\cos^2 t\right\}$ o.e.	M1 A1
	$\left\{\text{At } P\left(4\sqrt{3}, \frac{15}{2}\right),\ t = \frac{\pi}{3}\right\}$	
	$\frac{dy}{dx} = \frac{10\sqrt{3}\cos\left(\frac{2\pi}{3}\right)}{4\sec^2\left(\frac{\pi}{3}\right)}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	A1 **cso**
		(4)
1(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$	
	So $x = 4\tan\left(\frac{\pi}{4}\right)$, $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	M1
	Coordinates are $\left(4, 5\sqrt{3}\right)$	A1
		(2)
		(6 marks)

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2(a)	$x = 3t - 4,\ \ y = 5 - \frac{6}{t},\quad t > 0$	
	$\frac{\mathrm{d}x}{\mathrm{d}t} = 3,\quad \frac{\mathrm{d}y}{\mathrm{d}t} = 6t^{-2}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6t^{-2}}{3}\ \left\{= \frac{6}{3t^2} = 2t^{-2} = \frac{2}{t^2}\right\}$	M1
		A1 isw
		(2)
2(b)	$\left\{t = \frac{1}{2} \Rightarrow\right\}\ P\left(-\frac{5}{2}, -7\right)$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y - "-7" = "8"\left(x - "-\frac{5}{2}"\right)$ • $"-7" = ("8")("-\frac{5}{2}") + c$ So, $y = (\text{their } m_{\mathrm{T}})x + "c"$	M1
	T: $y = 8x + 13$	A1 **cso**
		(3)
2(c)	$\left\{t = \frac{x+4}{3}\ \Rightarrow\right\}\ y = 5 - \frac{6}{\left(\frac{x+4}{3}\right)}$	M1
		A1 o.e.
	$\Rightarrow\ y = 5 - \frac{18}{x+4}\ \Rightarrow\ y = \frac{5(x+4) - 18}{x+4}$	
	So, $y = \frac{5x+2}{x+4},\ \{x > -4\}$	A1 **cso**
		(3)
		(8 marks)

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Question	Scheme	Marks
3(a)	$x = 4t + 3,\ \ y = 4t + 8 + \frac{5}{2t}$	
	$\frac{dx}{dt} = 4,\ \ \frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$	B1
	So, $\frac{dy}{dx} = \frac{4 - \frac{5}{2}t^{-2}}{4}$ $\left\{= 1 - \frac{5}{8}t^{-2} = 1 - \frac{5}{8t^2}\right\}$	M1
	$\{\text{When } t = 2,\}\ \frac{dy}{dx} = \frac{27}{32}$	A1
		(3)
3(b)	$\left\{t = \frac{x-3}{4} \Rightarrow\right\}\ y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$	M1
	$y = x - 3 + 8 + \frac{10}{x-3}$	
	$y = \frac{(x-3)(x-3) + 8(x-3) + 10}{x-3}$ *or* $y(x-3) = (x-3)(x-3) + 8(x-3) + 10$ *or* $y = \frac{(x+5)(x-3) + 10}{x-3}$ *or* $y = \frac{(x+5)(x-3)}{x-3} + \frac{10}{x-3}$	dM1
	$\Rightarrow y = \frac{x^2 + 2x - 5}{x-3}$, $\{a = 2 \text{ and } b = -5\}$	A1 **cso**
		(3)
		(6 marks)

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Question	Scheme	Marks
4(a)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 2\sqrt{3}\cos 2t$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}t} = -8\cos t \sin t$	M1 A1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-8\cos t \sin t}{2\sqrt{3}\cos 2t}$	M1
	$= -\frac{4\sin 2t}{2\sqrt{3}\cos 2t}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{3}\sqrt{3}\tan 2t \qquad \left(k = -\frac{2}{3}\right)$	A1
		(5)
4(b)	When $t = \frac{\pi}{3}$ $\quad x = \frac{3}{2},\ y = 1$ can be implied	B1
	$m = -\frac{2}{3}\sqrt{3}\tan\left(\frac{2\pi}{3}\right) \quad (=2)$	M1
	$y - 1 = 2\left(x - \frac{3}{2}\right)$	M1
	$y = 2x - 2$	A1
		(4)
4(c)	$x = \sqrt{3}\sin 2t = \sqrt{3} \times 2\sin t \cos t$	M1
	$x^2 = 12\sin^2 t \cos^2 t = 12\left(1 - \cos^2 t\right)\cos^2 t$	
	$x^2 = 12\left(1 - \frac{y}{4}\right)\frac{y}{4}$ or equivalent	M1 A1
		(3)
		(12 marks)

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Question	Scheme	Marks
5	$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \text{„} \ t < 2\pi$	
	$\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right), \quad \frac{dy}{dt} = -6\sin 2t$	B1 B1
	So, $\frac{dy}{dx} = \frac{-6\sin 2t}{4\cos\left(t + \frac{\pi}{6}\right)}$	B1$\surd$ oe (3)
	$\left\{\frac{dy}{dx} = 0 \ \Rightarrow\right\} \ -6\sin 2t = 0$	M1 oe
	@ $t = 0, \quad x = 4\sin\left(\frac{\pi}{6}\right) = 2, \quad y = 3\cos 0 = 3 \quad \rightarrow (2, 3)$ @ $t = \frac{\pi}{2}, \ x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}, \ y = 3\cos\pi = -3 \rightarrow (2\sqrt{3}, -3)$ @ $t = \pi, \quad x = 4\sin\left(\frac{7\pi}{6}\right) = -2, \ y = 3\cos 2\pi = 3 \ \rightarrow (-2, 3)$ @ $t = \frac{3\pi}{2}, x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}, \ y = 3\cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	M1 A1 A1 A1
		(5)
		(8 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	C4 2016	5	7.2, 7.5, 5.7	Coordinate geometry, parametric differentiation	1.1b
2	C4 2017	1	3.3, 7.5	Coordinate geometry, Parametric differentiation	1.1b, 2.1
3	C4 2015	5	3.3, 7.5	Parametric curves, parametric differentiation	1.1b, 2.1
4	C4 2012	6	3.3, 7.5, 5.5, 5.6	Parametric curves and equations, Parametric differentiation	1.1b, 2.1, 3.1a
5	C4 Jan 2012	5	3.3, 7.5	Parametric curves and equations, Differentiation	1.1b, 2.1