Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

AS and A level Further Mathematics
Core Pure Mathematics

Practice Paper
Matrix algebra (part 2)

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (i)

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(*a*) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents an enlargement, scale factor –2, with centre the origin.

(*b*) Write down the matrix **B**.

(1)

(ii)

$$\mathbf{M} = \begin{pmatrix} 3 & k \\ -2 & 3 \end{pmatrix}, \qquad \text{where } k \text{ is a positive constant.}$$

Triangle T has an area of 16 square units.

Triangle T is transformed onto the triangle T' by the transformation represented by the matrix **M**.

Given that the area of the triangle T' is 224 square units, find the value of k.

(3)

(Total 6 marks)

2. $$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

The transformation represented by **B** followed by the transformation represented by **A** is equivalent to the transformation represented by **P**.

(*a*) Find the matrix **P**.

(2)

Triangle T is transformed to the triangle T' by the transformation represented by **P**.

Given that the area of triangle T' is 24 square units,

(*b*) find the area of triangle T.

(3)

Triangle T' is transformed to the original triangle T by the matrix represented by **Q**.

(*c*) Find the matrix **Q**.

(2)

(Total 7 marks)

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3. $$\mathbf{X} = \begin{pmatrix} 1 & a \\ 3 & 2 \end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(*a*) Find the value of a for which the matrix **X** is singular.

(2)

$$\mathbf{Y} = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}.$$

(*b*) Find $\mathbf{Y}^{-1}$.

(2)

The transformation represented by **Y** maps the point A onto the point B.

Given that B has coordinates $(1 - \lambda, 7\lambda - 2)$, where λ is a constant,

(*c*) find, in terms of λ, the coordinates of point A.

(4)

(Total 8 marks)

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4. (i) $$\mathbf{A} = \begin{pmatrix} 5k & 3k-1 \\ -3 & k+1 \end{pmatrix}, \quad \text{where } k \text{ is a real constant.}$$

Given that **A** is a singular matrix, find the possible values of k.

(4)

(ii) $$\mathbf{B} = \begin{pmatrix} 10 & 5 \\ -3 & 3 \end{pmatrix}$$

A triangle T is transformed onto a triangle T' by the transformation represented by the matrix **B**.

The vertices of triangle T' have coordinates (0, 0), (−20, 6) and ($10c$, $6c$), where c is a positive constant.

The area of triangle T' is 135 square units.

(a) Find the matrix $\mathbf{B}^{-1}$.

(2)

(b) Find the coordinates of the vertices of the triangle T, in terms of c where necessary.

(3)

(c) Find the value of c.

(3)

(Total 12 marks)

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5. (i) In each of the following cases, find a 2 × 2 matrix that represents

(a) a reflection in the line $y = -x$,

(b) a rotation of 135° anticlockwise about (0, 0),

(c) a reflection in the line $y = -x$ followed by a rotation of 135° anticlockwise about (0, 0).

(4)

(ii) The triangle T has vertices at the points (1, k), (3, 0) and (11, 0), where k is a constant. Triangle T is transformed onto the triangle T' by the matrix

$$\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix}$$

Given that the area of triangle T' is 364 square units, find the value of k.

(6)

(Total 10 marks)

6.

$$\mathbf{A} = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

and $\mathbf{I}$ is the 2×2 identity matrix.

(*a*) Prove that

$$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I}$$

(2)

(*b*) Hence show that

$$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I})$$

(2)

The transformation represented by $\mathbf{A}$ maps the point P onto the point Q.

Given that Q has coordinates $(2k + 8, -2k - 5)$, where k is a constant,

(*c*) find, in terms of k, the coordinates of P.

(4)

(Total 8 marks)

7.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}.$$

(*a*) Show that $\mathbf{A}$ is non-singular.

(2)

(*b*) Find $\mathbf{B}$ such that $\mathbf{BA}^2 = \mathbf{A}$.

(4)

(Total 6 marks)

8.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(*a*) Find det $\mathbf{A}$.

(1)

(*b*) Find $\mathbf{A}^{-1}$.

(2)

The triangle R is transformed to the triangle S by the matrix $\mathbf{A}$.

Given that the area of triangle S is 72 square units,

(*c*) find the area of triangle R.

(2)

The triangle S has vertices at the points (0, 4), (8, 16) and (12, 4).

(*d*) Find the coordinates of the vertices of R.

(4)

(Total 9 marks)

9. $$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}.$$

(*a*) Find det **M**.

(1)

The transformation represented by **M** maps the point $S(2a - 7, a - 1)$, where a is a constant, onto the point $S'(25, -14)$.

(*b*) Find the value of a.

(3)

The point R has coordinates (6, 0).

Given that O is the origin,

(*c*) find the area of triangle ORS.

(2)

Triangle ORS is mapped onto triangle $OR'S'$ by the transformation represented by **M**.

(*d*) Find the area of triangle $OR'S'$.

(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(*e*) describe fully the single geometrical transformation represented by **A**.

(2)

The transformation represented by **A** followed by the transformation represented by **B** is equivalent to the transformation represented by **M**.

(*f*) Find **B**.

(4)

(Total 14 marks)

TOTAL FOR PAPER: 100 MARKS