Year 2 Pure Chapter 2: Functions - Exam Questions (Answers)

1. (a) $fg(2) = f(-3) = |2(-3) - 5|; = |-11| = 11$ M1; A1

(b) $g(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$.

Hence $g_{min} = -3$ M1

Either $g_{min} = -3$

or $g(x) \geq -3$ B1

or $g(5) = 25 - 20 + 1 = 6$

$\underline{-3 \leq g(x) \leq 6}$ or $\underline{-3 \leq y \leq 6}$ A1

2. (a) Method for finding either coordinate of the lowest point (differentiating and equating to zero, completing the square, using symmetry). M1

$x = 3$ or $g(x) = -9$ A1

$g(x) \geq -9$ A1

(b) $fg(1) = f(-5)$ M1

$= -15$ A1

3. (a) $2(4x + a) = 8x + 2a$ M1 A1

(b) $3a = 8x + 2a$ M1

$x = a \div 8$ A1, A1

4. (a) $f^{-1}(x) = \frac{1}{2}x, \quad x \in \mathbb{R}$ B1 B1

(b) $gf^{-1}(x) = g(\frac{1}{2}x) = \frac{3}{4}x^2 + 2$ M1 A1

(c) Range $gf^{-1}(x) \geq 2$ B1

5. (a) $2 + \frac{3}{x+2} \left(= \frac{2(x+2)+3}{x+2}\right) = \underline{\underline{\frac{2x+7}{x+2}}}$ or $\frac{2(x+2)+3}{x+2}$ B1

(b) $y = 2 + \frac{3}{x+2}$ OR $y = \frac{2x+7}{x+2}$

$y - 2 = \frac{3}{x+2}$ $y(x + 2) = 2x + 7$ M1

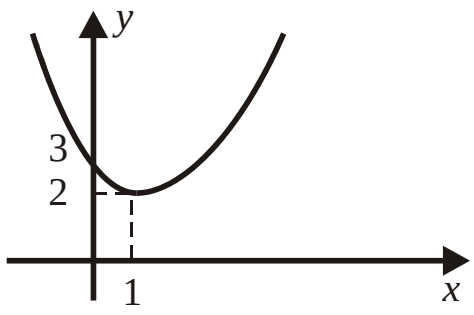
$yx - 2x = 7 - 2y$

$x + 2 = \frac{3}{y-2}$ $x(y - 2) = 7 - 2y$ M1

$x = \frac{3}{y-2} - 2$ $x = \frac{7-2y}{y-2}$

$\underline{\underline{f^{-1}(x) = \frac{3}{x-2} - 2}}$ $\underline{\underline{f^{-1}(x) = \frac{7-2x}{x-2}}}$ A1

6. (a) $x^2 - 2x + 3 = (x - 1)^2 + 2$ M1



f(4) = 3^2 + 2 = 11 f ≥ 2 A1

f ≤ 11 B1

(b) f(2) = 3

16 = gf(2)

16 = 3λ + 1 B1; M1

$\underline{\lambda = 5}$ A1 ft

7. (a) $x = 1 - 2y^3$

$y = \left(\frac{1-x}{2}\right)^{1/3}$ or $\sqrt[3]{\frac{1-x}{2}}$ M1A1

$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{1/3}$ Ignore domain

(b) $gf(x) = \frac{3}{1-2x^3} - 4$ M1A1

$= \frac{3-4(1-2x^3)}{1-2x^3}$ M1

$= \frac{8x^3-1}{1-2x^3}$ (*) A1

$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$

(c) $8x^3 - 1 = 0$ M1

$x = \frac{1}{2}$ Correct answer and no additional answers A1