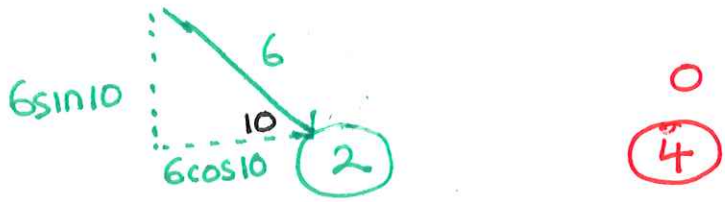
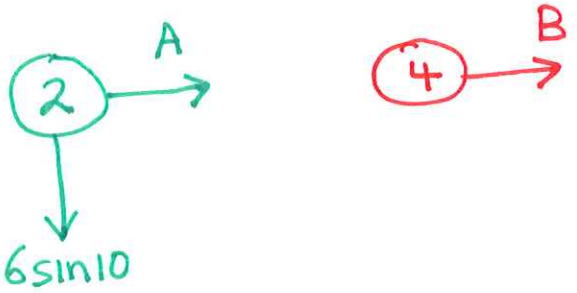
Ex 5C

Q1.

Before



After



com (↔)

$$(2)(6\sin 10) = 2A + 4B$$

$$12\sin 10 = 2A + 4B$$

$$6\sin 10 = A + 2B \quad (1)$$

e (↔)

$$\frac{1}{2} = \frac{B - A}{6\cos 10}$$

$$6\cos 10 = 2B - 2A$$

$$3\cos 10 = B - A$$

$$B = 3\cos 10 + A \quad (2)$$

sub (2) into (1)

$$6\cos 10 = A + 2(3\cos 10 + A)$$

$$6\cos 10 = A + 6\cos 10 + 2A$$

$$6\cos 10 = 3A + 6\cos 10$$

$$0 = 3A$$

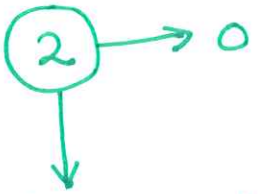
$$A = 0$$

$$B = 3\cos 10$$

Ex5C

Q1.

After



$6\sin 10 = 1.04 ms^{-1}$

parallel to common tangent



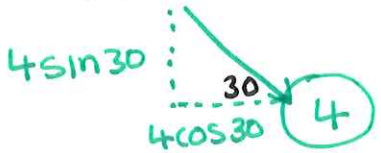
$3\cos 10 = 2.95 ms^{-1}$

parallel to the centre line

Ex5C

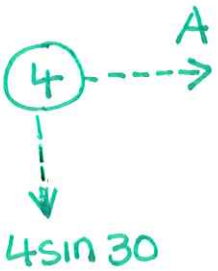
Q2.

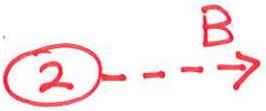
Before





After





com (↔)

$(4)(4\cos 30) = 4A + 2B$

$16\cos 30 = 4A + 2B$

$8\cos 30 = 2A + B$ ①

e

$\frac{1}{3} = \frac{B-A}{4\cos 30}$

$\frac{4}{3}\cos 30 = B - A$

$B = \frac{4}{3}\cos 30 + A$

sub ② into ①

$8\cos 30 = 2A + \frac{4}{3}\cos 30 + A$

$24\cos 30 = 6A + 4\cos 30 + 3A$

$20\cos 30 = 9A$

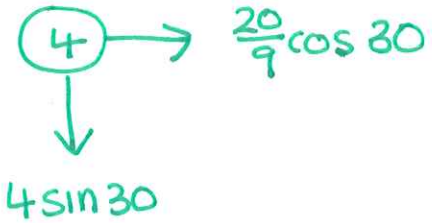
$A = \frac{20}{9}\cos 30$

$B = \frac{32}{9}\cos 30$

Ex5C

Q2.

After



$$V_A = \left(\frac{20}{9}\cos 30\right)^2 + \left(4\sin 30\right)^2$$

$$V_A = \frac{4\sqrt{39}}{9} = 2.78 \text{ ms}^{-1}$$

$$\alpha = \tan^{-1}\left(\frac{20/9\cos 30}{4\sin 30}\right)$$

After



$$= \frac{16\sqrt{3}}{9} = 3.08 \text{ ms}^{-1}$$

parallel to the line of centres

Ex 5C

Q3.

Before



After



com (⇔)

$$(3)(5\cos 45) = 3A + 4B$$

$$15\cos 45 = 3A + 4B \quad (1)$$

e $\quad \frac{1}{2} = \frac{B-A}{5\cos 45}$

$$5\cos 45 = 2B - 2A \quad (2)$$

(1) $15\cos 45 = 3A + 4B$

(2) $5\cos 45 = -2A + 2B$

$$15\cos 45 = 3A + 4B$$
$$10\cos 45 = -4A + 4B$$
$$5\cos 45 = 7A$$

$$A = \frac{5}{7}\cos 45$$

$$5\cos 45 = 2B - 2\left(\frac{5}{7}\cos 45\right)$$

$$5\cos 45 = 2B - \frac{10}{7}\cos 45$$

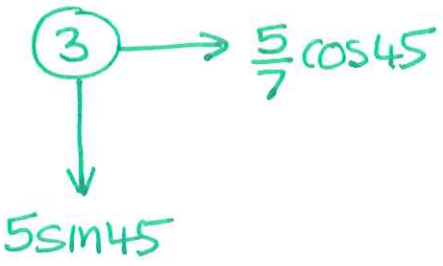
$$35\cos 45 = 14B - 10\cos 45$$

$$45\cos 45 = 14B$$

$$B = \frac{45}{14}\cos 45$$

Ex 5C
Q3.

After



$$V_A^2 = \left(\frac{5}{7}\cos 45\right)^2 + (5\sin 45)^2$$

$$V_A = \frac{25}{7}\ ms^{-1}$$

$$\alpha = \tan^{-1}\left(\frac{\frac{5}{7}\cos 45}{5\sin 45}\right)$$

$\alpha = 8.13^\circ$ to the common tangent

$$4 \rightarrow \frac{45}{14}\cos 45$$

$$= \frac{45\sqrt{2}}{28}$$

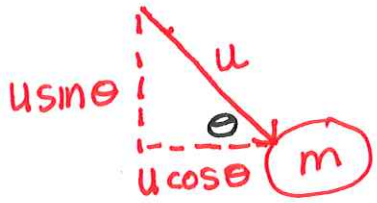
$$= 2.27\ ms^{-1}\ (3sf)$$

parallel to the line of centres

Ex5C

Q4.

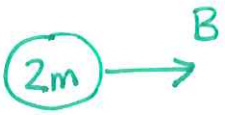
Before





After





com (↔)

$$um\cos\theta = -Vm\sin\theta + 2mB$$

$$u\cos\theta = -V\sin\theta + 2B \quad (1)$$

e (↔)

$$e = \frac{B - -V\sin\theta}{u\cos\theta}$$

$$u\cos\theta = B + V\sin\theta$$

$$B = u\cos\theta - V\sin\theta \quad (2)$$

Ex5C

Q4.

conti

sub ② into ①

$$u\cos\theta = -V\sin\theta + 2(ue\cos\theta - V\sin\theta)$$

$$u\cos\theta = -V\sin\theta + 2ue\cos\theta - 2V\sin\theta$$

$$u\cos\theta = -3V\sin\theta + 2ue\cos\theta$$

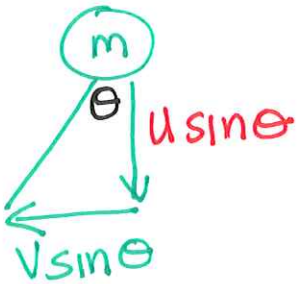
$$3V\sin\theta = 2ue\cos\theta - u\cos\theta$$

$$3V\sin\theta = u\cos\theta(2e - 1)$$

$$\frac{3V}{u}\frac{\sin\theta}{\cos\theta} = 2e - 1$$

$$\frac{V}{u}\tan\theta = \frac{2e-1}{3}$$

After



$$\tan\theta = \frac{V\sin\theta}{u\sin\theta}$$

$$\tan\theta = \frac{V}{u}$$

$$\tan\theta\tan\theta = \frac{2e-1}{3}$$

$$\tan^2\theta = \frac{2e-1}{3}$$

Ex5C

Q5.

Before



After



com (⇔)

$$mV = Am + Bm$$

$$V = A + B$$
$$A = V - B \quad (1)$$

e

$$\frac{2}{3} = \frac{A - B}{V}$$

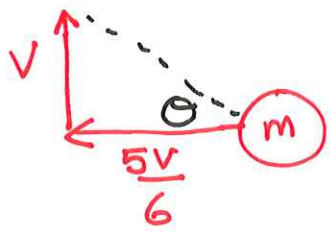
$$2V = 3A - 3B \quad (2)$$

sub (1) into (2)

$$2V = 3(V - B) - 3B$$
$$2V = 3V - 3B - 3B$$
$$6B = V$$
$$B = \frac{V}{6} \text{ ms}^{-1} \qquad A = V - \frac{V}{6} = \frac{5V}{6} \text{ ms}^{-1}$$

Ex5C
Q5.
conti

(A)

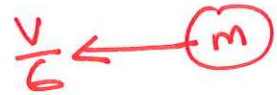


$$\tan^{-1}\left(\frac{V}{\frac{5V}{6}}\right)$$

$$= \tan^{-1}\left(\frac{6}{5}\right)$$

$$= 50.2^\circ \text{ to the line of centres}$$

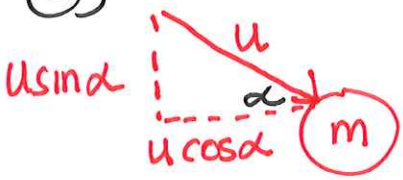
(B)



parallel to the line of centres

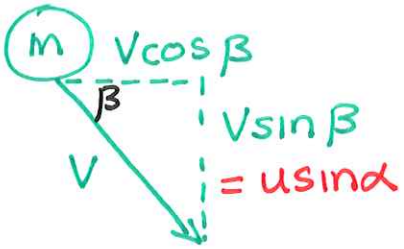
Ex5C

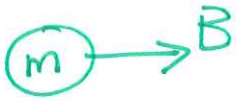
Q6. (a) Before





After





com (←→)

$$mu\cos\alpha = m\beta\cos\alpha + mB$$

$$u\cos\alpha = \beta\cos\alpha + B$$

$$B = u\cos\alpha - \beta\cos\alpha \quad (1)$$

e

$$e = \frac{B - \beta\cos V}{u\cos\alpha}$$

$$eu\cos\alpha = B - \beta\cos V \quad (2)$$

Ex5C

Q6.

(a) conti

sub ① into ②

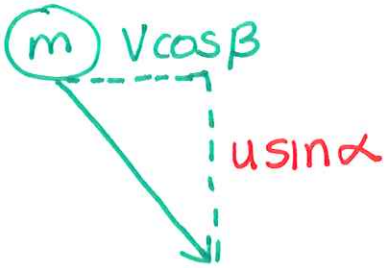
$$u\cos\alpha - V\cos\beta = eu\cos\alpha + V\cos\beta$$

$$u\cos\alpha - eu\cos\alpha = 2V\cos\beta$$

$$u\cos\alpha(1-e) = 2V\cos\beta$$

$$1-e = \frac{2V}{u}\frac{\cos\beta}{\cos\alpha} \quad ③$$

After



$$\tan\beta = \frac{u\sin\alpha}{V\cos\beta}$$

$$\frac{\tan\beta\cos\beta}{\sin\alpha} = \frac{u}{V}$$

$$\frac{\sin\alpha}{\tan\beta\cos\beta} = \frac{V}{u} \quad ④$$

sub ④ into ③

$$1-e = 2\left(\frac{\sin\alpha}{\tan\beta\cos\beta}\right)\frac{\cos\beta}{\cos\alpha}$$

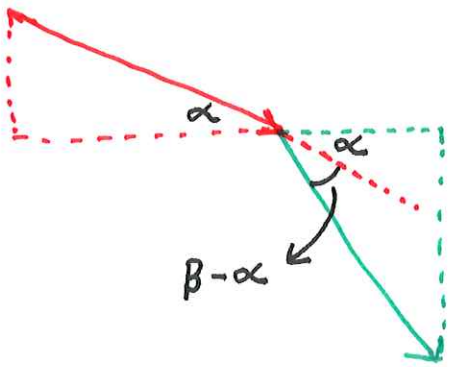
$$1-e = 2\frac{\tan\alpha}{\tan\beta}$$

$$\tan\beta = \frac{2\tan\alpha}{1-e}$$

Ex5C

Q6.

(b)



$\tan(\beta - \alpha)$

Addition Formula $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$$\tan(\beta - \alpha) = \frac{\tan\beta - \tan\alpha}{1 + \tan\beta \tan\alpha}$$

sub: $\tan\beta = \dfrac{2\tan\alpha}{1-e}$

$$\tan(\beta-\alpha) = \frac{\frac{2\tan\alpha}{1-e} - \tan\alpha}{1 + \frac{2\tan\alpha}{1-e}\tan\alpha}$$

$$\tan(\beta-\alpha) = \frac{\frac{2\tan\alpha}{1-e} - \frac{\tan\alpha(1-e)}{1-e}}{\frac{1-e}{1-e} + \frac{2\tan^2\alpha}{1-e}}$$

$$\tan(\beta-\alpha) = \frac{\frac{2\tan\alpha - \tan\alpha(1-e)}{1-e}}{\frac{(1-e) + 2\tan^2\alpha}{1-e}}$$

$$\tan(\beta-\alpha) = \frac{2\tan\alpha - \tan\alpha(1-e)}{(1-e) + 2\tan^2\alpha}$$

$$\tan(\beta-\alpha) = \frac{2\tan\alpha - \tan\alpha + e\tan\alpha}{(1-e) + 2\tan^2\alpha}$$

Ex5C

Q6.

(b) cont $\tan(\beta-\alpha)=\dfrac{\tan\alpha+e\tan\alpha}{(1-e)+2\tan^2\alpha}$

$$\tan(\beta-\alpha)=\frac{\tan\alpha(1+e)}{2\tan^2\alpha+1-e}$$

$$\beta-\alpha=\arctan\left(\frac{(1+e)\tan\alpha}{2\tan^2\alpha+1-e}\right)$$

Ex5C

Q7.

(a) Before



After



com

$$6\cos 45 + -2\cos 60 = 2A + B$$

$$\frac{6\sqrt{2}}{2} - 1 = 2A + B$$

$$3\sqrt{2} - 1 = 2A + B \quad (1)$$

e

$$\frac{\sqrt{2}}{3} = \frac{B - A}{3\cos 45 + 2\cos 60}$$

$$\frac{\sqrt{2}}{3} = \frac{B - A}{\frac{3\sqrt{2}}{2} + 1}$$

$$\frac{\sqrt{2}}{3}\left(\frac{3\sqrt{2}}{2} + 1\right) = B - A$$

$$\frac{6}{6} + \frac{\sqrt{2}}{3} = B - A$$

$$1 + \frac{\sqrt{2}}{3} = B - A$$

$$B = 1 + \frac{\sqrt{2}}{3} + A \quad (2)$$

Ex 5C

Q7.

(a) cont(i) Sub ② into ①

$$3\sqrt{2} - 1 = 2A + 1 + \frac{\sqrt{2}}{3} + A$$

$$3\sqrt{2} - \frac{\sqrt{2}}{3} - 2 = 3A$$

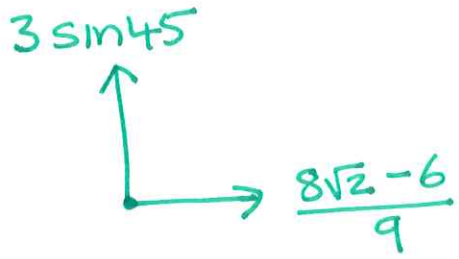
$$9\sqrt{2} - \sqrt{2} - 6 = 3A$$

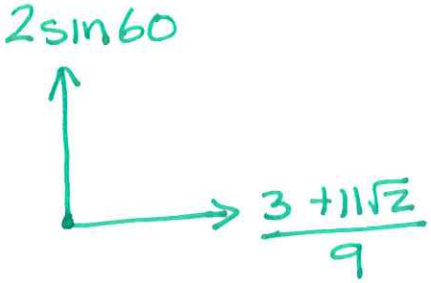
$$8\sqrt{2} - 6 = 3A$$

$$A = \frac{8\sqrt{2} - 6}{9}$$

$$B = \frac{3 + 11\sqrt{2}}{9}$$

After





$$V_A^2 = (3\sin 45)^2 + \left(\frac{8\sqrt{2} - 6}{9}\right)^2$$

$$V_A = 2.20195$$

$$V_B^2 = (2\sin 60)^2 + \left(\frac{3 + 11\sqrt{2}}{9}\right)^2$$

$$V_B = 2.6928$$

Ex5C

Q7.

(a)

conti (ii)

before A B

$\frac{1}{2}(2)(3)^2 = 9$ $\frac{1}{2}(1)(2)^2 = 2$

after

$\frac{1}{2}(2)(2.20195)^2 = 4.849$ $\frac{1}{2}(1)(2.6928)^2 = 3.626$

KE before = 11

KE after = 8.475

KE loss = 2.53 J (3sf)

(b)

B before

$-2\cos 60$

(1) ← momentum $(1)(-2\cos 60) = -1$

B after

momentum

(1) → $\frac{3+11\sqrt{2}}{9}$ $(1)\left(\frac{3+11\sqrt{2}}{9}\right) = 2.06$

Impulse = $2.06 - -1$

= 3.06 Ns

Ex5C

$\tan\alpha = \frac{4}{3}$ $\cos\alpha = \frac{3}{5}$ $\sin\alpha = \frac{4}{5}$

Q8.

(a)

Before



After



com (⟷)

$3m + -3m = -Am + Bm$

$0 = -A + B$

$A = B$ (1)

e

$\frac{2}{3} = \frac{B+A}{6}$

$4 = B + A$ (2)

Sub (1) into (2)

$4 = A + A$

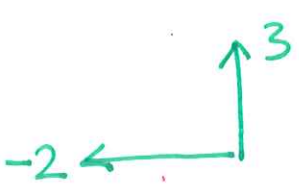
$A = 2$ $B = 2$

Ex 5C

Q8.

(a) conti

After





$V_A = \sqrt{2^2+3^2}$

$V_A = \sqrt{13}$ ms^{-1}

$V_A = \sqrt{4^2+2^2}$

$V_A = \sqrt{20}$ ms^{-1}

(b)

KE before

$$\frac{1}{2}m(3\sqrt{2})^2 + \frac{1}{2}m(5)^2 = \frac{43m}{2} \text{ J}$$

KE after

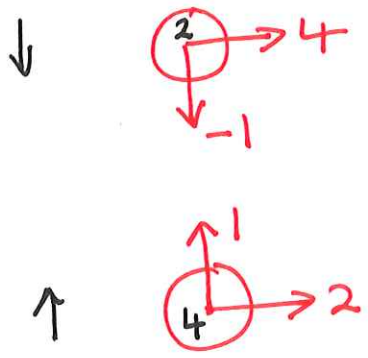
$$\frac{1}{2}m(\sqrt{13})^2 + \frac{1}{2}m(\sqrt{20})^2 = \frac{33m}{2} \text{ J}$$

Fraction Lost $\frac{43-33}{43} = \frac{10}{43}$

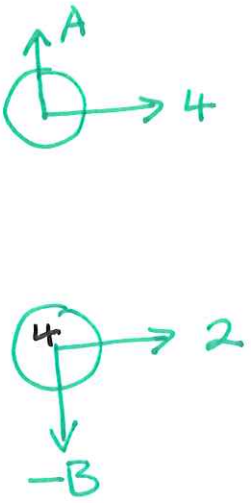
Ex5C

Q9.

Before



after



com (↕)

$$2(-1) + 4(1) = 2A - 4B$$

$$2 = 2A - 4B$$

$$1 = A - 2B$$

$$A = 1 + 2B \quad (1)$$

e

$$\frac{1}{2} = \frac{A+B}{2}$$

$$1 = A+B$$

sub (1) into (2)

$$1 = 1 + 2B + B$$

$$0 = 3B$$

$$B = 0$$

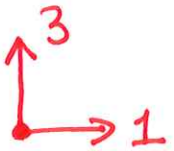
$$A = 1$$

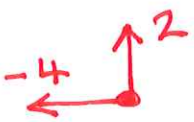
$$V_A = 4i + j$$

$$V_B = 2i$$

Ex 5C ③ → ← ①

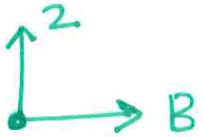
Q10. Before





After





com (⟷)

$$③(①) + (①)(-4) = -3A + B$$

$$-1 = -3A + B$$

$$B = 3A - 1 \quad ①$$

e

$$\frac{3}{4} = \frac{B + A}{5}$$

$$15 = 4B + 4A \quad ②$$

sub ① into ②

$$15 = 4(3A - 1) + 4A$$

$$15 = 12A - 4 + 4A$$

$$19 = 16A$$

$$A = \frac{19}{16}$$

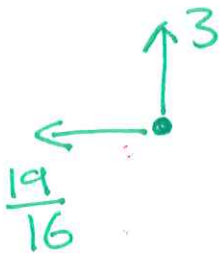
$$B = 3\left(\frac{19}{16}\right) - 1$$

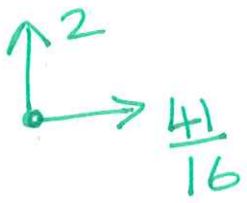
$$B = \frac{41}{16}$$

Ex5C

Q10

conti





$$V_A = \sqrt{\left(\frac{19}{16}\right)^2 + (3)^2}$$

$$V_B = \sqrt{\left(\frac{41}{16}\right)^2 + (2)^2}$$

$V_A = 3.23 \text{ ms}^{-1}$ (3sf)

$V_B = 3.25 \text{ ms}^{-1}$ (3sf)

Ex5C

Q11.



Before

After



com (↔)

$(2)(1) + (2)(-1) = -A + 2B$

$0 = -A + 2B$

$A = 2B \quad (1)$

e

$$\frac{3}{5} = \frac{B + A}{3}$$

$9 = 5B + 5A \quad (2)$

Sub (1) into (2)

$9 = 5B + 5(2B)$

$9 = 5B + 10B$

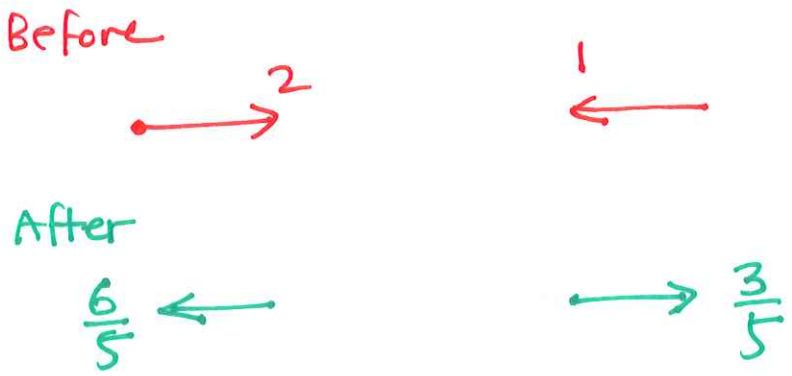
$9 = 15B$

$B = \frac{9}{15} = \frac{3}{5}$

$A = \frac{18}{15} = \frac{6}{5}$

Ex5C

Q11.



KE before $\frac{1}{2}(2)^2(1) + \frac{1}{2}(2)(1)^2 = 3\text{J}$

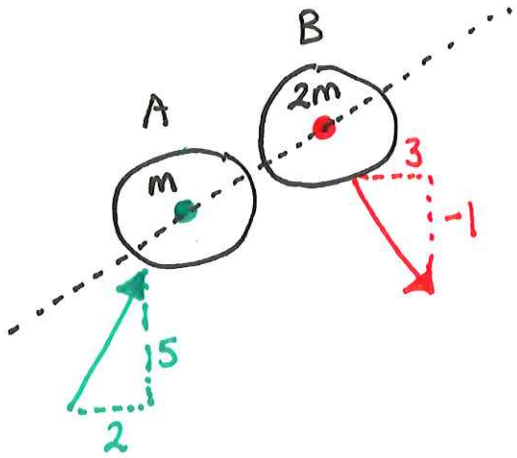
KE after $\frac{1}{2}(1)\left(\frac{6}{5}\right)^2 + \frac{1}{2}(2)\left(\frac{3}{5}\right)^2 = 1.08\text{ J}$

KE Lost $3 - 1.08 = \underline{\underline{1.92\text{J}}}$

Ex 5C

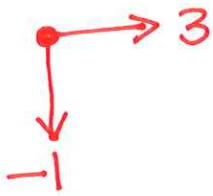
Q12.

(a)

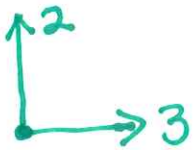


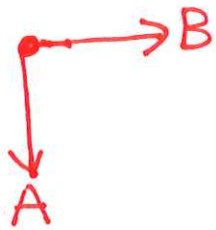
Before





After





com (↔)

$(2)(m) + (3)(2m)$

$=$

$(3)(m) + (2m)(B)$

$2m + 6m = 3m + 2mB$

$8 = 3 + 2B$

$B = 2.5$

com (↕)

$(5)(m) + (-1)(2m)$

$=$

$(2)(m) + (A)(2m)$

$5m - 2m = 2m + 2mA$

$3 = 2 + 2A$

$\frac{1}{2} = A$

$V = 2.5i + \frac{1}{2}j$

Ex 5C

Q12

(b) Impulse is parallel to the line of centres

Impulse exerted on A

momentum before

$$m(2i + 5j) = 2mi + 5mj$$

momentum after

$$m(3i + 2j) = 3mi + 2mj$$

$$\text{Impulse} = 3mi + 2mj - (2mi + 5mj)$$

$$= mi - 3mj$$

$$= m(i - 3j)$$

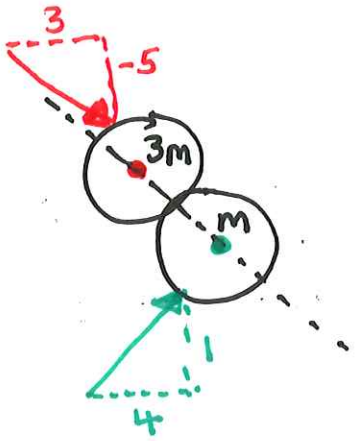
unit vector $\sqrt{1^2 + (-3)^2} = \sqrt{10}$

$$\frac{1}{\sqrt{10}}(i - 3j)$$

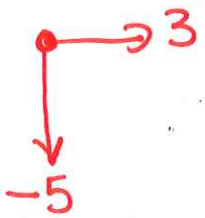
Ex5C

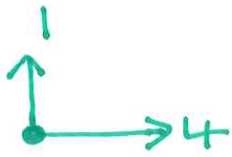
Q13.

(a)

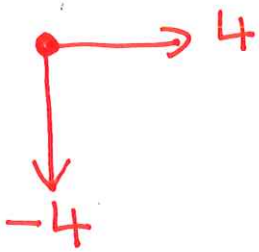


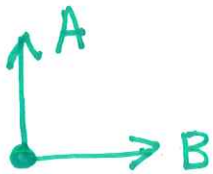
Before





After





com (⇔)

$3m(3) + m(4) = 3m(4) + mB$

$13 = 12 + B$

$B = 1$

com (⇕)

$3m(-5) + m(1) = 3m(-4) + m(A)$

$-14 = -12 + A$

$A = -2$

$$V_B = \sqrt{1^2 + (-2)^2} = \sqrt{5}$$

Ex 5C

Q13.

(a) conti

KE before

$$\frac{1}{2}(3m)\left(\sqrt{3^2+5^2}\right)^2 + \frac{1}{2}(m)\left(\sqrt{1^2+4^2}\right)^2$$

$$= \quad 51m \quad + \quad 8.5m$$

$$= \quad 59.5m$$

KE after

$$\frac{1}{2}(3m)\left(\sqrt{4^2+4^2}\right)^2 + \frac{1}{2}(m)\left(\sqrt{5}\right)^2$$

$$= \quad 48m \quad + \quad 2.5$$

$$= \quad 50.5$$

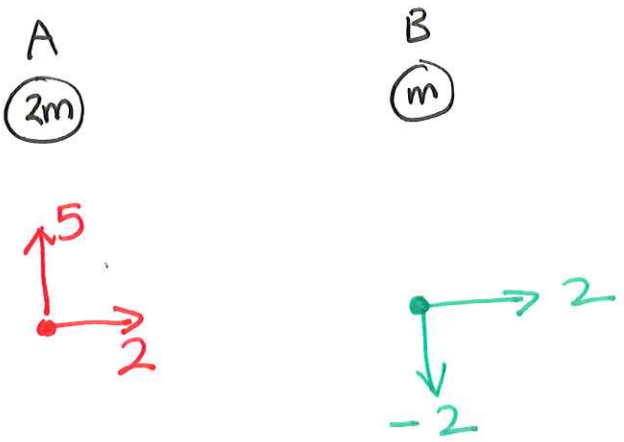
KE lost

$59.5 - 50.5 = \underline{\underline{9\ J}}$

Ex5C

Q14.

(a) Before



After



com (↔)

$2m(2) + m(2)$

$=$

$2m(3) + mB$

$4 + 2 = 6 + B$

$B = 0$

com (↕)

$2m(5) + m$

$=$

$2m(4) + m(A)$

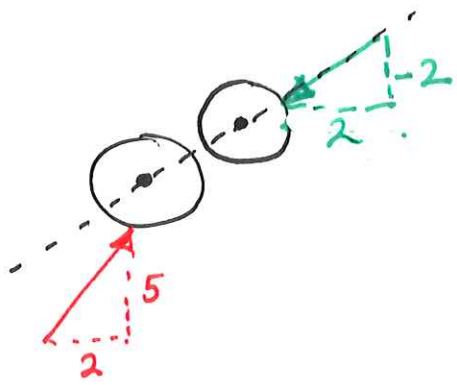
$10 - 2 = 8 + A$

$A = 0$

$V_B = 0i + 0j$

Ex5C

Q14
(b)



B is brought to a halt
∴ the centre line must be parallel to the original direction of B

Unit vector to the line of centres

$$\sqrt{2^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$

Unit vector parallel to line of centre $= \frac{1}{2\sqrt{2}}(2i - 2j) = \frac{1}{\sqrt{2}}(i - j)$

Speed of A <u>before</u> (in the direction of the line of centre)

$$\frac{1}{\sqrt{2}}(i - j).(2i + 5j)$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}.\begin{pmatrix}2\\5\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}(2 + -5) = \frac{-3}{\sqrt{2}}$$

Speed of A <u>after</u> (in the direction of the line of centre)

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}.\begin{pmatrix}3\\4\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}(3 + -4) = \frac{-1}{\sqrt{2}}$$

Ex5C

Q14

(b) conti

before

Speed of B (in direction of the line of centre)

$$\sqrt{2^2 + (-2)^2} = \sqrt{4}$$

$$= 2\sqrt{2}$$

after

Speed of B

0

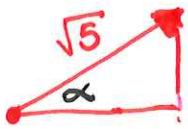
$$e = \frac{\frac{1}{\sqrt{2}}}{2\sqrt{2} - \frac{-3}{\sqrt{2}}}$$

$$e = \frac{\frac{1}{\sqrt{2}}}{\frac{7\sqrt{2}}{2}}$$

$$e = \frac{1}{7}$$

Ex5C
Q15.

A
(2)

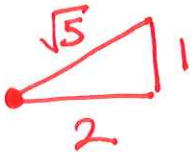


$\tan\alpha = \frac{1}{2}$

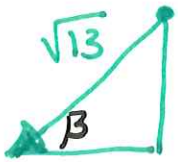
$1^2+2^2=\sqrt{5}$

$\sin\alpha = \frac{1}{\sqrt{5}}$

$\cos\alpha = \frac{2}{\sqrt{5}}$



B
(1)

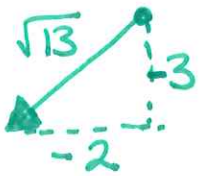


$\tan\beta = \frac{3}{2}$

$3^2+2^2=\sqrt{13}$

$\sin\beta = \frac{3}{\sqrt{13}}$

$\cos\beta = \frac{2}{\sqrt{13}}$



Ex5C

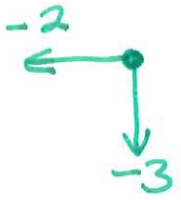
Q15.

conti

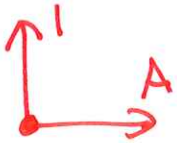
A (2) B (1)

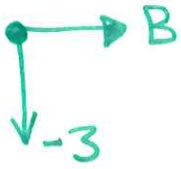
Before





After





com (↔)

$$(2)(2) + (-2)(1) = 2A + B$$

$$2 = 2A + B \quad (1)$$

e

$$\frac{1}{2} = \frac{B-A}{4}$$

$$2 = B - A$$

$$B = 2 + A \quad (2)$$

sub (2) into (1)

$$2 = 2A + 2 + A$$

$$2 = 3A + 2$$

$$0 = 3A$$

$$A = 0$$

$$B = 2$$

$V_A = \sqrt{1^2 + 0^2}$

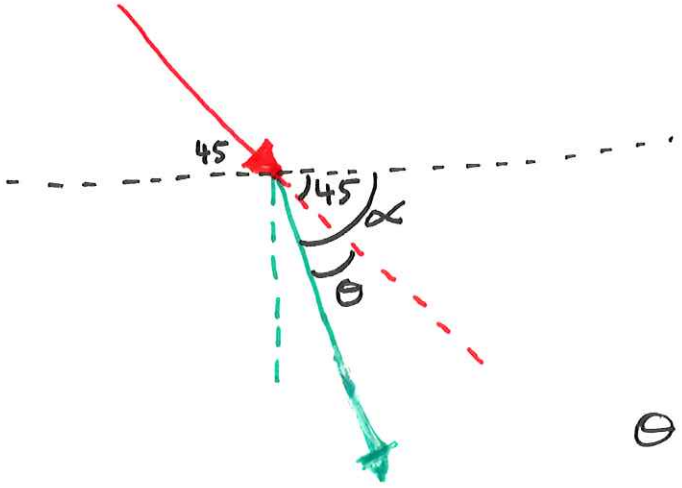
$= 1 \text{ ms}^{-1}$

$V_B = \sqrt{2^2 + (-3)^2}$

$= \sqrt{13} \text{ ms}^{-1}$

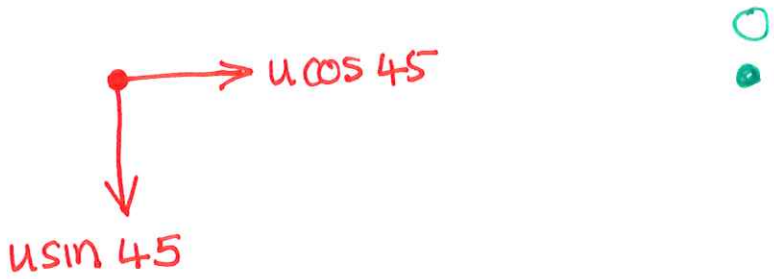
Ex5C

Q16.



$\theta = \alpha - 45$

Before



After



Com (↔)

$mu\cos 45 = Am + Bm$

$u\cos 45 = A + B$ (1)

e

$$e = \frac{B - A}{u\cos 45}$$

$eu\cos 45 = B - A$

$B = eu\cos 45 + A$ (2)

Ex 5C

Q16.

conti.

(i) Sub ② into ①

$$u\cos 45 = A + eu\cos 45 + A$$

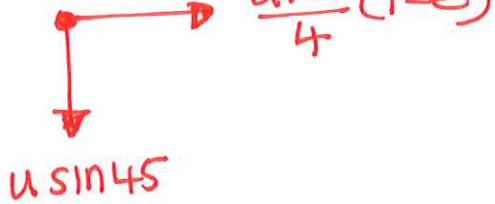
$$u\cos 45 - eu\cos 45 = 2A$$

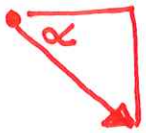
$$u\cos 45\,(1-e) = 2A$$

$$\frac{u\sqrt{2}}{2}(1-e) = 2A$$

$$A = \frac{u\sqrt{2}}{4}(1-e)$$

After





$$\tan\alpha = \frac{u\sin 45}{\frac{u\sqrt{2}}{4}(1-e)}$$

$$\tan\alpha = \frac{\frac{u\sqrt{2}}{2}}{\frac{u\sqrt{2}}{4}(1-e)}$$

$$\tan\alpha = \frac{2}{1-e}$$

Ex5C

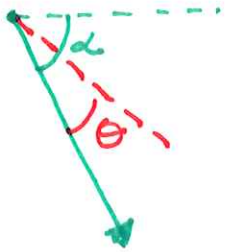
Q16

conti

(ii)

Addition Rule

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$



$\theta = \alpha - 45$

$$\tan(\alpha - 45) = \frac{\tan\alpha - \tan 45}{1 + \tan\alpha \tan 45}$$

sub $\tan\alpha = \frac{2}{1-e}$

$$\tan(\alpha - 45) = \frac{\frac{2}{1-e} - 1}{1 + \frac{2}{1-e}}$$

$$\tan(\alpha - 45) = \frac{\frac{1+e}{1-e}}{\frac{3-e}{1-e}}$$

$$\tan(\alpha - 45) = \frac{1+e}{3-e}$$

$$\tan\theta = \frac{1+e}{3-e}$$