Name:

Level 2 Further Maths

Stationary Points

Corbettmaths

Ensure you have: Pencil or pen

Answers

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/

1. A curve has equation $y = x^2 - 4x + 21$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} = 2x - 4$

$2x - 4$

(2)

(b) Hence, find the coordinates of the minimum point.

$2x - 4 = 0$

$2x = 4$

$x = 2$

$y = 2^2 - 4 \times 2 + 21 = 17$

$(2, 17)$

(3)

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2. A curve has equation $y = 1 + x - x^2$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} = 1 - 2x$

$1 - 2x$

(2)

(b) Hence, find the coordinates of the maximum point.

$1 - 2x = 0$

$2x = 1$

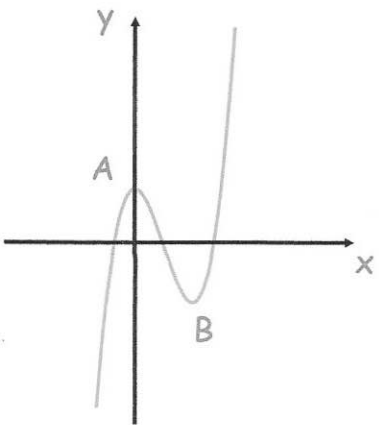
$x = \frac{1}{2}$

$y = 1 + 0.5 - 0.5^2 = 1.25$

$(\frac{1}{2}, \frac{5}{4})$

(3)

3. A curve has equation $y = x^3 - 3x^2 + 1$



Work out the coordinates of the stationary points, A and B.

$\frac{dy}{dx} = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$\therefore \quad x = 0 \quad \text{or} \quad x = 2$

$y = 1 \qquad y = -3$

$y = 0^3 - 3 \times 0^2 + 1 = 1$

$y = 2^3 - 3 \times 2^2 + 1 = -3$

A = (0, 1)

B = (2, -3)

(5)

4. $y = 2x^3 - 9x^2 + 12x - 9$

(a) Work out $\frac{dy}{dx}$

$\frac{dy}{dx} = 6x^2 - 18x + 12$

.....

(3)

(b) Hence, work out the coordinates of the stationary points of

$y = 2x^3 - 9x^2 + 12x - 9$

$\frac{dy}{dx} = 6x^2 - 18x + 12$

$0 = 6x^2 - 18x + 12$

$0 = x^2 - 3x + 2$

$0 = (x - 1)(x - 2)$

$x = 1$ or $x = 2$

$y = -4$ $y = -5$

$y = 2 - 9 + 12 - 9 = -4$

$y = 2 \times 2^3 - 9 \times 2^2 + 12 \times 2 - 9 = -5$

(1, -4) and (2, -5)

(3)

5. (a) Work out the stationary points on the curve $y = x^3 - 3x$

$\frac{dy}{dx} = 3x^2 - 3$

$0 = 3x^2 - 3$

$0 = 3(x^2 - 1)$

$0 = 3(x-1)(x+1)$

$x = 1$ or $x = -1$

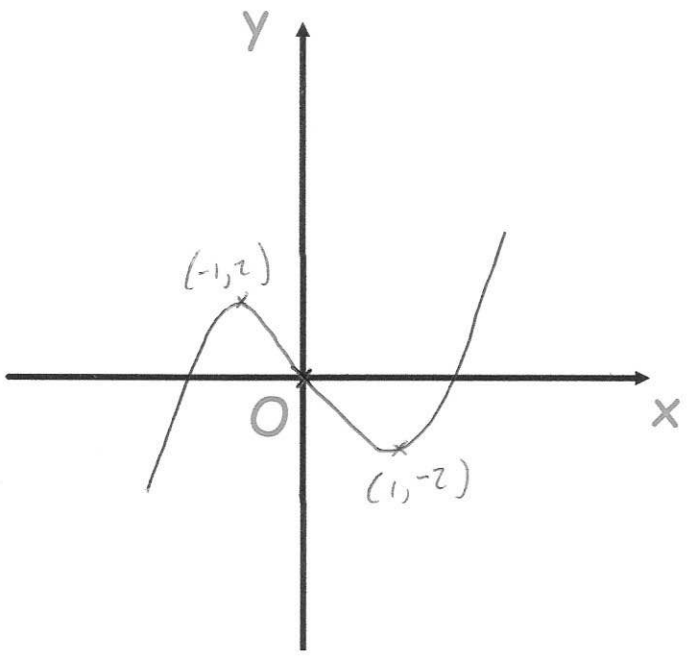
$y = 1^3 - 3 \times 1 = -2$

$y = (-1)^3 - 3 \times (-1) = 2$

(-1, 2) and (1, -2)

(4)

(b) Sketch $y = x^3 - 3x$



(3)

6. The curve C has equation $y = 3x^4 - 12x^3 + 2$

(a) Find $\frac{dy}{dx}$

$\frac{dy}{dx} = 12x^3 - 36x^2$

.....

(2)

(b) Find $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = 36x^2 - 72x$

.....

(1)

(c) Show that C has a stationary point when $x = 3$

$12(3)^3 - 36(3)^2$

$324 - 324 = 0$

$\therefore$ stationary point

(2)

(d) Determine the nature of this stationary point, giving a reason for your answer

$36(3)^2 - 72(3)$

$= 324 - 216$

$\therefore \frac{d^2y}{dx^2} > 0$, $\therefore$ it is a minimum

(2)

7. The curve C has equation $y = -x^3 + 12x^2 - 36x$

Work out the coordinates of any stationary point on this curve and determine their nature

$$\frac{dy}{dx} = -3x^2 + 24x - 36$$

$$0 = -3x^2 + 24x - 36$$

$$0 = 3x^2 - 24x + 36$$

$$0 = x^2 - 8x + 12$$

$$0 = (x - 6)(x - 2)$$

$$x = 6 \quad \text{or} \quad x = 2$$

$$y = -2^3 + 12(2)^2 - 36(2) = -32$$

$$y = -6^3 + 12(6)^2 - 36(6) = 0$$

$(2, -32)$ & $(6, 0)$

$$\frac{d^2y}{dx^2} = -6x + 24$$

when $x = 2$ $\frac{d^2y}{dx^2} = 12$ since $\frac{d^2y}{dx^2} > 0$, $(2, -32)$ is a minimum

when $x = 6$ $\frac{d^2y}{dx^2} = -12$ since $\frac{d^2y}{dx^2} < 0$, $(6, 0)$ is a maximum

(6)

8. The equation of a curve is $y = x^3 - \frac{1}{2}x^2 + ax + 1$ where a is a constant

The curve has a maximum point at $\left(-\frac{2}{3}, \frac{49}{27}\right)$

The curve has a minimum point at $(1, -0.5)$

Work out the value of a

$$\frac{dy}{dx} = 3x^2 - x + a$$

$$0 = 3x^2 - x + a$$

since $x = 1$ & $x = -\frac{2}{3}$

$$(x-1)(3x+2) = 0$$

$$3x^2 + 2x - 3x - 2 = 0$$

$$3x^2 - x - 2 = 0$$

$$a = -2$$

.....

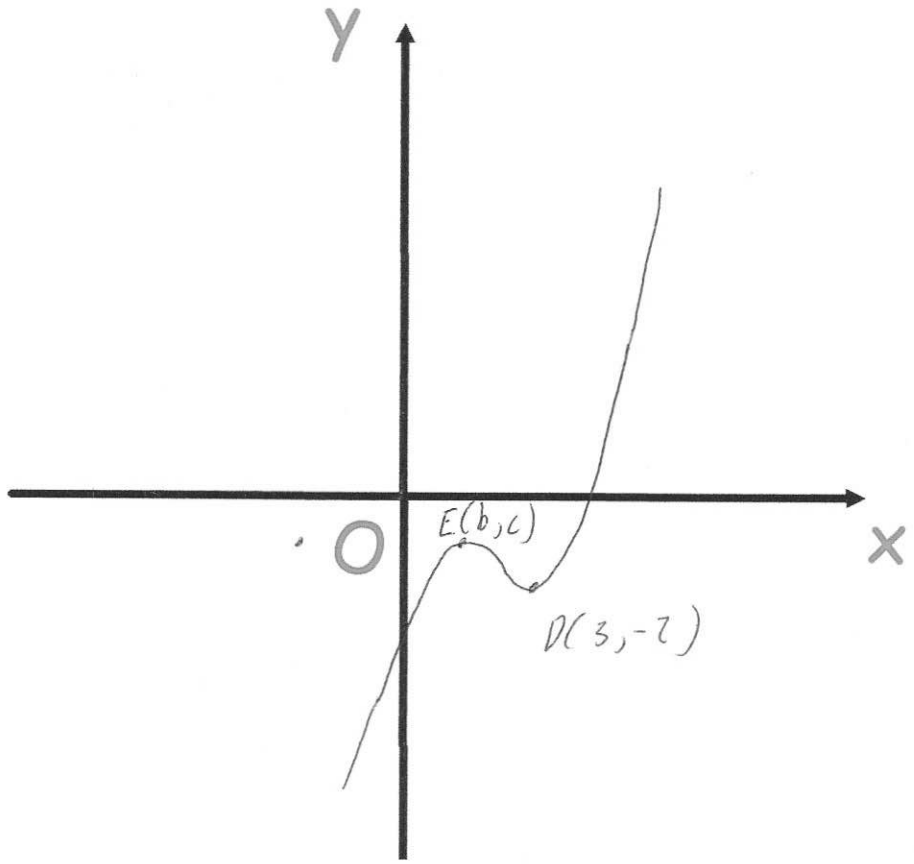
(6)

9. y = f(x) has exactly two stationary points.

The stationary points are

a minimum at D $(3, -2)$ $0 < b < 3$
a maximum at E (b, c) where and $-2 < c < 0$

Sketch the curve and label D and E.



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