Dreams Pack

Maths Crib Sheets

Applied Year 2

THE STUFF
THAT DREAMS
ARE
MADE OF

Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

Statistics

Probability

$\mathrm{P}(A') = 1 - \mathrm{P}(A)$

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A)$

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(B \mid A)\mathrm{P}(A)}{\mathrm{P}(B \mid A)\mathrm{P}(A) + \mathrm{P}(B \mid A')\mathrm{P}(A')}$$

For independent events A and B,

$\mathrm{P}(B \mid A) = \mathrm{P}(B)$
$\mathrm{P}(A \mid B) = \mathrm{P}(A)$
$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$

Standard deviation

Standard deviation = √(Variance)

Interquartile range = $\mathrm{IQR} = \mathrm{Q}_3 - \mathrm{Q}_1$

For a set of n values $x_1, x_2, \ldots x_i, \ldots x_n$

$$\mathrm{S}_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

Standard deviation = $\sqrt{\dfrac{\mathrm{S}_{xx}}{n}}$ or $\sqrt{\dfrac{\sum x^2}{n} - \bar{x}^2}$

Discrete distributions

Distribution of X	$\mathrm{P}(X = x)$	Mean	Variance
Binomial $\mathrm{B}(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$

Sampling distributions

For a random sample of n observations from $\mathrm{N}(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0, 1)$$

Statistical Tables

Binomial Cumulative Distribution Function

The tabulated value is $P(X \leqslant x)$, where X has a binomial distribution with index n and parameter p.

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 5, x = 0$	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0312
1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000
3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
$n = 6, x = 0$	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094
2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438
3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563
4	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906
5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
$n = 7, x = 0$	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375
6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922
$n = 8, x = 0$	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367
5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
$n = 9, x = 0$	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898
3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
$n = 10, x = 0$	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 12, x = 0$	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002
1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032
2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193
3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730
4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938
5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872
6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128
7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062
8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270
9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
$n = 15, x = 0$	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000
1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005
2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037
3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176
4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592
5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509
6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036
7	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000
8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964
9	1.0000	1.0000	1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9937	0.9824
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$n = 20, x = 0$	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000
1	0.7358	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000
2	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002
3	0.9841	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013
4	0.9974	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059
5	0.9997	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207
6	1.0000	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577
7	1.0000	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316
8	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517
9	1.0000	1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119
10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881
11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483
12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	0.8684
13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9935	0.9786	0.9423
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9936	0.9793
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 40, x = 0$	0.1285	0.0148	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.3991	0.0805	0.0121	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.6767	0.2228	0.0486	0.0079	0.0010	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.8619	0.4231	0.1302	0.0285	0.0047	0.0006	0.0001	0.0000	0.0000	0.0000
4	0.9520	0.6290	0.2633	0.0759	0.0160	0.0026	0.0003	0.0000	0.0000	0.0000
5	0.9861	0.7937	0.4325	0.1613	0.0433	0.0086	0.0013	0.0001	0.0000	0.0000
6	0.9966	0.9005	0.6067	0.2859	0.0962	0.0238	0.0044	0.0006	0.0001	0.0000
7	0.9993	0.9581	0.7559	0.4371	0.1820	0.0553	0.0124	0.0021	0.0002	0.0000
8	0.9999	0.9845	0.8646	0.5931	0.2998	0.1110	0.0303	0.0061	0.0009	0.0001
9	1.0000	0.9949	0.9328	0.7318	0.4395	0.1959	0.0644	0.0156	0.0027	0.0003
10	1.0000	0.9985	0.9701	0.8392	0.5839	0.3087	0.1215	0.0352	0.0074	0.0011
11	1.0000	0.9996	0.9880	0.9125	0.7151	0.4406	0.2053	0.0709	0.0179	0.0032
12	1.0000	0.9999	0.9957	0.9568	0.8209	0.5772	0.3143	0.1285	0.0386	0.0083
13	1.0000	1.0000	0.9986	0.9806	0.8968	0.7032	0.4408	0.2112	0.0751	0.0192
14	1.0000	1.0000	0.9996	0.9921	0.9456	0.8074	0.5721	0.3174	0.1326	0.0403
15	1.0000	1.0000	0.9999	0.9971	0.9738	0.8849	0.6946	0.4402	0.2142	0.0769
16	1.0000	1.0000	1.0000	0.9990	0.9884	0.9367	0.7978	0.5681	0.3185	0.1341
17	1.0000	1.0000	1.0000	0.9997	0.9953	0.9680	0.8761	0.6885	0.4391	0.2148
18	1.0000	1.0000	1.0000	0.9999	0.9983	0.9852	0.9301	0.7911	0.5651	0.3179
19	1.0000	1.0000	1.0000	1.0000	0.9994	0.9937	0.9637	0.8702	0.6844	0.4373
20	1.0000	1.0000	1.0000	1.0000	0.9998	0.9976	0.9827	0.9256	0.7870	0.5627
21	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991	0.9925	0.9608	0.8669	0.6821
22	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9970	0.9811	0.9233	0.7852
23	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9989	0.9917	0.9595	0.8659
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9966	0.9804	0.9231
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9988	0.9914	0.9597
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9966	0.9808
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9988	0.9917
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9968
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9989
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$p =$	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
$n = 50, x = 0$	0.0769	0.0052	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.2794	0.0338	0.0029	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.5405	0.1117	0.0142	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.7604	0.2503	0.0460	0.0057	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.8964	0.4312	0.1121	0.0185	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000
5	0.9622	0.6161	0.2194	0.0480	0.0070	0.0007	0.0001	0.0000	0.0000	0.0000
6	0.9882	0.7702	0.3613	0.1034	0.0194	0.0025	0.0002	0.0000	0.0000	0.0000
7	0.9968	0.8779	0.5188	0.1904	0.0453	0.0073	0.0008	0.0001	0.0000	0.0000
8	0.9992	0.9421	0.6681	0.3073	0.0916	0.0183	0.0025	0.0002	0.0000	0.0000
9	0.9998	0.9755	0.7911	0.4437	0.1637	0.0402	0.0067	0.0008	0.0001	0.0000
10	1.0000	0.9906	0.8801	0.5836	0.2622	0.0789	0.0160	0.0022	0.0002	0.0000
11	1.0000	0.9968	0.9372	0.7107	0.3816	0.1390	0.0342	0.0057	0.0006	0.0000
12	1.0000	0.9990	0.9699	0.8139	0.5110	0.2229	0.0661	0.0133	0.0018	0.0002
13	1.0000	0.9997	0.9868	0.8894	0.6370	0.3279	0.1163	0.0280	0.0045	0.0005
14	1.0000	0.9999	0.9947	0.9393	0.7481	0.4468	0.1878	0.0540	0.0104	0.0013
15	1.0000	1.0000	0.9981	0.9692	0.8369	0.5692	0.2801	0.0955	0.0220	0.0033
16	1.0000	1.0000	0.9993	0.9856	0.9017	0.6839	0.3889	0.1561	0.0427	0.0077
17	1.0000	1.0000	0.9998	0.9937	0.9449	0.7822	0.5060	0.2369	0.0765	0.0164
18	1.0000	1.0000	0.9999	0.9975	0.9713	0.8594	0.6216	0.3356	0.1273	0.0325
19	1.0000	1.0000	1.0000	0.9991	0.9861	0.9152	0.7264	0.4465	0.1974	0.0595
20	1.0000	1.0000	1.0000	0.9997	0.9937	0.9522	0.8139	0.5610	0.2862	0.1013
21	1.0000	1.0000	1.0000	0.9999	0.9974	0.9749	0.8813	0.6701	0.3900	0.1611
22	1.0000	1.0000	1.0000	1.0000	0.9990	0.9877	0.9290	0.7660	0.5019	0.2399
23	1.0000	1.0000	1.0000	1.0000	0.9996	0.9944	0.9604	0.8438	0.6134	0.3359
24	1.0000	1.0000	1.0000	1.0000	0.9999	0.9976	0.9793	0.9022	0.7160	0.4439
25	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991	0.9900	0.9427	0.8034	0.5561
26	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9955	0.9686	0.8721	0.6641
27	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9981	0.9840	0.9220	0.7601
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993	0.9924	0.9556	0.8389
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9966	0.9765	0.8987
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9986	0.9884	0.9405
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9947	0.9675
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9978	0.9836
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9923
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9967
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9987
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Critical Values for Correlation Coefficients

These tables concern tests of the hypothesis that a population correlation coefficient ρ is 0. The values in the tables are the minimum values which need to be reached by a sample correlation coefficient in order to be significant at the level shown, on a one-tailed test.

Product Moment Coefficient					Sample size, n	Spearman's Coefficient		
Level						Level		
0.10	0.05	0.025	0.01	0.005		0.05	0.025	0.01
0.8000	0.9000	0.9500	0.9800	0.9900	4	1.0000	–	–
0.6870	0.8054	0.8783	0.9343	0.9587	5	0.9000	1.0000	1.0000
0.6084	0.7293	0.8114	0.8822	0.9172	6	0.8286	0.8857	0.9429
0.5509	0.6694	0.7545	0.8329	0.8745	7	0.7143	0.7857	0.8929
0.5067	0.6215	0.7067	0.7887	0.8343	8	0.6429	0.7381	0.8333
0.4716	0.5822	0.6664	0.7498	0.7977	9	0.6000	0.7000	0.7833
0.4428	0.5494	0.6319	0.7155	0.7646	10	0.5636	0.6485	0.7455
0.4187	0.5214	0.6021	0.6851	0.7348	11	0.5364	0.6182	0.7091
0.3981	0.4973	0.5760	0.6581	0.7079	12	0.5035	0.5874	0.6783
0.3802	0.4762	0.5529	0.6339	0.6835	13	0.4835	0.5604	0.6484
0.3646	0.4575	0.5324	0.6120	0.6614	14	0.4637	0.5385	0.6264
0.3507	0.4409	0.5140	0.5923	0.6411	15	0.4464	0.5214	0.6036
0.3383	0.4259	0.4973	0.5742	0.6226	16	0.4294	0.5029	0.5824
0.3271	0.4124	0.4821	0.5577	0.6055	17	0.4142	0.4877	0.5662
0.3170	0.4000	0.4683	0.5425	0.5897	18	0.4014	0.4716	0.5501
0.3077	0.3887	0.4555	0.5285	0.5751	19	0.3912	0.4596	0.5351
0.2992	0.3783	0.4438	0.5155	0.5614	20	0.3805	0.4466	0.5218
0.2914	0.3687	0.4329	0.5034	0.5487	21	0.3701	0.4364	0.5091
0.2841	0.3598	0.4227	0.4921	0.5368	22	0.3608	0.4252	0.4975
0.2774	0.3515	0.4133	0.4815	0.5256	23	0.3528	0.4160	0.4862
0.2711	0.3438	0.4044	0.4716	0.5151	24	0.3443	0.4070	0.4757
0.2653	0.3365	0.3961	0.4622	0.5052	25	0.3369	0.3977	0.4662
0.2598	0.3297	0.3882	0.4534	0.4958	26	0.3306	0.3901	0.4571
0.2546	0.3233	0.3809	0.4451	0.4869	27	0.3242	0.3828	0.4487
0.2497	0.3172	0.3739	0.4372	0.4785	28	0.3180	0.3755	0.4401
0.2451	0.3115	0.3673	0.4297	0.4705	29	0.3118	0.3685	0.4325
0.2407	0.3061	0.3610	0.4226	0.4629	30	0.3063	0.3624	0.4251
0.2070	0.2638	0.3120	0.3665	0.4026	40	0.2640	0.3128	0.3681
0.1843	0.2353	0.2787	0.3281	0.3610	50	0.2353	0.2791	0.3293
0.1678	0.2144	0.2542	0.2997	0.3301	60	0.2144	0.2545	0.3005
0.1550	0.1982	0.2352	0.2776	0.3060	70	0.1982	0.2354	0.2782
0.1448	0.1852	0.2199	0.2597	0.2864	80	0.1852	0.2201	0.2602
0.1364	0.1745	0.2072	0.2449	0.2702	90	0.1745	0.2074	0.2453
0.1292	0.1654	0.1966	0.2324	0.2565	100	0.1654	0.1967	0.2327

Regression (Non-Linear)

Non-Linear Model

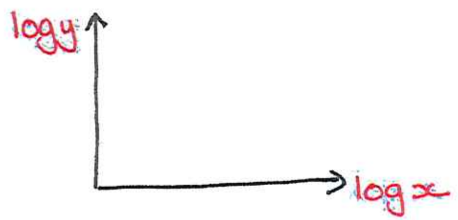
$y = ax^n$

Code to Linear Model

$\log y = \log ax^n$

$\log y = \log a + \log x^n$

$\log y = \log a + n \log x$



Compare with y=mx+c

$\log y = n \log x + \log a$

$y = m \quad x \quad + \quad c$

gradient = n

y-intercept = $\log a$

Example:

Find the exponential equation that can model the data in the form $y = ax^n$ given coding to linear model gives

$y = 0\cdot0792x - 0\cdot2215$

code to linear model

$y = ax^n$

$\log y = \log a + n \log x$

y-intercept

$\log a = -0\cdot2215$

$\log_{10} a = -0.2215$

$10^{-0.2215} = a$

$a = 0.600$ (3sf)

gradient

$n = 0.0792$

$y = (0.6)(x)^{0.0792}$

Non-Linear Model

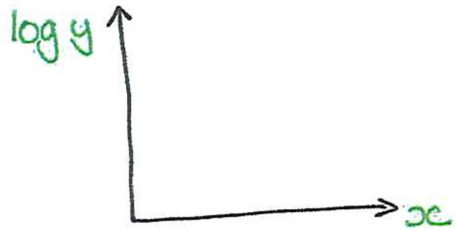
$y = ab^x$

Code to Linear Model

$\log y = \log ab^x$

$\log y = \log a + \log b^x$

$\log y = \log a + x \log b$



Compare with y=mx+c

$\log y = \log b \, x + \log a$

$y = m \quad x \quad + \quad c$

gradient = $\log b$

y-intercept = $\log a$

Example:

Find the exponential equation that can model the data in the form $g = ab^t$ given coding to linear model gives

$y = 0\cdot0792x - 0\cdot2215$

code to linear model

$g = ab^t$

$\log g = \log a + t \log b$

y-intercept

$\log a = -0\cdot2215$

$\log_{10} a = -0.2215$

$10^{-0.2215} = a$

$a = 0.600$ (3sf)

gradient

$\log b = 0.0792$

$\log_{10} b = 0.0792$

$10^{0.0792} = b$

$b = 1.20$ (3sf)

$g = (0.6)(1.2)^t$

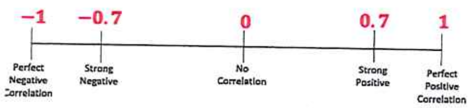
Year 2 Applied Chp 1 – Regression and Correlation

PMCC

Product Moment Correlation Coefficient is the Measurement of Linear Correlation

$$\text{PMCC} = r$$

$$-1 \le r \le 1$$



Instructions for **Casio ClassWiz:**

6: Statistics	Press MODE then select 'Statistics'.
$y = a + bx$	We want to measure **linear** correlation, so select $y = a + bx$
Data Entry	
PMCC	Press OPTN then choose "Regression Calc" to obtain r.

(You should obtain $r = 0.868$)

Hypothesis Testing for Correlation

Step 1) Write a hypothesis:

$H_0:\ \rho = 0$ $H_1:\ \rho > 0$	$H_0:\ \rho = 0$ $H_1:\ \rho < 0$	$H_0:\ \rho = 0$ $H_1:\ \rho \neq 0$
H_0: No correlaton *H_1: Positve correlation*	*H_0: No correlaton* *H_1: Negative correlation*	*H_0: No correlaton* *H_1: There is correlation*

r denotes the PMCC for a sample,
ρ denoted the PMCC for a whole population,
It is the Greek letter rho.

Step 2) Use a PMCC table to identify the minimum value needed to make the correlation statistically significant:

Product Moment Coefficient					
Level					Sample size, n
0.10	0.05	0.025	0.01	0.005	
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.5509	0.6694	0.7545	0.8329	0.8745	7
0.5067	0.6215	0.7067	0.7887	0.8343	8
0.4716	0.5822	0.6664	0.7498	0.7977	9
0.4428	0.5494	0.6319	0.7155	0.7646	10
0.4187	0.5214	0.6021	0.6851	0.7348	11
0.3981	0.4973	0.5760	0.6581	0.7079	12
0.3802	0.4762	0.5529	0.6339	0.6835	13
0.3646	0.4575	0.5324	0.6120	0.6614	14
0.3507	0.4409	0.5140	0.5923	0.6411	15
0.3383	0.4259	0.4973	0.5742	0.6226	16
0.3271	0.4124	0.4821	0.5577	0.6055	17
0.3170	0.4000	0.4683	0.5425	0.5897	18
0.3077	0.3887	0.4555	0.5285	0.5751	19
0.2992	0.3783	0.4438	0.5155	0.5614	20
0.2914	0.3687	0.4329	0.5034	0.5487	21
0.2841	0.3598	0.4227	0.4921	0.5368	22
0.2774	0.3515	0.4133	0.4815	0.5256	23
0.2711	0.3438	0.4044	0.4716	0.5151	24
0.2653	0.3365	0.3961	0.4622	0.5052	25
0.2598	0.3297	0.3882	0.4534	0.4958	26
0.2546	0.3233	0.3809	0.4451	0.4869	27
0.2497	0.3172	0.3739	0.4372	0.4785	28
0.2451	0.3115	0.3673	0.4297	0.4705	29
0.2407	0.3061	0.3610	0.4226	0.4629	30
0.2070	0.2638	0.3120	0.3665	0.4026	40
0.1843	0.2353	0.2787	0.3281	0.3610	50
0.1678	0.2144	0.2542	0.2997	0.3301	60
0.1550	0.1982	0.2352	0.2776	0.3060	70
0.1448	0.1852	0.2199	0.2597	0.2864	80
0.1364	0.1745	0.2072	0.2449	0.2702	90
0.1292	0.1654	0.1966	0.2324	0.2565	100

Step 3) Accept or Reject

$H_0:\ \rho = 0$ $H_1:\ \rho > 0$	Table PMCC: **0.35** Sample PMCC: **0.25**	Accept H_0
$H_0:\ \rho = 0$ $H_1:\ \rho < 0$	Table PMCC: **-0.5** Sample PMCC: **-0.7**	Reject H_0 Accept H_1
$H_0:\ \rho = 0$ $H_1:\ \rho \neq 0$	Table PMCC: **0.4** Sample PMCC: **0.5**	Reject H_0 Accept H_1

Example:
A scientist takes 30 observations of the masses of two reactants in an experiment.

She calculates a product moment correlation coefficient of r=−0.45.

The scientist believes there is no correlation between the masses of the two reactants.

Test at the 10% level of significance, the scientist's claim, stating your hypotheses clearly.

Two-tailed so 5% at each tail.

Product Moment Coefficient					
Level					Sample size, n
0.10	0.05	0.025	0.01	0.005	
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.2992	0.3783	0.4438	0.5155	0.5614	20
0.2914	0.3687	0.4329	0.5034	0.5487	21
0.2841	0.3598	0.4227	0.4921	0.5368	22
0.2774	0.3515	0.4133	0.4815	0.5256	23
0.2711	0.3438	0.4044	0.4716	0.5151	24
0.2653	0.3365	0.3961	0.4622	0.5052	25
0.2598	0.3297	0.3882	0.4534	0.4958	26
0.2546	0.3233	0.3809	0.4451	0.4869	27
0.2497	0.3172	0.3739	0.4372	0.4785	28
0.2451	0.3115	0.3673	0.4297	0.4705	29
0.2407	0.3061	0.3610	0.4226	0.4629	30
0.2070	0.2638	0.3120	0.3665	0.4026	40
0.1843	0.2353	0.2787	0.3281	0.3610	50
0.1678	0.2144	0.2542	0.2997	0.3301	60

$$H_0: \rho = 0$$
$$H_1: \rho \neq 0$$

Sample size = 30

Critical value at 5% significance: 0.306

$$-0.45 < -0.3061$$

Accept H_1

There is evidence, that there is a correlation between the masses of the two reactants.

Year 2 Applied Chp 2 - Conditional Probability

Set Notation

$P(A)$ = Probability of A

$P(A')$ = Probabiliy of Not A

$P(A \cup B)$ = Probability of A or B

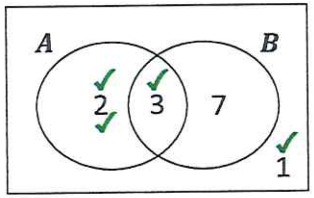
$P(A \cap B)$ = Probability of A and B

Venn Diagrams

$P(A$ **or** $Not\ B)$

Tick A

Tick Not B



$$P(A \textbf{ or } Not\ B) = \frac{6}{13}$$

A **or** B = Include ✓ **or** ✓✓

A **and** B = Include Only ✓✓

Probability Laws

If A and B are **mutually exclusive** then:

$$P(A \cup B) = P(A) + P(B)$$

If A and B are **independent** then:

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional Probability

$(B|A)$

The '|' symbol reads **"given that"**

$P(B|A)$

Find the probability of event B given that A has happened.

Example

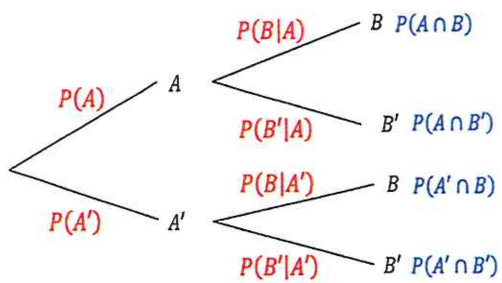
	French	Spanish
Girls	14	38
Boys	26	22

Determine the $P(G|S)$

Step 1: Highlight Spanish $\frac{\quad}{60}$

Step 2: Highlight Girls of what you have highlighted. $\frac{38}{60}$

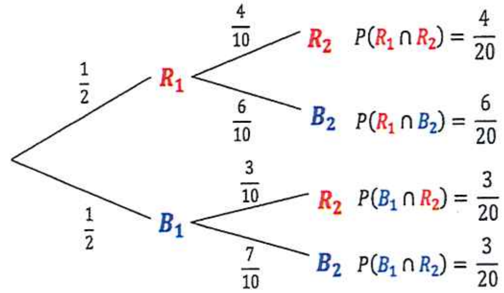
Probability Trees



You have two bags, the first with 5 red balls and 5 blue balls, and the second with 3 red balls and 6 blue balls.

You first pick a ball from the first bag, and place it in the second.

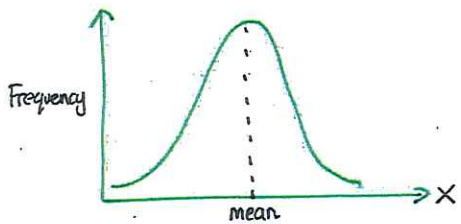
You then pick a ball from the second bag. Draw a tree diagram.



Year 2 Applied Chp 3 - Normal Distribution

Normal Distribution

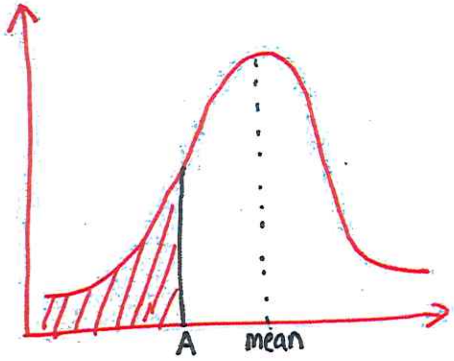
$X \sim N(\mu, \sigma^2)$



Normal CD

(Find the Area)

Find $P(X < A)$

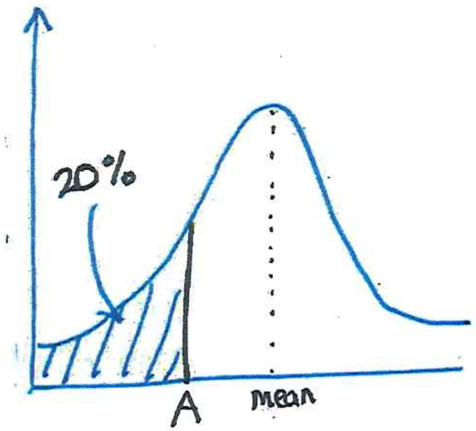


Normal CD
Lower: -10000000000
Upper: A
μ: mean
σ: standard deviation

Inverse Normal

(Find the Boarderline)

$P(X < A) = 0{\cdot}2$



Inverse Normal
Area: 0.2
μ: mean
σ: standard deviation

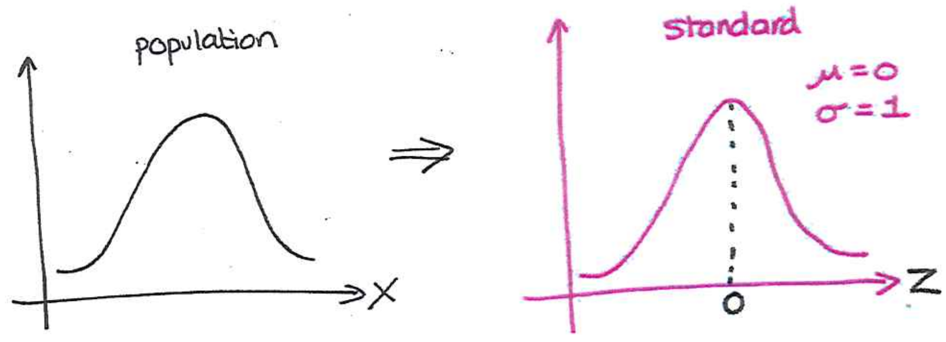
$P(X > A) = 0{\cdot}3$

has the same as boarder as

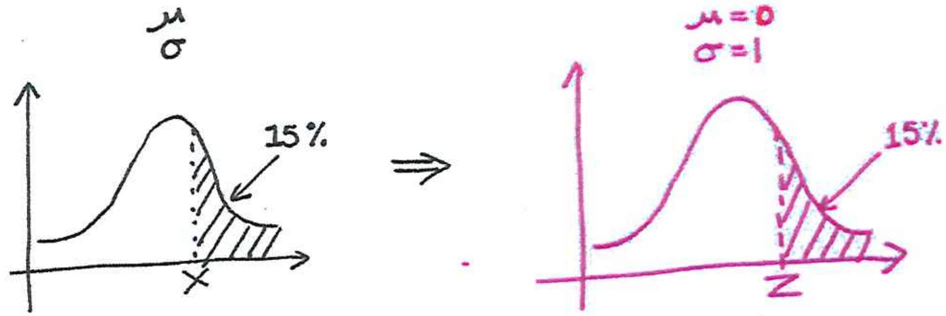
$P(X < A) = 0{\cdot}7$

Standard Normal

Missing mean or standard devaiton then convert the Population Distribution to a Standard Distribution



Convert



then use

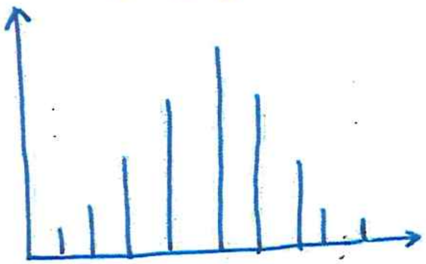
$$Z = \frac{X - \mu}{\sigma}$$

Year 2 Applied Chp 3 – Normal Distribution

Approximating Binomial Distribution

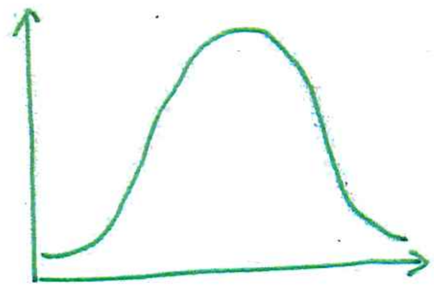
Binomial Distribution

$X \sim B(n, p)$

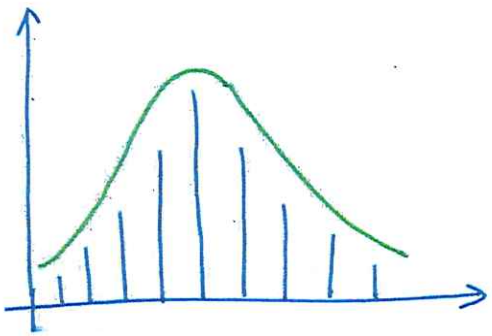


Normal Distribution

$X \sim N(\mu, \sigma^2)$



n is large
p is close to 0.5
then a
binomial distribution
can be approximated by a
normal distribution



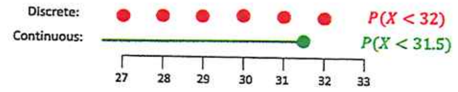
Approximating

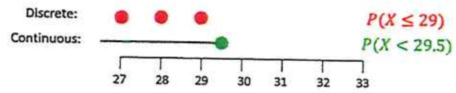
$Y \sim N(np, np(1-p))$

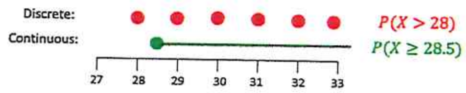
$Mean =$
np

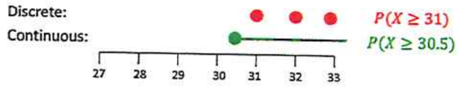
Standard Deviation $=$
$\sqrt{np(1-p)}$

Contitinuty Correction
(Discrete to Continuous)









Year 2 Applied Chp 3 - Normal Distribution

Hypothesis Testing on a Sample Mean

Population Distribution

$$X \sim N(\mu, \sigma^2)$$

Sample Distribution

$$X \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Sample size $= n$

Standard deviation $= \sqrt{\frac{\sigma^2}{n}}$

Hypothesis Testing Steps:

Step 1. State you hypotheses

Step 2. State you distribution for the sample

Step 3. Calculate the probability (from the normal distribution) of your sample result happening

Step 4. Compare this probability with the significant level

Step 5. Accept or reject your hypotheses H_o

Example:

A certain company sells fruit juice in cartons. The amount of juice in a carton has a normal distribution with a standard deviation of 3ml. The company claims that the mean amount of juice per carton, μ, is 60ml. A trading inspector has received complaints that the company is overstating the mean amount of juice per carton and wishes to investigate this complaint. The trading inspector takes a random sample of 16 cartons and finds that the mean amount of juice per carton is 59.1ml.

Using a 5% level of significance, and stating your hypotheses clearly, test whether or not there is evidence to uphold this complaint.

Step 1: State your hypotheses

$$H_0: \mu = 60$$
$$H_1: \mu < 60$$

Step 2: State your distribution for the sample

Population

$X \sim N(60, 3^2)$

$\sigma = \sqrt{\frac{3^2}{16}} = 0.75$

Sample (Size 16)

$\bar{X} \sim N(60, 0.75^2)$

Step 3: Calculate the probability of your sample result happening

Use NORMAL CD

Lower $= 0$

Upper $= 59.1$

$\mu = 60$

$\sigma = 0.75$

$$P(\bar{X} \leq 59.1) = 0.1151$$

Step 4. Compare this probability with the significant level

$$0.1151 > 0.05$$

Step 5. Accept or reject your hypotheses H_o

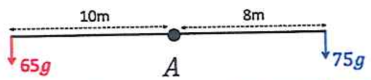
Insufficient evidence to reject H_0

You cannot conclude that the mean of the whole population is less than 60 ml.

Year 2 Applied Chp 4 - Moments

Moments

Moments = Force × Perpendicular Distance



Anti-Clockwise Moments
M(A)
$= 65g \times 10$
$= 6370$ ***Nm***

Clockwise Moments
M(A)
$= 75g \times 8$
$= 5880$ ***Nm***

For moment questions set up two equations.

Vertical Forces

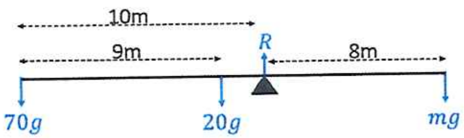
Forces Up = Forces Down

Moments

ACW Moments = CW Moments

(Choice a pivot point that gives you the least amount of work)

Uniform Rods



Moments (R)

$$ACW\ Moments = CW\ Moments$$

$$(20g \times 1) + (70g \times 10) = (mg \times 8)$$

$$720g = 8mg$$

$$m = 90 \text{ kg}$$

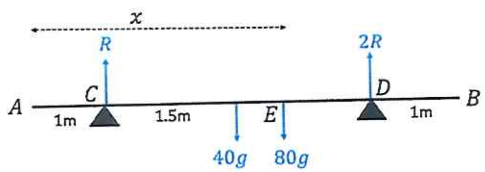
Vertical Direction

$$R = 70g + 20g + mg$$

$$R = 1764N$$

Non-Uniform Rods

(Always take the distance to the end of the rod)



Vertical Direction

$$R + 2R = 40g + 80g$$
$$3R = 120g$$
$$R = 40g$$

Moments (A)

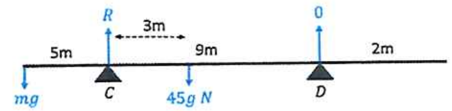
$$(40g \times 2.5) + (80g \times x) = (40g \times 1) + (80g \times 4)$$
$$x = \mathbf{3.25\ m}$$

Tilting (Supports)

If a beam is on the point of tilting about A, then there is no reactional force at B and the system is still in equilibrium.

Example:

A uniform beam AB, of mass 45kg and length 16m, rests horizontally on supports C and D where $AC = 5$ m and $CD = 9$ m. When Lewis stands at A, the beam is on the point of tilting about C. Determine Lewis' mass.



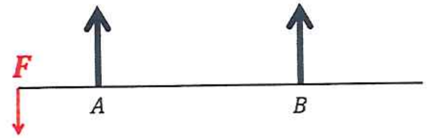
Moments (C)

$$mg \times 5 = 3 \times 45g$$

$$5mg = 135g$$

$$m = 27$$

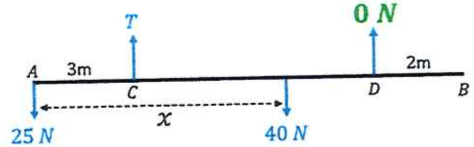
Tilting (Suspended Systems)



If a beam is on the point of tilting about A, then there is no reactional force at B and the system is still in equilibrium.

Example:

A non-uniform rod AB, of length 10 m and weight 40 N, is suspended from a pair of light cables attached to C and D where $AC = 3$ m and $BD = 2$ m. When a weight of 25 N is hung from A the rod is on the point of rotating. Find the distance of the centre of mass of the rod from A.



Moments (C)

$$25 \times 3 = 40(x - 3)$$

$$x = \mathbf{4.875}$$

Year 2 Applied Chp 5 – Forces and Friction

Formula

$$F = ma$$

F is the resultant force
(i.e. driving forces – resistance forces)

$$Friction = \mu R$$

$0 \le \mu \le 1$

Friction forces will **always** oppose the direction of motion.

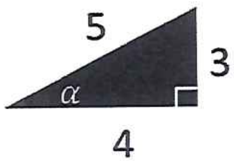
Solving

Set up two equations:

Resolve vertical to the slope

Resolve parallel to the slope

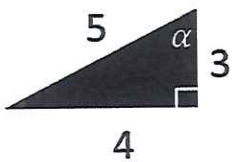
Calculating with Angles



$$\sin\alpha = \frac{opp}{hyp} = \frac{3}{5}$$

$$\cos\alpha = \frac{adj}{hyp} = \frac{4}{5}$$

$$\tan\alpha = \frac{opp}{adj} = \frac{3}{4}$$



$$\sin\alpha = \frac{opp}{hyp} = \frac{4}{5}$$

$$\cos\alpha = \frac{adj}{hyp} = \frac{3}{5}$$

$$\tan\alpha = \frac{opp}{adj} = \frac{4}{3}$$

Now simplify your calculations:

$$10 \times 2g\cos\alpha$$

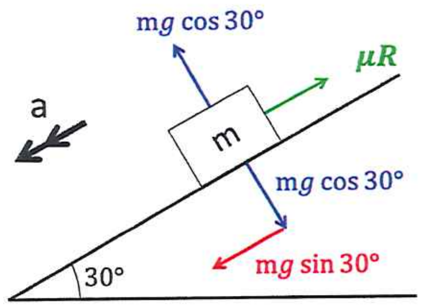
Becomes

$$20g\frac{3}{5}$$

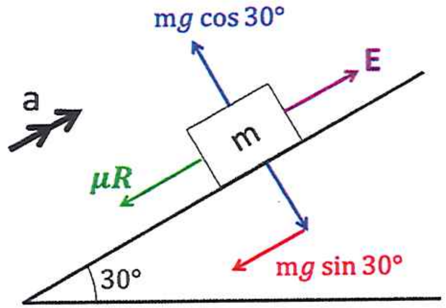
$$= 12g$$

Inclined Slopes

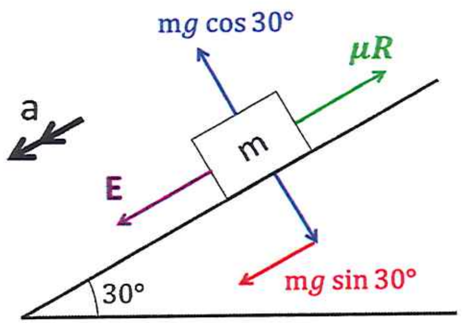
No External Forces
Sliding down



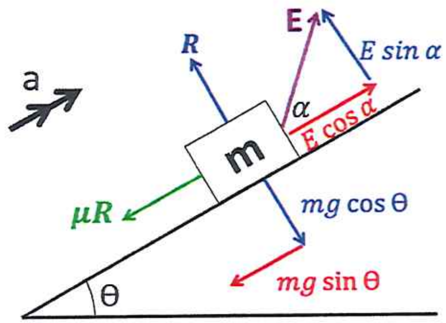
External Force - Parralel to Slope
Sliding Up



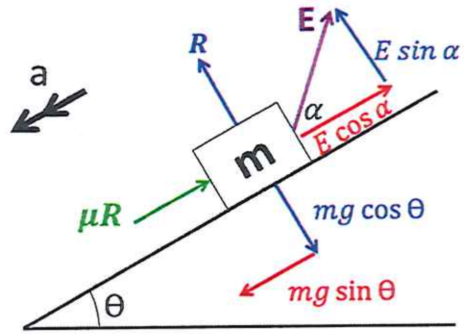
External Force - Parralel to Slope
Sliding Down



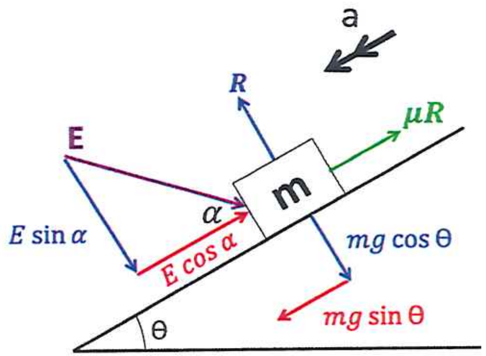
External Force - Not Parralel to Slope
Sliding Up



External Force - Not Parralel to Slope
Sliding Down



External Force - Not Parralel to Slope
Sliding Down



Year 2 Applied Chp 6 - Projectiles

Projectiles

Vertical Motion

SUVAT formula

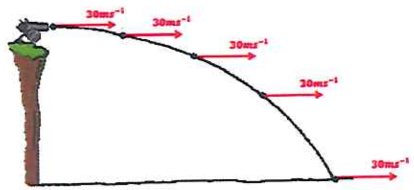
S = vertical displacement
U_v = vertical component of U
V_v = vertical component of V
A = −9.8
T = time

Horizontal Motion

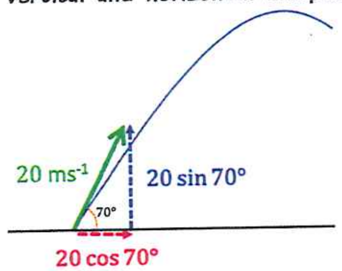
$$Speed = \frac{Distance}{Time}$$

Distance = horizontal displacement
Speed = horizontal component
Time = time

Horizontal speed will remain constant.

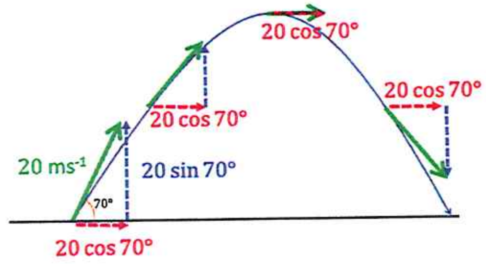


Breakdown speeds at given points into vertical and horizontal components



Maximum Height

The vertical speed will be zero.



Solving

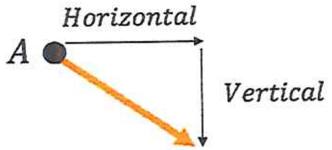
Resolve Vertically
and
Resolve Horizontally

Velocity at a Point

Find the horizontal velocity at point A

Find the vertical velocity at point A

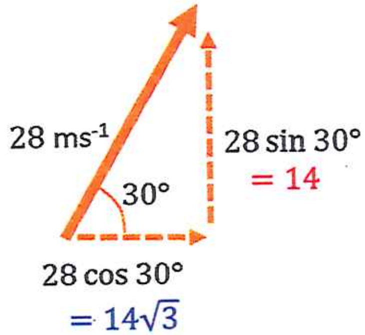
Then find the resultant velocity at point A using Pythagoras.



$$\sqrt{(Horizontal)^2 + (Vertical)^2}$$

Example:

A particle P is projected from a point O on a horizontal plane with speed $28\ ms^{-1}$ and with angle of elevation 30°. After projection, the particle moves freely under gravity until it strikes the plane at a point A.
Find:
a) the greatest height above the plane reached by P
b) the time of flight of P
c) the distance OA



Vertical
S =
U = 14
V = 0
A = −9.8
T =

Horizontal
Distance =
Speed = $14\sqrt{3}$
Time =

a) *Resolve Vertial*

$v^2 = u^2 + 2as$

$0 = 14^2 + 2 \times (-9.8) \times s$

$s = 10$

b) *Resolve Vertical*

S = 0
U = 14
V = 0
A = −9.8
T =

$s = ut + \frac{1}{2}at^2$

$0 = 14t - 4.9t^2$

$t = 2.857$

c) *Resolve Horizontial*

Distance =
Speed = $14\sqrt{3}$
Time = 2.857

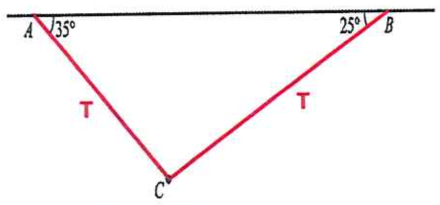
Distance = Speed x Time
= 28 cos 30° × 2.857
= 69.28

Year 2 Applied Chp 7 - Application of Forces

Inextensible String

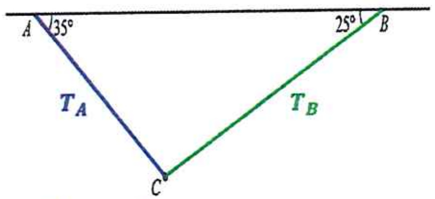
No matter how much you pull on it, it won't extend.

One continuous piece of string.



T and T same tension

Two seperate pieces of string.



T_A and T_A different tensions

Solving

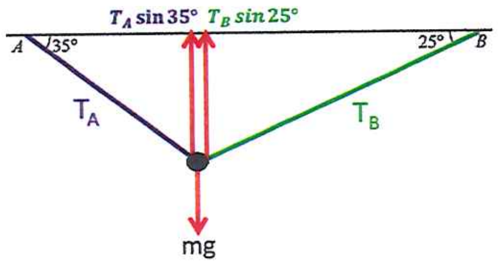
Write two equations.

One for vertical equations.
Forces up = Forces down

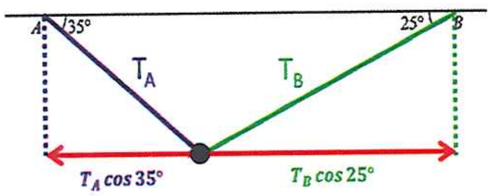
One for horitional equations.
Forces right = Forces left

Then solve the two equations simultaneous if needed.

Vertical Forces

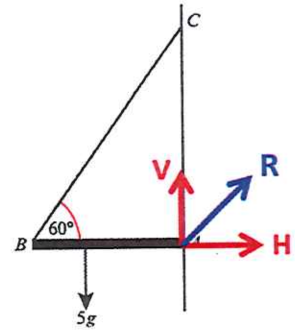


Horizontial Forces



Hinges

Freely hinged means that a rod can rotate about the point it is hinged.



At a hinge there will be a vertical and horizontal forces producing a resultant force.

$\sqrt{(Vertical)^2 + (Horizontial)^2}$
$= Resultant$

Solving

Write three equations.

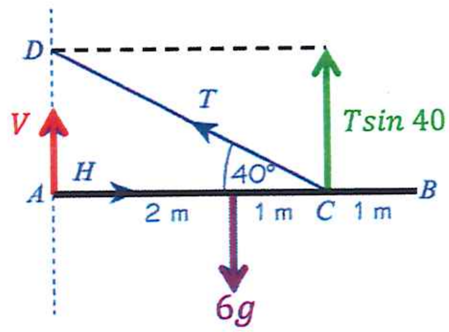
One for vertical equations.
Forces up = Forces down

One for horitional equations.
Forces right = Forces left

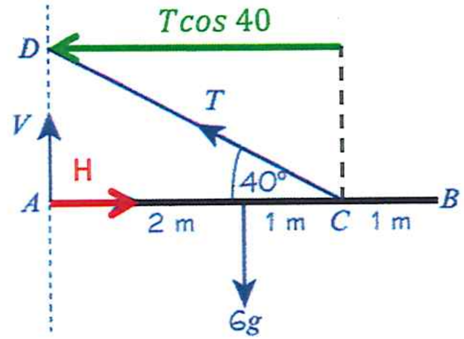
One for Moments
(often use the hinge as the pivot point)

Then solve the two equations simultaneous if needed.

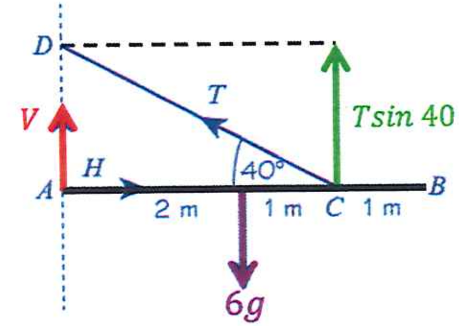
Vertical Forces



Horizontial Forces



Moments



Pivot A to find T

Pivot C to find V

Year 2 Applied Chp 7 - Application of Forces

Ladders and Solving

Write three equations.

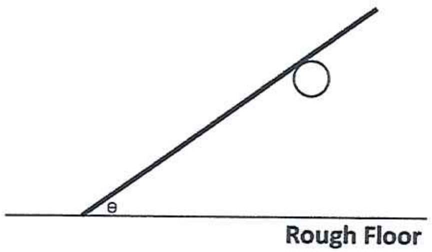
One for vertical equations.
Forces up = Forces down

One for horitional equations.
Forces right = Forces left

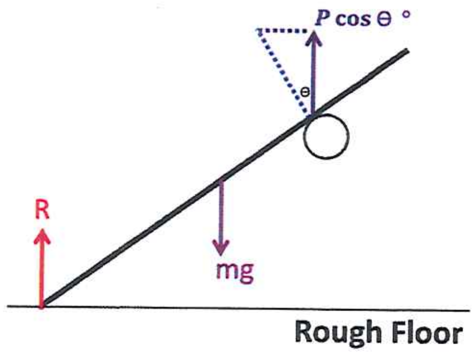
One for Moments
(often use the hinge as the pivot point)

Then solve the two equations simultaneous if needed.

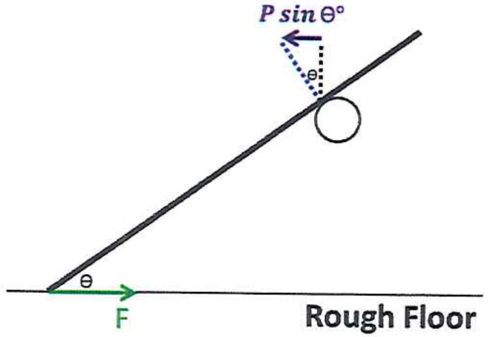
Ladders and pegs



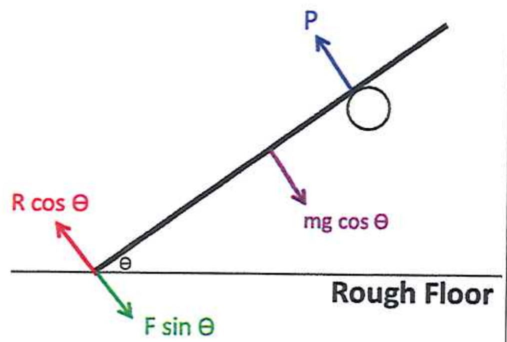
Vertical Forces



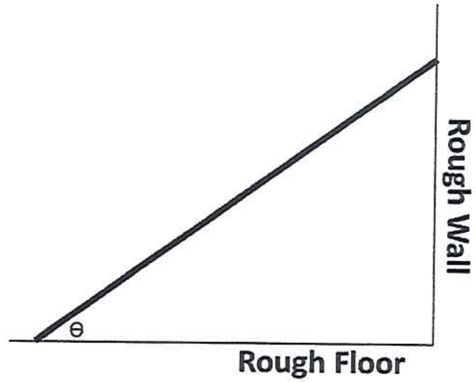
Horizontial Forces



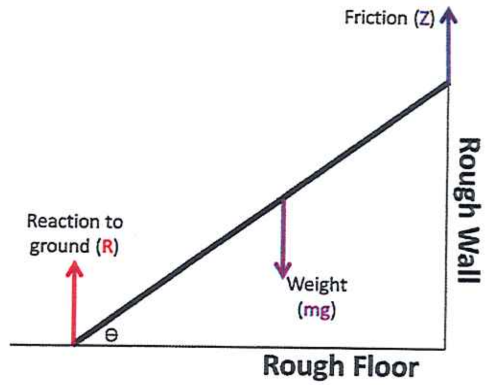
Moments



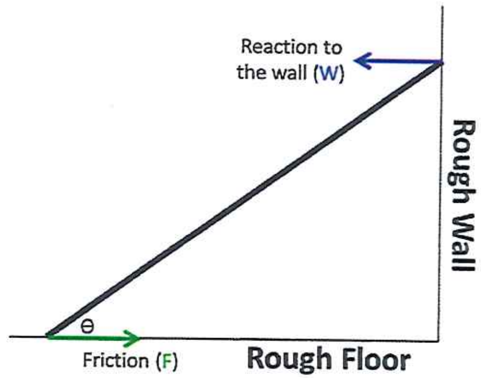
Ladders and Walls



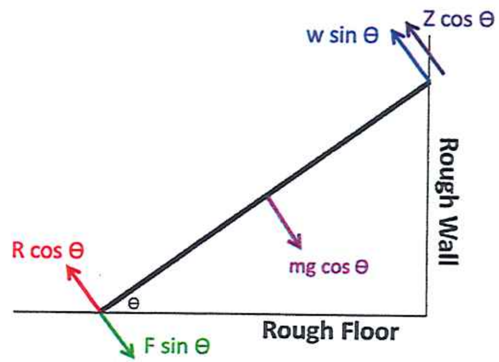
Vertical Forces



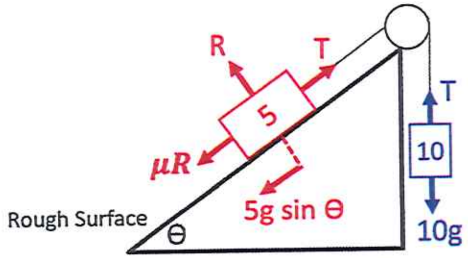
Horizontial Forces



Moments



Connected Particles



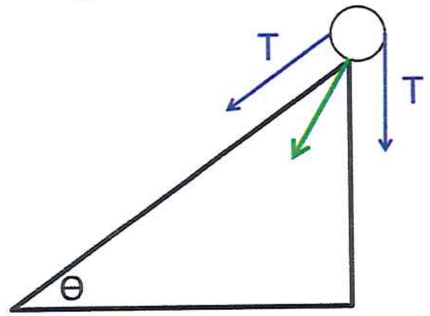
Write an **F = ma** equation for each practical.

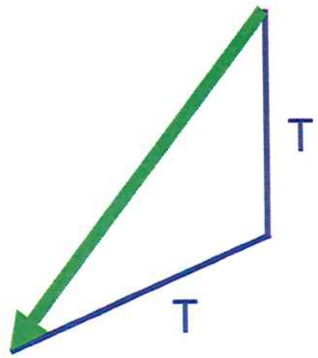
Then solve simultaneously.

$F = ma$
$T - \mu R - 5g \sin \theta = 5a$

$F = ma$
$10g - T = 10a$

Force Exerted on a Pully





Force Exerted on Pully
$= 2T \cos\left(\frac{90 - \theta}{2}\right)$

Year 2 Applied Chp 8 – Further Kinematics

Vectors + Location

To describe the motion of a particle you can use the equation:

$$r = r_0 + vt$$

r_0 = initial position (vector)
v = velocity (vector)
t = time (scalar)

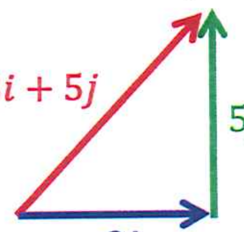
Vectors + SUVAT

use r instead of s for displacement

$$r = ut + \frac{1}{2}at^2$$

$$v = u + at$$

Vectors + Projectiles



Resultant Velocity $= 3i + 5j\ ms^{-1}$

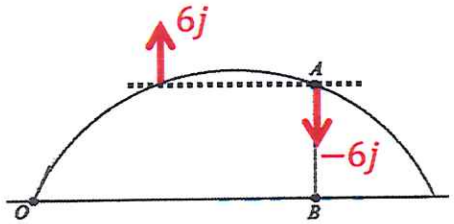
Resultant Speed $= \sqrt{3^2 + 5^2}$
$= \sqrt{34}\ ms^{-1}$

Horizontal Velocity $= 3\ ms^{-1}$

Vertical Velocity $= 5\ ms^{-1}$

Vertical Speed

Up and Down



Variable Acceleration

If you have variable acceleration, then **do not use** SUVAT formulae.

$$\begin{tikzcd} s \arrow[d, bend left, "\frac{ds}{dt}"] \\ v \arrow[u, bend left, "\int v\,dt"] \arrow[d, bend left, "\frac{dv}{dt}"] \\ a \arrow[u, bend left, "\int a\,dt"] \end{tikzcd}$$

Remember:

$$\frac{d}{dx}e^{kx} = ke^{kx}$$

Distance Travelled

$$= \int_{start\ time}^{finish\ time} v\,dt$$

Variable Acceleration

Differentiate the
i and j components
separately.

$$s = 3t\boldsymbol{i} + 4t^2\boldsymbol{j}$$

$$v = 3\ \boldsymbol{i} + 8t\ \boldsymbol{j}$$

Integrate the
i and j components
separately.
Both components will have
their own C value.

$$\int 3t\boldsymbol{i} + \frac{1}{2}t^2\boldsymbol{j}$$

$$= (\frac{3}{2}t^2 + C)\boldsymbol{i} + (\frac{1}{6}t^3 + C)\boldsymbol{j}$$