Year 2 Pure Chapter 5: Radians (Trig Equations) – Exam Questions (Answers)

1)

Solve the following equations for θ, giving your answers to 3 significant figures where appropriate, in the intervals indicated.

a $3\sin\theta = 2,\ 0 \leqslant \theta \leqslant \pi$ **b** $\sin\theta = -\cos\theta,\ -\pi \leqslant \theta \leqslant \pi$

a $3\sin\theta = 2,\ 0 \leqslant \theta \leqslant \pi$

$$\sin\theta = \frac{2}{3}$$

$$\theta = 0.730, 2.41$$

b $\sin\theta = -\cos\theta,\ -\pi \leqslant \theta \leqslant \pi$

$$\tan\theta = -1$$

$$\theta = -\frac{\pi}{4}, \frac{3\pi}{4}$$

1) Continued

Solve the following equations for θ, giving your answers to 3 significant figures where appropriate, in the intervals indicated.

c $\tan\theta + \frac{1}{\tan\theta} = 2,\ 0 \leqslant \theta \leqslant 2\pi$

d $2\sin^2\theta - \sin\theta - 1 = \sin^2\theta,\ -\pi \leqslant \theta \leqslant \pi$

c

$$\tan\theta + \frac{1}{\tan\theta} = 2, 0 \leqslant \theta \leqslant 2\pi$$

$$\tan^2\theta + 1 = 2\tan\theta$$

$$\tan^2\theta - 2\tan\theta + 1 = 0$$

$$(\tan\theta - 1)^2 = 0$$

$$\tan\theta = 1$$

$$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$$

d $2\sin^2\theta - \sin\theta - 1 = \sin^2\theta,\ -\pi \leqslant \theta \leqslant \pi$

$$\sin^2\theta - \sin\theta - 1 = 0$$

$$\sin\theta = \frac{1 \pm \sqrt{5}}{2}$$

$$\sin\theta = 1.618 \text{ (no solutions)}$$

$$\text{or } \sin\theta = -0.618$$

$$\Rightarrow \theta = -0.666, -2.48$$

2)

Find the values of x in the interval $0 < x < \frac{3\pi}{2}$ which satisfy the equation

$$\frac{\sin 2x + 0.5}{1 - \sin 2x} = 2$$ **(6 marks)**

$$\frac{\sin 2x + 0.5}{1 - \sin 2x} = 2, \quad 0 < x < \frac{3\pi}{2}$$

$$\sin 2x + 0.5 = 2 - 2\sin 2x$$

$$3\sin 2x = 1.5$$

$$\sin 2x = 0.5$$

$$\text{Let } X = 2x$$

$$\sin X = 0.5, \quad 0 < X < 3\pi$$

$$X = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

3)

a Show that the equation

$5\sin x = 1 + 2\cos^2 x$

can be written in the form

$2\sin^2 x + 5\sin x - 3 = 0$ **(2 marks)**

b Solve, for $0 \leqslant x < 2\pi$,

$2\sin^2 x + 5\sin x - 3 = 0$ **(4 marks)**

a $5\sin x = 1 + \cos^2 x$

$5\sin x = 1 + 2(1 - \sin^2 x)$

$2\sin^2 x + 5\sin x - 3 = 0$

b $2\sin^2 x + 5\sin x - 3 = 0, 0 \leqslant x \leqslant 2\pi$

$(2\sin x - 1)(\sin x + 3) = 0$

$\sin x = 0.5 \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

or $\sin x = -3$ (no solution)