

# Pearson Edexcel Level 3

## GCE Mathematics

### Advanced Subsidiary

### Paper 1: Pure Mathematics

Wednesday 16 May 2018

Time: 2 hours

Paper Reference(s)

**8MA0/01**

**You must have:**

**Mathematical Formulae and Statistical Tables, calculator**

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

#### Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

#### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question*.

#### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**Answer ALL questions. Write your answers in the spaces provided.**

1. Find

$$\int \left( \frac{2}{3}x^3 - 6\sqrt{x} + 1 \right) dx,$$

giving your answer in its simplest form.

(4)

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2. (i) Show that  $x^2 - 8x + 17 > 0$  for all real values of  $x$ .

(3)

- (ii) “If I add 3 to a number and square the sum, the result is greater than the square of the original number.”

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

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3. Given that the point  $A$  has position vector  $4\mathbf{i} - 5\mathbf{j}$  and the point  $B$  has position vector  $-5\mathbf{i} - 2\mathbf{j}$ ,

- (a) find the vector  $\overrightarrow{AB}$ ,

(2)

- (b) find  $|\overrightarrow{AB}|$ .

Give your answer as a simplified surd.

(2)

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4. The line  $l_1$  has equation  $4y - 3x = 10$ .

The line  $l_2$  passes through the points  $(5, -1)$  and  $(-1, 8)$ .

Determine, giving full reasons for your answer, whether lines  $l_1$  and  $l_2$  are parallel, perpendicular or neither.

(4)

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5. A student's attempt to solve the equation  $2 \log_2 x - \log_2 \sqrt{x} = 3$  is shown below.

$$2 \log_2 x - \log_2 \sqrt{x} = 3$$

$$2 \log_2 \left( \frac{x}{\sqrt{x}} \right) = 3$$

using the subtraction law for logs

$$2 \log_2 \sqrt{x} = 3$$

simplifying

$$\log_2 x = 3$$

using the power law for logs

$$x = 3^2 = 9$$

using the definition of a log

- (a) Identify two errors made by this student, giving a brief explanation of each.

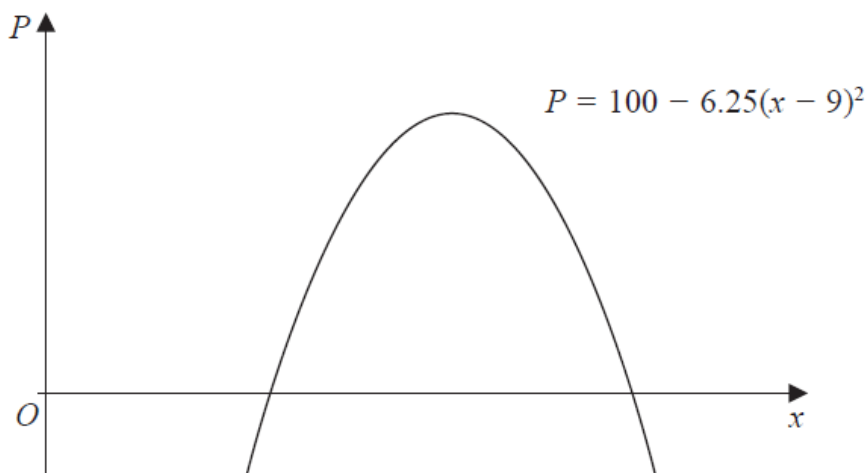
(2)

- (b) Write out the correct solution.

(3)

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6.



**Figure 1**

A company makes a particular type of children's toy.

The annual profit made by the company is modelled by the equation

$$P = 100 - 6.25(x - 9)^2,$$

where  $P$  is the profit measured in thousands of pounds and  $x$  is the selling price of the toy in pounds.

A sketch of  $P$  against  $x$  is shown in Figure 1.

Using the model,

- (a) explain why £15 is not a sensible selling price for the toy. (2)

Given that the company made an annual profit of more than £80 000,

- (b) find, according to the model, the least possible selling price for the toy. (3)

The company wishes to maximise its annual profit.

State, according to the model,

- (c) (i) the maximum possible annual profit,  
(ii) the selling price of the toy that maximises the annual profit. (2)
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7. In a triangle  $ABC$ , side  $AB$  has length 10 cm, side  $AC$  has length 5 cm, and angle  $BAC = \theta$ , where  $\theta$  is measured in degrees. The area of triangle  $ABC$  is  $15 \text{ cm}^2$ .

(a) Find the two possible values of  $\cos \theta$ .

(4)

Given that  $BC$  is the longest side of the triangle,

(b) find the exact length of  $BC$ .

(2)

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8. A lorry is driven between London and Newcastle.

In a simple model, the cost of the journey  $\pounds C$  when the lorry is driven at a steady speed of  $v$  kilometres per hour is

$$C = \frac{1500}{v} + \frac{2v}{11} + 60.$$

(a) Find, according to this model,

(i) the value of  $v$  that minimises the cost of the journey,

(ii) the minimum cost of the journey.

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

(6)

(b) Prove, by using  $\frac{d^2C}{dv^2}$ , that the cost is minimised at the speed found in part (a)(i).

(2)

(c) State one limitation of this model.

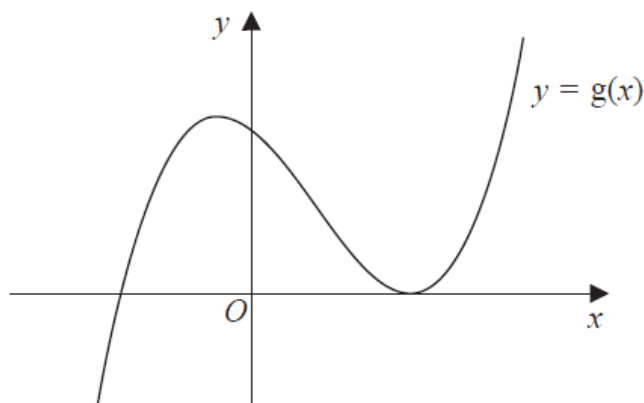
(1)

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9.  $g(x) = 4x^3 - 12x^2 - 15x + 50.$

(a) Use the factor theorem to show that  $(x + 2)$  is a factor of  $g(x)$ . (2)

(b) Hence show that  $g(x)$  can be written in the form  $g(x) = (x + 2)(ax + b)^2$ , where  $a$  and  $b$  are integers to be found. (4)



**Figure 2**

Figure 2 shows a sketch of part of the curve with equation  $y = g(x)$ .

(c) Use your answer to part (b), and the sketch, to deduce the values of  $x$  for which

(i)  $g(x) \leq 0$ ,

(ii)  $g(2x) = 0$ . (3)

10. Prove, from first principles, that the derivative of  $x^3$  is  $3x^2$ . (4)

11. (a) Find the first 3 terms, in ascending powers of  $x$ , of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9,$$

giving each term in its simplest form.

(4)

$$f(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the first two terms, in ascending powers of  $x$ , in the series expansion of  $f(x)$  are 128 and  $36x$ ,

- (b) find the value of  $a$ ,

(2)

- (c) find the value of  $b$ .

(2)

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12. (a) Show that the equation

$$4 \cos \theta - 1 = 2 \sin \theta \tan \theta$$

can be written in the form

$$6 \cos^2 \theta - \cos \theta - 2 = 0.$$

(4)

- (b) Hence solve, for  $0 \leq x < 90^\circ$ ,

$$4 \cos 3x - 1 = 2 \sin 3x \tan 3x,$$

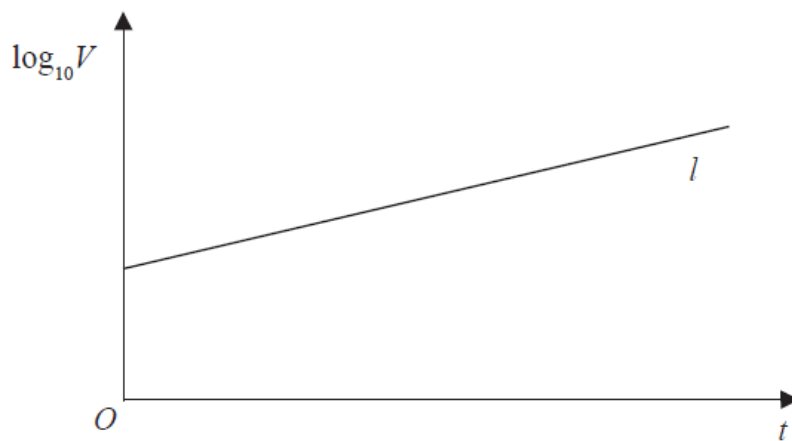
giving your answers, where appropriate, to one decimal place.

*(Solutions based entirely on graphical or numerical methods are not acceptable.)*

(4)

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13.



**Figure 3**

The value of a rare painting, £ $V$ , is modelled by the equation  $V = pq^t$ , where  $p$  and  $q$  are constants and  $t$  is the number of years since the value of the painting was first recorded on 1st January 1980.

The line  $l$  shown in Figure 3 illustrates the linear relationship between  $t$  and  $\log_{10} V$  since 1st January 1980.

The equation of line  $l$  is  $\log_{10} V = 0.05t + 4.8$ .

- (a) Find, to 4 significant figures, the value of  $p$  and the value of  $q$ . (4)
- (b) With reference to the model, interpret
- (i) the value of the constant  $p$ ,
  - (ii) the value of the constant  $q$ . (2)
- (c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds. (2)
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14. The circle  $C$  has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0.$$

- (a) Find

- (i) the coordinates of the centre of  $C$ ,
- (ii) the radius of  $C$ .

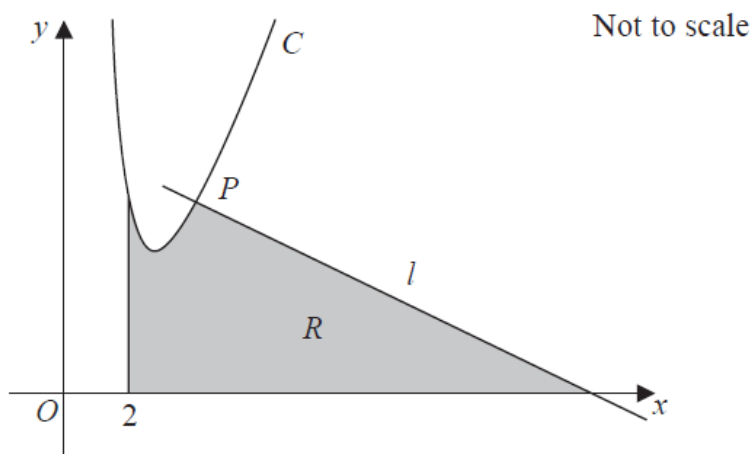
(3)

The line with equation  $y = kx$ , where  $k$  is a constant, cuts  $C$  at two distinct points.

- (b) Find the range of values for  $k$ .

(6)

15.



**Figure 4**

Figure 4 shows a sketch of part of the curve  $C$  with equation

$$y = \frac{32}{x^2} + 3x - 8, \quad x > 0.$$

The point  $P(4, 6)$  lies on  $C$ . The line  $l$  is the normal to  $C$  at the point  $P$ .

The region  $R$ , shown shaded in Figure 4, is bounded by the line  $l$ , the curve  $C$ , the line with equation  $x = 2$  and the  $x$ -axis.

Show that the area of  $R$  is 46.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(10)

**TOTAL FOR THE PAPER: 100 MARKS**

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