

**AS and A level Mathematics Practice Paper – Differentiation in context – Mark scheme**

| Question    | Scheme                                                                                                                                                                                                              | Marks                              |
|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------|
| <b>1(a)</b> | $\frac{1}{2}(9x + 6x)4x$ or $2x \times 15x$ or $\left( \frac{1}{2}4x \times (9x - 6x) + 6x \times 4x \right)$ or $6x^2 + 24x^2$ or $\left( 9x \times 4x - \frac{1}{2}4x \times (9x - 6x) \right)$ or $36x^2 - 6x^2$ | <b>M1A1cso</b>                     |
|             | $\Rightarrow 30x^2 y = 9600 \Rightarrow y = \frac{9600}{30x^2} \Rightarrow y = \frac{320}{x^2} *$                                                                                                                   | <b>(2)</b>                         |
| <b>1(b)</b> | $(S =) \frac{1}{2}(9x + 6x)4x + \frac{1}{2}(9x + 6x)4x + 6xy + 9xy + 5xy + 4xy$                                                                                                                                     | <b>M1A1</b>                        |
|             | $y = \frac{320}{x^2} \Rightarrow (S =) 30x^2 + 30x^2 + 24x\left(\frac{320}{x^2}\right)$                                                                                                                             | <b>M1</b>                          |
|             | So, $(S =) 60x^2 + \frac{7680}{x} *$                                                                                                                                                                                | <b>A1* cso</b>                     |
|             |                                                                                                                                                                                                                     | <b>(4)</b>                         |
| <b>1(c)</b> | $\frac{dS}{dx} = 120x - 7680x^{-2} \left\{ = 120x - \frac{7680}{x^2} \right\}$                                                                                                                                      | <b>M1</b>                          |
|             |                                                                                                                                                                                                                     | <b>A1 aef</b>                      |
|             | $120x - \frac{7680}{x^2} = 0$ $\Rightarrow x^3 = \frac{7680}{120}; = 64 \Rightarrow x = 4$                                                                                                                          | <b>M1A1cso</b>                     |
|             | $\{x = 4,\} \quad S = 60(4)^2 + \frac{7680}{4} = 2880 \text{ (cm}^2\text{)}$                                                                                                                                        | <b>ddM1<br/>A1 cao<br/>and cso</b> |
|             |                                                                                                                                                                                                                     | <b>(6)</b>                         |
| <b>1(d)</b> | $\frac{d^2S}{dx^2} = 120 + \frac{15360}{x^3} > 0$ $\Rightarrow \text{Minimum}$                                                                                                                                      | <b>M1A1ft</b>                      |
|             | <b>Note parts (c) and (d) can be marked together.</b>                                                                                                                                                               | <b>(2)</b>                         |
|             |                                                                                                                                                                                                                     | <b>(14 marks)</b>                  |

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|-------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------|
| <b>2(a)</b> | $\text{Area}(FEA) = \frac{1}{2}x^2 \left( \frac{2\pi}{3} \right); = \frac{\pi x^2}{3}$                                                                                                               | M1 A1             |
|             |                                                                                                                                                                                                      | <b>(2)</b>        |
| <b>2(b)</b> | $\{A=\} \frac{1}{2}x^2 \sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$                                                                                                                                     | M1 A1             |
|             | $1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$<br>$\Rightarrow y = \frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3}) *$ | A1 *              |
|             |                                                                                                                                                                                                      | <b>(3)</b>        |
| <b>2(c)</b> | $\{P=\} x + x\theta + y + 2x + y \left\{ = 3x + \frac{2\pi x}{3} + 2y \right\}$                                                                                                                      | B1ft              |
|             | $\dots 2y = + 2 \left( \frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3}) \right)$                                                                                                                       | M1                |
|             | $P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$                                     |                   |
|             | $\Rightarrow P = \frac{1000}{x} + \frac{x}{12}(4\pi + 36 - 3\sqrt{3}) *$                                                                                                                             | A1 *              |
|             |                                                                                                                                                                                                      | <b>(3)</b>        |
| <b>2(d)</b> | $\frac{dP}{dx} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}; = 0$                                                                                                                                | M1 A1 M1          |
|             | $\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}} (= 16.63392808\dots)$                                                                                                                 | A1                |
|             | $\left\{ P = \frac{1000}{(16.63\dots)} + \frac{(16.63\dots)}{12}(4\pi + 36 - 3\sqrt{3}) \right\} \Rightarrow P = 120.236\dots (\text{m})$                                                            | A1                |
|             |                                                                                                                                                                                                      | <b>(5)</b>        |
| <b>2(e)</b> | $\frac{d^2P}{dx^2} = \frac{2000}{x^3} > 0 \Rightarrow \text{Minimum}$                                                                                                                                | M1 A1ft           |
|             |                                                                                                                                                                                                      | <b>(2)</b>        |
|             |                                                                                                                                                                                                      | <b>(15 marks)</b> |

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| Question    | Scheme                                                                                                             |                                                                                              | Marks             |
|-------------|--------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------|-------------------|
| <b>3(a)</b> | $\{V =\} \quad 2x^2y = 81$                                                                                         | $2x^2y = 81$                                                                                 | B1 oe             |
|             | $\{L = 2(2x + x + 2x + x) + 4y \Rightarrow L = 12x + 4y\}$                                                         |                                                                                              |                   |
|             | $y = \frac{81}{2x^2} \Rightarrow L = 12x + 4\left(\frac{81}{2x^2}\right)$                                          | Making $y$ the subject of their expression and substitute this into the correct $L$ formula. | M1                |
|             | So, $L = 12x + \frac{162}{x^2}$ <b>AG</b>                                                                          | Correct solution only.                                                                       | A1 <b>cs</b>      |
|             |                                                                                                                    |                                                                                              | <b>(3)</b>        |
| <b>3(b)</b> | $\frac{dL}{dx} = 12 - \frac{324}{x^3} \quad \{= 12 - 324x^{-3}\}$                                                  | Either $12x \rightarrow 12$ or $\frac{162}{x^2} \rightarrow \frac{\pm \lambda}{x^3}$         | M1                |
|             |                                                                                                                    | Correct differentiation (need not be simplified).                                            | A1 aef            |
|             | $\left\{\frac{dL}{dx} =\right\} 12 - \frac{324}{x^3} = 0 \Rightarrow x^3 = \frac{324}{12}; = 27 \Rightarrow x = 3$ | $L' = 0$ and “their $x^3 = \pm$ value” or “their $x^{-3} = \pm$ value”                       | M1;               |
|             |                                                                                                                    | $x = \sqrt[3]{27}$ or $x = 3$                                                                | A1 <b>cs</b>      |
|             | $\{x = 3,\} \quad L = 12(3) + \frac{162}{3^2} = 54 \text{ (cm)}$                                                   | Substitute candidate’s value of $x$ ( $\neq 0$ ) into a formula for $L$ .                    | ddM1              |
|             |                                                                                                                    | 54                                                                                           | A1 <b>cao</b>     |
|             |                                                                                                                    |                                                                                              | <b>(6)</b>        |
| <b>3(c)</b> | $\{\text{For } x = 3\}, \quad \frac{d^2L}{dx^2} = \frac{972}{x^4} > 0 \Rightarrow \text{Minimum}$                  | Correct ft $L''$ and considering sign.                                                       | M1                |
|             |                                                                                                                    | $\frac{972}{x^4}$ and $> 0$ and conclusion.                                                  | A1                |
|             |                                                                                                                    |                                                                                              | <b>(2)</b>        |
|             |                                                                                                                    |                                                                                              | <b>(11 marks)</b> |

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| Question    | Scheme                                                                                                                                                                                                         |                                                                                   | Marks             |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|-------------------|
| <b>4</b>    | $\frac{dV}{dt} = 80\pi,$<br>$V = 4\pi h(h + 4) = 4\pi h^2 + 16\pi h,$                                                                                                                                          |                                                                                   |                   |
|             | $\frac{dV}{dh} = 8\pi h + 16\pi$                                                                                                                                                                               | $\pm \alpha h \pm \beta, \alpha \neq 0, \beta \neq 0$                             | M1                |
|             |                                                                                                                                                                                                                | $8\pi h + 16\pi$                                                                  | A1                |
|             | $\left\{ \frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow \right\}$<br>$(8\pi h + 16\pi) \frac{dh}{dt} = 80\pi$                                                                                  | $\left( \text{Candidate's } \frac{dV}{dh} \right) \times \frac{dh}{dt} = 80\pi$   | M1 oe             |
|             | $\left\{ \frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow \right\} \quad \frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi} \quad \text{or} \quad 80\pi \div \text{Candidate's } \frac{dV}{dh}$ |                                                                                   |                   |
|             | When $h = 6,$<br>$\left\{ \frac{dh}{dt} = \right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi \left\{ = \frac{80\pi}{64\pi} \right\}$                                                                              | <b>dependent on the previous M1</b>                                               | dM1               |
|             | $\frac{dh}{dt} = \underline{\quad\quad} \text{ (cms}^{-1}\text{)}$                                                                                                                                             | $1.25 \text{ or } \frac{5}{4} \text{ or } \frac{10}{8} \text{ or } \frac{80}{64}$ | A1 oe             |
|             |                                                                                                                                                                                                                |                                                                                   | <b>(5 marks)</b>  |
| <b>5(a)</b> | $P = \frac{800e^0}{1+3e^0}, = \frac{800}{1+3} = 200$                                                                                                                                                           |                                                                                   | M1 A1             |
|             |                                                                                                                                                                                                                |                                                                                   | <b>(2)</b>        |
| <b>5(b)</b> | $250 = \frac{800e^{0.1t}}{1+3e^{0.1t}}$                                                                                                                                                                        |                                                                                   |                   |
|             | $250(1+3e^{0.1t}) = 800e^{0.1t} \Rightarrow 50e^{0.1t} = 250, \Rightarrow e^{0.1t} = 5$                                                                                                                        |                                                                                   | M1 A1             |
|             |                                                                                                                                                                                                                | $t = \frac{1}{0.1} \ln(5)$                                                        | M1                |
|             |                                                                                                                                                                                                                | $t = 10 \ln(5)$                                                                   | A1                |
|             |                                                                                                                                                                                                                |                                                                                   | <b>(4)</b>        |
| <b>5(c)</b> | $P = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow \frac{dP}{dt} = \frac{(1+3e^{0.1t}) \times 800 \times 0.1e^{0.1t} - 800e^{0.1t} \times 3 \times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$                                |                                                                                   | M1 A1             |
|             |                                                                                                                                                                                                                |                                                                                   |                   |
|             | At $t=10$<br>$\frac{dP}{dt} = \frac{(1+3e) \times 80e - 240e^2}{(1+3e)^2} = \frac{80e}{(1+3e)^2}$                                                                                                              |                                                                                   | M1 A1             |
|             |                                                                                                                                                                                                                |                                                                                   | <b>(4)</b>        |
| <b>5(d)</b> | $P = \frac{800e^{0.1t}}{1+3e^{0.1t}} = \frac{800}{e^{-0.1t} + 3} \Rightarrow P_{\max} = \frac{800}{3} = 266. \text{ Hence P cannot be 270}$                                                                    |                                                                                   | B1                |
|             |                                                                                                                                                                                                                |                                                                                   | <b>(1)</b>        |
|             |                                                                                                                                                                                                                |                                                                                   | <b>(11 marks)</b> |

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| Question    | Scheme                                                                                                            | Marks            |
|-------------|-------------------------------------------------------------------------------------------------------------------|------------------|
| <b>6(a)</b> | $P_0 = \frac{100}{1+3} + 40 = 65$                                                                                 | B1               |
|             |                                                                                                                   | <b>(1)</b>       |
| <b>6(b)</b> | $\frac{d}{dt} e^{kt} = Ce^{kt}$                                                                                   | M1               |
|             | $\frac{dP}{dt} = \frac{(1+3e^{-0.9t}) \times -10e^{-0.1t} - 100e^{-0.1t} \times -2.7e^{-0.9t}}{(1+3e^{-0.9t})^2}$ | M1 A1            |
|             |                                                                                                                   | <b>(3)</b>       |
| <b>6(c)</b> | At maximum $-10e^{-0.1t} - 30e^{-0.1t} \times e^{-0.9t} + 270e^{-0.1t} \times e^{-0.9t} = 0$                      |                  |
|             | $e^{-0.1t} (-10 + 240e^{-0.9t}) = 0$                                                                              |                  |
|             | $e^{-0.9t} = \frac{10}{240}$ oe $e^{0.9t} = 24$                                                                   | M1               |
|             | $-0.9t = \ln\left(\frac{1}{24}\right) \Rightarrow t = \frac{10}{9} \ln(24) = 3.53$                                | M1,A1            |
|             | Sub $t = 3.53 \Rightarrow P_T = 102$                                                                              | A1               |
|             |                                                                                                                   | <b>(4)</b>       |
| <b>6(d)</b> | 40                                                                                                                | B1               |
|             |                                                                                                                   | <b>(1)</b>       |
|             |                                                                                                                   | <b>(9 marks)</b> |
| <b>7</b>    | $\frac{dI}{dt} = -16 \ln(0.5) 0.5^t$                                                                              | M1 A1            |
|             | At $t = 3$ $\frac{dI}{dt} = -16 \ln(0.5) 0.5^3$                                                                   | M1               |
|             | $= -2 \ln 0.5 = \ln 4$                                                                                            | M1 A1            |
|             |                                                                                                                   | <b>(5 marks)</b> |

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| Question    | Scheme                                                                                                                                                                                              | Marks            |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------|
| <b>8(a)</b> | $V = x^3 \Rightarrow \frac{dV}{dx} = 3x^2$ * <span style="float: right;">CSO</span>                                                                                                                 | B1               |
|             |                                                                                                                                                                                                     | <b>(1)</b>       |
| <b>8(b)</b> | $\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{0.048}{3x^2}$                                                                                                                           | M1               |
|             | At $x = 8$ , $\frac{dx}{dt} = \frac{0.048}{3(8^2)} = 0.00025$ ( $\text{cm s}^{-1}$ ) <span style="float: right;"><math>2.5 \times 10^{-4}</math></span>                                             | A1               |
|             |                                                                                                                                                                                                     | <b>(2)</b>       |
| <b>8(c)</b> | $S = 6x^2 \Rightarrow \frac{dS}{dx} = 12x$                                                                                                                                                          | B1               |
|             | $\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt} = 12x \left( \frac{0.048}{3x^2} \right)$                                                                                                        | M1               |
|             | At $x = 8$ $\frac{dS}{dt} = 0.024$ ( $\text{cm}^2 \text{ s}^{-1}$ )                                                                                                                                 | A1               |
|             |                                                                                                                                                                                                     | <b>(3)</b>       |
|             |                                                                                                                                                                                                     | <b>(6 marks)</b> |
| <b>9(a)</b> | $\frac{dV}{dh} = \frac{1}{2} \pi h - \pi h^2$ <span style="float: right;">or equivalent</span>                                                                                                      | M1 A1            |
|             | At $h = 0.1$ , $\frac{dV}{dh} = \frac{1}{2} \pi (0.1) - \pi (0.1)^2 = 0.04\pi$ <span style="float: right;"><math>\frac{\pi}{25}</math></span>                                                       | M1 A1            |
|             |                                                                                                                                                                                                     | <b>(4)</b>       |
| <b>9(b)</b> | $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{\pi}{800} \times \frac{1}{\frac{1}{2} \pi h - \pi h^2}$ <span style="float: right;">or <math>\frac{\pi}{800} \div</math> their (a)</span> | M1               |
|             | At $h = 0.1$ , $\frac{dh}{dt} = \frac{\pi}{800} \times \frac{25}{\pi} = \frac{1}{32}$ <span style="float: right;">awrt 0.031</span>                                                                 | A1               |
|             |                                                                                                                                                                                                     | <b>(2)</b>       |
|             |                                                                                                                                                                                                     | <b>(6 marks)</b> |

# AS and A level Mathematics Practice Paper – Differentiation in context – Mark scheme

|   | Source paper | Question number | New spec references          | Question description                                        | New AOs                         |
|---|--------------|-----------------|------------------------------|-------------------------------------------------------------|---------------------------------|
| 1 | C2 2014      | 10              | 5.7                          | Differentiation                                             | 1.1b, 2.1, 2.2a, 2.4, 3.1b, 3.4 |
| 2 | C2 2016      | 9               | 5.1, 7.1, 7.2 and 7.3        | Trigonometry, Differentiation                               | 1.1b, 2.1, 2.2a, 2.4, 3.1b, 3.4 |
| 3 | C2 2011      | 8               | 7.1, 7.2 and 7.3             | Differentiation                                             | 1.1b, 2.1                       |
| 4 | C4 June 2014 | 4               | 7.4                          | Differentiation                                             | 1.1b, 3.1b                      |
| 5 | C3 June 2014 | 8               | 6.1, 6.2, 6.3, 6.7, 7.4      | Exponential growth, Differentiation of exponential function | 1.1b, 3.1a, 3.4, 3.5b           |
| 6 | C3 2017      | 8               | 6.1, 6.2, 6.3, 6.5, 6.7, 7.4 | Real life exponential problem, quotient rule                | 1.1b, 2.1, 3.1a                 |
| 7 | C4 Jan 2011  | 2               | 7.2, 6.4                     | Differentiation                                             | 1.1b, 2.1                       |
| 8 | C4 2012      | 2               | 7.4                          | Differentiation                                             | 1.1b, 3.1a                      |
| 9 | C4 2011      | 3               | 7.4                          | Differentiation                                             | 1.1b                            |

