AS Level / Year 1
Edexcel Maths / Paper 1

March 2018 Mocks

Question	Scheme		AO	Marks
1				
	$4x^2+4x+3=4\left(x^2+x\right)+3$	Takes out a factor of 4 from first two terms or whole expression	AO1.1a	M1
	$=4\left[\left(x+\frac{1}{2}\right)^2-\frac{1}{4}\right]+3$	Completes the square correctly on their x^2+x	AO1.1a	M1
		Correct unsimplified expression oe	AO1.1b	A1
	$=4\left(x+\frac{1}{2}\right)^2+2$	Cao	AO1.1b	A1
				4

Question 1 Notes

1st M1 – for $4x^2+4x+3=4\left(x^2+x\right)+3$ or $4x^2+4x+3=4\left(x^2+x+\frac{3}{4}\right)$

2nd M1 – completes the square correctly on their x^2+x, must have a leading coefficient = 1

1st A1 – for $=4\left[\left(x+\frac{1}{2}\right)^2-\frac{1}{4}\right]+3$ (or equivalent, i.e. $=4\left[\left(x+\frac{1}{2}\right)^2-\frac{1}{4}+\frac{3}{4}\right]$ etc.)

2nd A1 – cao

Question	Scheme		AO	Marks
2				
	$\int \frac{1}{\sqrt{x^3}}dx = \int x^{-\frac{3}{2}}dx = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} \quad \{+c\}$	Writes $\frac{1}{\sqrt{x^3}} = x^{-\frac{3}{2}}$ and attempts to integrate	AO1.1a	M1
		Correct unsimplified indefinite integration (ignore constants)	AO1.1b	A1
	$\Rightarrow \int x^{-\frac{3}{2}}dx = -2x^{-\frac{1}{2}} = -\frac{2}{\sqrt{x}} \quad \{+c\}$			
	$\int_1^3 \frac{1}{\sqrt{x^3}}dx = -\frac{2}{\sqrt{x}}\Big\|_1^3 = 2 - \frac{2}{\sqrt{3}}$	Substitutes limits into their expression the correct way around (no simplifying necessary)	AO1.1a	M1
	$= \frac{2\sqrt{3}-2}{\sqrt{3}} = \frac{\sqrt{3}\left(2\sqrt{3}-2\right)}{3} = \frac{2}{3}\left(3-\sqrt{3}\right)$ *	Rationalises the denominator (expression must be correct)	AO1.1a	dM1
		Convincing proof, all steps shown, with answer in given form or correct value of *k* stated	AO1.1b	A1
				5

Question 2 Notes

1st M1 – writes the integrand in index form and attempts to integrate (adds 1 to power, divides by resulting power)

1st A1 – correct unsimplified integration (ignore constants)

2nd M1 – substitutes the limits into **their** indefinite integral the correct way around

3rd M1 – rationalises the denominator correctly. This is dependent on the 1st and 2nd M1 and **requires the expression to be correct**.

2nd A1 – complete and convincing proof, with all steps shown and no errors seen. Answer should be given in required form or *k* stated.

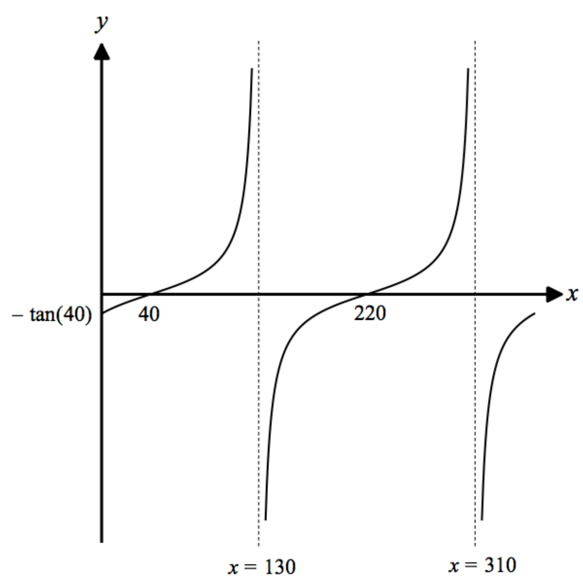
Question	Scheme		AO	Marks
3				
	$\overrightarrow{AB} = \begin{pmatrix} a \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} a-3 \\ 11 \end{pmatrix}$	Attempts to find expression for $\overrightarrow{AB}$	AO1.1a	M1
	$\left\lvert\overrightarrow{AB}\right\rvert = 5\sqrt{5} \Rightarrow \sqrt{(a-3)^2+121} = 5\sqrt{5}$	Forms the correct equation oe	AO3.1a	A1
	$\Rightarrow a^2-6a+9+121=125$ $\Rightarrow a^2-6a+5=0$ $\Rightarrow (a-5)(a-1)=0$ $\Rightarrow a_{\max}=5$	Attempts to solve their 3TQ for *a* Correct maximum value of *a* stated	AO1.1a AO1.1b	dM1 A1
				4

Question 3 Notes

1st M1 – considers $\overrightarrow{OB} - \overrightarrow{OA}$

2nd M1 – correct method used to solve their 3TQ for *a*

2nd A1 – correct maximum value of *a* stated. Condone *a* = 5 stated (can assume they are referring to the maximum).

Question	Scheme		AO	Marks
4				
(a)	$x = 40°,\ 220°$	Correct values of *x*. One mark for each correct value. See notes for guidance if extra values given	AO1.1b AO1.1b	B1 B1 **[2]**
(b)	$(0,\ \tan(-40))$	Correct coordinates oe. Accept exact value of *y* or decimal equivalents	AO1.1b	B1 **[1]**
(c)	$x = 130°, x = 310°$	Correct asymptotes. One mark for each correct value. See notes for guidance if extra equations given	AO1.1b AO1.1b	B1 B1 **[2]**
(d)		Correct shape *x*, *y* intercepts and asymptotes ft their (a), (b) and (c) shown on sketch correctly	AO1.2 AO1.1b	B1 B1FT **[2]**
				7

	Question 4 Notes

(a) One B mark for each correct x intercept. If additional **correct** values (ignoring range) are given, ignore these values and give the B marks. If additional **incorrect** values are given (in or out of range), deduct 1 B mark for each incorrect value given (max – 2 B marks).

(b) Cao, accept decimal or exact value of – tan(40) = 0.839… . Condone just y = – tan(40), i.e. coordinate form given.

(c) One B mark for each correct asymptote. If additional **correct** asymptotes (ignoring range) are given, ignore these values and give the B marks. If additional **incorrect** asymptotes are given deduct 1 B mark for each incorrect asymptote given (max – 2 B marks).

(d) 1st B1 – for the correct shape of the graph

2nd B1 – x, y intercepts and asymptotes ft their (a), (b) and (c) shown on their sketch correctly

Question	Scheme		AO	Marks
5				
(a) Method 1	$x-2\overline{)2x^3-1x^2-25x-12}$ with quotient $2x^2+3x^2-19$ $\underline{2x^3-4x^2}$ $3x^2-25x \quad (*)$ $\underline{3x^2-6x}$ $-19x-12$ $\underline{-19x+38}$ -50 so the remainder is – 50	Attempts long division (all steps correct up to $*$) Correct remainder	AO1.1a AO1.1b	M1 A1 **[2]**
(a) Method 2	$\text{f}(2)=2(2)^3-(2)^2-25(2)-12$ $=-50$ so the remainder is – 50	Substitutes 2 into f(x) Correct remainder	AO1.1a AO1.1b	M1 A1 **[2]**
(b) Method 1	$x+3\overline{)2x^3-1x^2-25x-12}$ with quotient $2x^2-7x^2-4$ $\underline{2x^3+6x^2}$ $-7x^2-25x \quad (*)$ $\underline{-7x^2-21x}$ $-4x-12$ $\underline{-4x-12}$ 0 Therefore, $(x+3)$ is a factor of $\text{f}(x)$	Attempts long division (all steps correct up to $*$) Completely correct long division, with conclusion at the end	AO1.1a AO1.1b	M1 A1 **[2]**

(b) Method 2	$\mathrm{f}(-3)=2(-3)^3-(-3)^2-25(-3)-12$ $=-54-9+75-12$ $=0$ Therefore, $(x+3)$ is a factor of $\mathrm{f}(x)$	Substitutes –3 into f(x) Complete and convincing proof with no errors seen. Must see the terms/groups of terms evaluated	AO1.1a AO1.1b	M1 A1 **[2]**
(c)	Other factor is $2x^2-7x^2-4$	Method to find other factor of f(x) **If candidates use Method 1 in (b), award M1 A1 for correct workings in (b)**	AO1.1a AO1.1b	M1 A1
	$\Rightarrow \mathrm{f}(x)=(x+3)(2x^2-7x^2-4)=(x+3)(2x+1)(x-4)$			
	Solutions are $x=-3,\ -\frac{1}{2},\ 4$	Correct solutions	AO1.1b	A1 **[3]**
				7
	Question 5 Notes			

(a) Method 1 and 2: A1 – no statements are necessary, but the final answer must be clearly identified, i.e. underlined etc.

(b) Method 1: A1 – completely correct long division with conclusive statement, i.e. 'therefore $(x+3)$ is a factor of $\mathrm{f}(x)$'

Method 2: A1 – complete and convincing proof. This is a show that question, so we need to see some explicit evaluation of the terms before they conclude f(–3) is equal to 0.

(c) 1st M1 – uses a correct method to find the other factor, i.e. inspection (implied by obtaining one correct coefficient) or long division.

1st A1 – obtains the correct factor

2nd A1 – correct solutions. Cao

Note: If candidates use Method 1 in (b), then they can score M1 A1 for correct workings in (b)

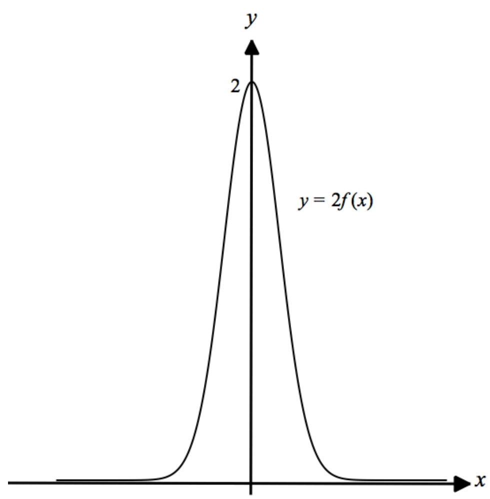
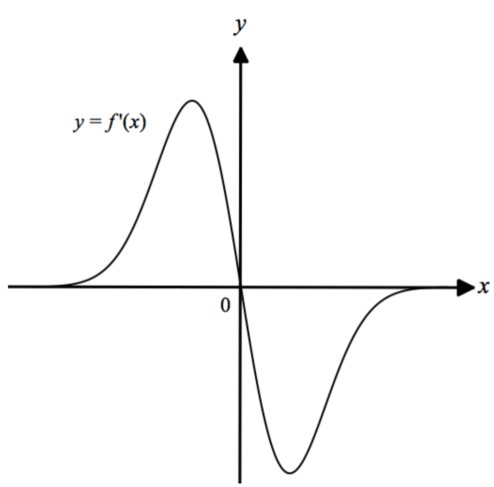
Question	Scheme		AO	Marks
6				
(a) Method 1	$\lim_{h\to 0}\frac{(m(x+h)+c)-(mx+c)}{h}=\lim_{h\to 0}\frac{mh}{h}=\lim_{h\to 0}m=m$	Considers $\frac{(m(x+h)+c)-(mx+c)}{h}$	AO1.2	M1
	so the gradient of the line $y=mx+c$ is m	Complete and convincing proof with correct limiting process seen	AO2.1	A1 **[2]**
(a) Method 2	$\lim_{x'\to x}\frac{(mx'+c)-(mx+c)}{x'-x}=\lim_{x'\to x}\frac{m(x'-x)}{x'-x}=\lim_{x'\to x}m=m$	Considers $\frac{(mx'+c)-(mx+c)}{x'-x}$	AO1.2	M1
	so the gradient of the line $y=mx+c$ is m	Complete and convincing proof with correct limiting process seen	AO2.1	A1 **[2]**
(b) Method 1	$\lim_{h\to 0}\frac{6(x+h)^3-6x^3}{h}=\lim_{h\to 0}\frac{6x^3+18x^2h+18xh^2+6h^3-6x^3}{h}$	Considers $\frac{6(x+h)^3-6x^3}{h}$	AO1.2	M1
	$=\lim_{h\to 0}\frac{18x^2h+18xh^2+6h^3}{h}$	Attempts to simplify numerator	AO1.1a	dM1
	$=\lim_{h\to 0}\left(18x^2+18xh+6h^2\right)$ $=18x^2$	Complete and convincing proof with correct limiting process seen	AO2.1	A1 **[3]**
(b) Method 2	$\lim_{x'\to x}\frac{6x'^3-6x^3}{x'-x}=\lim_{x'\to x}\frac{6(x'-x)(x'^2+xx'+x^2)}{(x'-x)}$	Considers $\frac{6x'^3-6x^3}{x'-x}$	AO1.2	M1
	$=6\lim_{x'\to x}\left(x'^2+xx'+x^2\right)$	Obtains a factor of $x'^2+xx'+x^2$	AO1.1a	dM1
	$=6(x^2+x^2+x^2)$ $=18x^2$	Complete and convincing proof with correct limiting process seen	AO2.1	A1 **[3]**
				5

Question	Scheme		AO	Marks
7				
(a)	$\frac{1}{2}(10)(12)\sin 60 = 52.0\ \text{cm}^2$	Attempts to find area Correct area to 1dp. Condone 52 cm^2	AO1.1a AO1.1b	M1 A1 **[2]**
(b)	$a^2 = 10^2 + 12^2 - 2(10)(12)\cos 60 = 124$ $\Rightarrow a = \sqrt{124} = 11.1(355...)\ \text{cm}$	Uses cosine rule Correct value of a. Accept any degree of accuracy	AO1.1a AO1.1b	M1 A1 **[2]**
(c)	e.g. $\frac{\sin x}{10} = \frac{\sin 60}{a} \Rightarrow \sin x = \frac{10\sin 60}{\sqrt{124}} = \frac{5\sqrt{93}}{62}$	Uses the sine rule with correct combinations of sides	AO1.1b	B1 **[1]**
(d/i)	$x = \sin^{-1}\left(\frac{5\sqrt{93}}{62}\right)$	Identifies correct value of x oe	AO3.2a	B1 **[1]**
(d/ii)	$180 - \sin^{-1}\left(\frac{5\sqrt{93}}{62}\right) = 128.94...$ $128.94...+60 > 180$, so this angle would violates the property that angles in a triangle add together to give 180.	Convincing illustration about why the other angle fails	AO3.2a	B1 **[1]**
				7

Question	Scheme		AO	Marks
8				
(a)	$5^{x^2} = 4^{k-6x}$	Equates $5^{x^2} = 4^{k-6x}$	AO3.1a	M1
	$\log\left(5^{x^2}\right) = \log\left(4^{k-6x}\right)$	Takes logs to both sides (accept any base or ln)	AO1.1a	dM1
	$x^2 \log 5 = (k-6x)\log 4$	Uses power rule for logs correctly	AO1.1b	dM1
	$x^2 \log 5 = k\log 4 - (6\log 4)x$ $x^2 \log 5 + (6\log 4)x - k\log 4 = 0$ *	Complete and convincing proof with no errors seen	AO2.1	A1 **[4]**
(b)	$(6\log 4)^2 - 4(\log 5)(-k\log 4) < 0$	Uses the discriminant (only interested in LHS here, ignore their choice of inequality/equality symbol)	AO3.1a	M1
	$-4k\log 5\log 4 > 36\log^2 4$ $\Rightarrow k < -\dfrac{36\log^2 4}{4\log 5\log 4}$ $\Rightarrow k < -\dfrac{9\log 4}{\log 5}$ * $\Rightarrow k < \dfrac{9\log 4^{-1}}{\log 5} = \dfrac{9\log 0.25}{\log 5}$	Attempts to re-arrange to get *k* on one side (ft using their chosen inequality/equality symbol). Ignore sign and inequality preservation errors also Complete and convincing proof with no errors seen. All stages must be correct and clearly shown. **NB: Look out for fudged workings that appear to lead to the right answer but come from mishandlings of negatives and inequality signs**	AO1.1a AO2.1	dM1 A1 **[3]**
				7

Question	Scheme		AO	Marks
9				
(a/i)	$m_{AB} = \dfrac{-2-(-1)}{-3-0} = \dfrac{1}{3}$	Attempts to find gradient of *AB*	AO1.1a AO1.1b	M1 A1
	$\Rightarrow$ gradient of perpendicular bisector = – 3	Correct gradient of perp bisector ft their gradient of *AB*	AO1.1b	A1ft **[3]**
(a/ii)	Midpoint of $AB = \left(\dfrac{-3}{2}, -\dfrac{3}{2}\right)$	Correct coordinates of midpoint of *AB*	AO1.1b	B1
	$\Rightarrow y+\dfrac{3}{2} = -3\left(x+\dfrac{3}{2}\right)$ $\Rightarrow 6x+2y+12=0$	Attempts to use their midpoint and (a/i) to find equation of perp. bisector Correct equation oe	AO1.1a AO1.1b	M1 A1 **[3]**
(b) Method 1	Gradient of normal to C at *A* is $-\dfrac{1}{2}$	Correct gradient of normal to *C* at *A*. Seen or implied	AO3.1a	M1
	So equation of normal is $x+2y+7=0$	Correct equation of normal to *C* at *A*	AO3.1a	A1
	x coordinate of centre given by solution to $5x+5=0 \Rightarrow x=-1$	Attempts to solve simultaneously their equation of the normal and their (a/i) Obtains one coordinate correctly	AO3.1a AO2.1	dM1 A1
	$\therefore -1+2y+7=0 \Rightarrow 2y=-6 \Rightarrow y=-3$ $\therefore$ coordinates of *C* are (–1, –3)	Obtains the second coordinate correctly (must show that *x* = –1 $\Rightarrow$ *y* = –3)	AO2.1	A1 **[5]**

(b) Method 2	Gradient of normal to C at A is $-\frac{1}{2}$	Correct gradient of normal to C at A. Seen or implied	AO3.1a	M1
	So equation of normal is $x+2y+7=0$	Correct equation of normal to C at A	AO3.1a	A1
	Now $-1+2(-3)+7=-7+7=0$, so (–1, –3) lies on the normal	Substitutes (–1, –3) into their normal and their (a/ii)	AO3.1a	dM1
	$6(-1)+2(-3)+12=-12+12=0$, so (–1, –3) lies the perpendicular bisector of *AB*	Shows that (–1, –3) lies on their normal OR their (a/ii)	AO2.1	A1
	∴ centre of *C* must have the coordinates (–1, –3)	Shows that (–1, –3) lies on the normal and perp bisector and concludes that therefore the centre must be at (–1, –3)	AO2.1	A1 **[5]**
(c)	e.g. $r^2=(-1--3)^2+(-3--2)^2=5 \Rightarrow r=\sqrt{5}$	Correct radius oe	AO1.1a AO1.1b	M1 A1 **[2]**
(d)	$(x+1)^2+(y+3)^2=5$	1st B1 : LHS correct 2nd B1ft : RHS correct ft their (d)	AO1.2 AO1.2	B1 B1ft **[2]**
				15
	Question 9 Notes			
(c) M1 – attempts to use Pythagoras (or equivalently, distance between two points formula) to find an expression for *r* or r^2. Condone one sign error.				

Question	Scheme	AO	Marks
10			
(a)	 Correct shape Correct y intercept at (0,2)	AO1.2 AO1.1b	B1 B1 **[2]**
(b)	 Asymptotes to x axis as $x \to \pm\infty$ Passes through the origin Maximum and minimum points shown clearly and in the correct places	AO2.2 AO2.2 AO2.2	B1 B1 B1 **[3]**
			5

Question	Scheme		AO	Marks
11				
(a)	${}^{10}C_3 = \dfrac{10!}{3!(10-3)!}$	Uses the definition. Accept 7! instead of (10 – 3)! explicitly shown	AO2.1	M1
	$= \dfrac{10\times 9\times 8}{3\times 2\times 1} = \dfrac{720}{6} = 120$ *	Convincing proof	AO2.1	A1 **[2]**
(b) Method 1	$\begin{pmatrix} n \\ m \end{pmatrix}\begin{pmatrix} n-m \\ r-m \end{pmatrix} = \dfrac{n!}{m!(n-m!)} \times \dfrac{(n-m)!}{(r-m)!(n-r)!}$	Uses the definition. Accept equivalent expressions for $n-r$, i.e. $n-m-r+m$. <u>Condone</u> $n-m-r-m$	AO2.1	M1
	$= \dfrac{n!}{m!(n-m!)} \times \dfrac{(n-m)!}{(r-m)!(n-r)!}$ $=$ $= \dfrac{n!}{m!} \times \dfrac{1}{(r-m)!(n-r)!} \times \dfrac{r!}{r!}$	Introduces $\dfrac{r!}{r!}$	AO2.1	dM1
	$= \dfrac{n!}{r!(n-r)!} \times \dfrac{r!}{m!(r-m)!}$ $= \begin{pmatrix} n \\ r \end{pmatrix}\begin{pmatrix} r \\ m \end{pmatrix}$	Complete and convincing proof with no errors seen	AO2.1	A1 **[3]**

(b) Method 2	$\binom{n}{r}\binom{r}{m}=\frac{n!}{r!(n-r!)}\times\frac{r!}{m!(r-m)!}$	Uses the definition	AO2.1	M1
	$=\frac{n!}{r!(n-r!)}\times\frac{r!}{m!(r-m)!}$ $=\frac{n!}{m!}\times\frac{(n-m)!}{(n-m)!}\times\frac{1}{(r-m)!(n-m-r+m)!}$	Introduces $\frac{(n-m)!}{(n-m)!}$	AO2.1	dM1
	$=\frac{n!}{m!(n-m)!}\times\frac{(n-m)!}{(r-m)!(n-m-r+m)!}$ $=\binom{n}{m}\binom{n-m}{r-m}$	Complete and convincing proof with no errors seen	AO2.1	A1 **[3]**
(c)	Let $m=1$ in (b), then $\binom{n}{r}\binom{r}{1}=\binom{n}{1}\binom{n-1}{r-1}$ $\Rightarrow\binom{n}{r}r=n\binom{n-1}{r-1}$ $\Rightarrow\binom{n}{r}=\frac{n}{r}\binom{n-1}{r-1}$	Substitutes $m=1$ into (b) Clearly uses $\binom{r}{1}=r$, $\binom{n}{1}=n$ to give a complete and convincing proof	AO2.2 AO2.1	M1 A1 **[2]**
				7
	Question 11 Notes			
(b) Special case: combinatorial proofs should be sent to review.				

Question	Scheme		AO	Marks
12				
(i)	$\frac{dy}{dx}=\frac{x^3}{x^2}-\frac{x^{\frac{1}{2}}}{x^2}=x-x^{-\frac{3}{2}}$	Attempts to write the gradient function in index form	AO3.1a AO1.1b	M1 A1
	$y=\int(x-x^{-\frac{3}{2}})dx=\frac{1}{2}x^2+2x^{-\frac{1}{2}}+c$	Attempts to integrate indefinitely Correct indefinite integration, including constant	AO3.1a AO1.1b	dM1 A1
	When $x=1,y=4\Rightarrow\ 4=\frac{1}{2}+2+c\Rightarrow c=\frac{3}{2}$	Substitutes initial conditions into their expression for *y* to find *c*	AO1.1b	dM1
	$\therefore y=\frac{1}{2}x^2+2x^{-\frac{1}{2}}+\frac{3}{2}$	States *y* in terms of *x*. NB: value of *c* alone is <u>**not**</u> enough	AO1.1b	A1 **[6]**
(ii)	e.g. let $f(x)=x,\ g(x)=x^2$, then $\int_0^1 f(x)g(x)dx=\int_0^1 x^3\,dx=\frac{1}{4}$	Choose two functions for f and g and attempts to compute $\int_0^1 f\times g$ or $\int_0^1 f\times\int_0^1 g$	AO2.1	M1
	$\int_0^1 f(x)dx\times\int_0^1 g(x)dx=\int_0^1 x\,dx\times\int_0^1 x^2\,dx=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$ $\frac{1}{4}\neq\frac{1}{6}$, so therefore Jessie is wrong.	Attempts to compute the other integral Both integrals computed correctly and shown not to be equal + conclusion, i.e. 'therefore Jessie is wrong' or ' therefore $\int_0^1 f\times\int_0^1 g\neq\int_0^1 fg$	AO2.1 AO2.1	M1 A1 **[3]**
				9

	Question 12 Notes

(a) 1st M1 – attempts to write the gradient function in index form. M mark should be awarded for clearly separating the fraction into two terms and attempting to use relevant index law

3rd A1 – we need to see *y* expressed in terms of *x*. In other words, candidates who find the value of *c* and leave their answer there cannot access the final A mark.

Special case:

1st M and A mark – as per scheme. Then:

$$\int_4^y dy' = \int_1^x (x' - x'^{-\frac{3}{2}})\,dx'$$

$$\Rightarrow y - 4 = \frac{1}{2}x^2 + 2x^{\frac{1}{2}} - \frac{1}{2} - 2$$

$$\Rightarrow y = \frac{1}{2}x^2 + 2x^{\frac{1}{2}} + \frac{3}{2}$$

No primes needed. 2nd M1 for integrating both sides, 2nd A1 for correct integration, 3rd M1 for correct limits, 3rd A1 – correct answer oe

(b) 1st M1 – chooses two functions for f and g (need not be distinct) and attempts to compute one of the integrals

2nd M1 – attempts to compute the other integral

A1 – both integrals correctly evaluated, shown to be different and a conclusion.

Question	Scheme		AO	Marks
13				
(a)	$f'(x)=2x-3\left(\frac{3}{2}\right)x^{\frac{1}{2}}=2x-\frac{9}{2}x^{\frac{1}{2}}$	Attempts to differentiate f wrt *x*	AO3.1a	M1
	$f'(4)=2(4)-\frac{9}{2}(4)^{\frac{1}{2}}=8-9=-1$	Substitutes 4 into their gradient function	AO1.1a	dM1
	So gradient of l is 1	Correct gradient of the line l	AO1.1b	A1
	At $x=4,\ f(4)=4^2-3\sqrt{4^3}+4=-4$	Correct y coordinate when $x=4$. Seen or implied	AO1.1a	B1
	So equation of l is $y=x-8$	Cao	AO1.1b	A1 **[5]**
(b)	$g'(x)=12x^2+2qx-2$	Differentiates g wrt *x* correctly	AO3.1a	M1
	$g'(-1)=1\Rightarrow 12-2q-2=1$	Forms correct equation using their g'	AO3.1a	dM1
	$\Rightarrow q=\frac{9}{2}$	Correct value of *q*	AO2.1	A1 **[3]**
				8

Question	Scheme		AO	Marks
14	**** Scheme change: (a/i) has changed to 1 mark (from 2), while (a/ii) is now 2 marks (from 1) ****			
(a/i)	$120 = \pi r^2 h + \frac{4}{3}\pi r^3 \Rightarrow \pi r^2 h = 120 - \frac{4}{3}\pi r^3 \Rightarrow h - \frac{120}{\pi r^2} - \frac{4r}{3}$ *	Convincing proof	AO2.1	B1 **[1]**
(a/ii)	$A = 4\pi r^2 + 2\pi rh$	Writes down correct formula for surface area of the solid	AO1.2	M1
	$A = 4\pi r^2 + 2\pi r\left(\frac{120}{\pi r^2} - \frac{4r}{3}\right)$ $\left\{\Rightarrow A = \frac{4}{3}\pi r^2 + \frac{240}{r}\right\}$	Correct expression oe (no need to simplify)	AO1.1b	A1 **[2]**
(b)	$\frac{dA}{dr} = \frac{8}{3}\pi r - \frac{240}{r^2}$	Attempts to differentiate their *A* wrt *r* Correct differentiation	AO3.1b AO1.1b	M1 A1
	$\frac{dA}{dr} = 0 \Rightarrow \frac{8}{3}\pi r = \frac{240}{r^2} \Rightarrow r = \ldots$	Sets their derivative equal to 0 and attempts to re-arrange for *r*	AO3.1b	dM1
	$\therefore r^3 = \frac{90}{\pi} \Rightarrow r = \left(\frac{90}{\pi}\right)^{\frac{1}{3}}$, so *A* is minimised when *r* is $\left(\frac{90}{\pi}\right)^{\frac{1}{3}}$	Convincing proof	AO2.1	A1 **[4]**

(c)	$\frac{d^2A}{dr^2} = \frac{8}{3}\pi + \frac{480}{r^3}$ Attempts to differentiate their $\frac{dA}{dr}$ again wrt r	AO1.1a	M1
	$\left.\frac{d^2A}{dr^2}\right\vert_{r=\left(\frac{90}{\pi}\right)^{\frac{1}{3}}} = \frac{8}{3}\pi + \frac{480}{\left(\frac{90}{\pi}\right)}\{= 8\pi\} > 0$ Considers $\left.\frac{d^2A}{dr^2}\right\vert_{r=\left(\frac{90}{\pi}\right)^{\frac{1}{3}}}$ and shows that it is greater than 0 (see notes). **NB: this mark requires their second derivative to be correct**	AO2.1	A1
	$\therefore$ Since $\left.\frac{d^2A}{dr^2}\right\vert_{r=\left(\frac{90}{\pi}\right)^{\frac{1}{3}}}$ > 0, the surface area is minimised when $\left(\frac{90}{\pi}\right)^{\frac{1}{3}}$ Conclusion	AO2.4	A1 **[3]**
			10
	Question 14 Notes		

(b) 2nd A1 – no conclusion is OK here.

(c) 1st A1 – substitutes $\left(\frac{90}{\pi}\right)^{\frac{1}{3}}$ into $\frac{d^2A}{dr^2}$ (must be correct) and shows that it is greater than 0. Candidates can either show the substitution or state the outcome – they don't need to do both, as the positivity is fairly 'obvious', but they must state that it is positive. Can be implied in the conclusion.

Marks breakdown by AO

AO	Number of marks	%
AO1	61	61
AO2	24	24
AO3	15	15