Pearson Edexcel

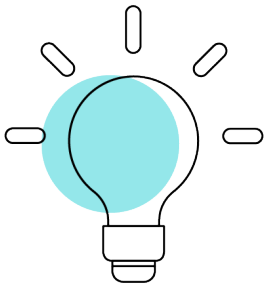
Topic Test
Summer 2022

Pearson Edexcel GCE Mathematics (9MA0)

Paper 3 – Mechanics

Topic 5: Moments

Contents



Mark Scheme

Question T5_Q1

Question	Scheme	Marks	AOs
7(a)	Resolve vertically	M1	3.1b
	$R+40\sin\alpha=20g$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$40\cos\alpha-F=20a$	A1	1.1b
	$F = 0.14R$	B1	1.2
	$a = 0.396$ or 0.40 (m s^{-2})	A1	2.2a
		(6)	

(a)

M1: Resolve vertically with usual rules applying

A1: Correct equation. Neither g nor $\sin\alpha$ need to be substituted

M1: Apply $F = ma$ horizontally, with usual rules

A1: Neither F nor $\cos\alpha$ need to be substituted

B1: $F = 0.14R$ seen (e.g. on a diagram)

A1: Either answer

Question T5_Q2

Question	Scheme	Marks	AOs
9(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin\alpha = Mga + 3Mgx$	A1	1.1b
	$T = \dfrac{Mg(a+3x)}{2a \times \frac{3}{5}} = \dfrac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\dfrac{5Mg(3x+a)}{6a}\cos\alpha = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \dfrac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \dfrac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin\alpha$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\dfrac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1ft	1.1b
	$Y = \dfrac{5Mg}{2}$ N.B. May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \dfrac{Y}{X}$ or $\dfrac{R\sin\beta}{R\cos\beta} = \dfrac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \dfrac{5}{4}$	A1	2.2a
		(5)	
(d)	$\dfrac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \dfrac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\dfrac{5a}{3}$ from A oe Or just: $x \le \dfrac{5a}{3}$, if no incorrect statement seen. N.B. **If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 **A1**	2.4
		(3)	
			(13 marks)

Notes:

(a)

M1: Using M(A), with usual rules, or any other complete method to obtain an equation in a, M, x and T only.

A1: Correct equation

A1*: Correct PRINTED ANSWER, correctly obtained, need to see $\sin\alpha = \frac{3}{5}$ used.

(b)

M1: Using an appropriate strategy to find x. e.g. Resolve horizontally with usual rules applying OR Moments about C. Must use the given expression for T.

A1: Accept $0.67a$ or better

(c)

M1: Using a complete method to find Y (or $R\sin\beta$) e.g. resolve vertically or Moments about B, with usual rules

A1 ft: Correct equation with their x substituted in T expression or using $T = \frac{2Mg}{\cos\alpha}$

A1: Y (or $R\sin\beta$) $= \frac{5Mg}{2}$ or $2.5Mg$ or $2.50Mg$

M1: For finding an equation **in tan β only** using $\tan\beta = \frac{Y}{X}$ or $\tan\beta = \frac{X}{Y}$

This is independent but must have found a Y.

A1: Accept $\frac{-5}{4}$ if it follows from their working.

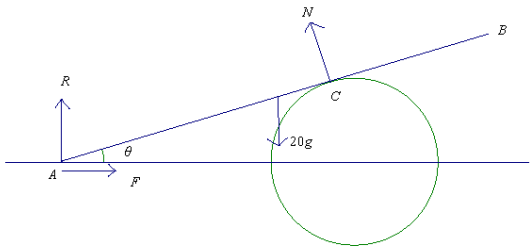
(d)

M1: Allow $T = 5Mg$ or $T < 5Mg$ and solves for x, showing all necessary steps (M0 for $T > 5Mg$)

A1: Allow $x = \frac{5a}{3}$ or $x < \frac{5a}{3}$. Accept $1.7a$ or better.

B1: **Treat as A1.** For any appropriate equivalent fully correct comment or statement. E.g. maximum value of x is $\frac{5a}{3}$

Question T5_Q3

Question	Scheme	Marks	AO
4(a)	Drum **smooth**, or no friction, (therefore reaction is perpendicular to the ramp)	B1	2.4
		(1)	
(b)	**N.B.** In (b), for a moments equation, if there is an extra $\sin\theta$ or $\cos\theta$ on a length, give M0 for the equation e.g. M(A): $20g\times 4\cos\theta = 5N\sin\theta$ would be given M0A0		
			
	Possible equns $(\nearrow): F\cos\theta + R\sin\theta = 20g\sin\theta$ $(\nwarrow): N + R\cos\theta = 20g\cos\theta + F\sin\theta$ $(\uparrow) R + N\cos\theta = 20g$ $(\rightarrow): F = N\sin\theta$ M(A): $20g\times 4\cos\theta = 5N$ M(B): $3N + R\times 8\cos\theta = F\times 8\sin\theta + 20g\times 4\cos\theta$ M(C): $R\times 5\cos\theta = F\times 5\sin\theta + 20g\times\cos\theta$ M(G): $R\times 4\cos\theta = F\times 4\sin\theta + N$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $F = 42.14784$; $R = 51.49312$)		
	Alternative 1: using cpts along ramp (X) and perp to ramp(Y) Possible equations: $(\nearrow): X = 20g\sin\theta$ $(\nwarrow): Y + N = 20g\cos\theta$ $(\uparrow): X\sin\theta + Y\cos\theta + N\cos\theta = 20g$ $(\rightarrow): X\cos\theta = Y\sin\theta + N\sin\theta$ M(A): $20g\times 4\cos\theta = 5N$ M(B): $20g\times 4\cos\theta = 8Y + 3N$ M(C): $20g\times\cos\theta = 5Y$ M(G): $4Y = N\times 1$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $X = 54.88$; $Y = 37.632$)		

	Alternative 2: using horizontal cpt (H) and cpt perp to ramp (S)		
	$(\nearrow): H\cos\theta = 20g\sin\theta$	M1	3.3
	$(\nwarrow): S + N = H\sin\theta + 20g\cos\theta$	A1	1.1b
	$(\uparrow): S\cos\theta + N\cos\theta = 20g$	M1	3.4
	$(\rightarrow): H = S\sin\theta + N\sin\theta$	A1	1.1b
	$\text{M}(A): 20g \times 4\cos\theta = 5N$	M1	3.4
	$\text{M}(B): 20g \times 4\cos\theta + H \times 8\sin\theta = 8S + 3N$	A1	1.1b
	$\text{M}(C): 20g \times \cos\theta + H \times 5\sin\theta = 5S$		
	$\text{M}(G): 4S = N \times 1 + H \times 4\sin\theta$		
	(The values of the 3 unknowns are: $N = 150.528$; $H = 57.1666\ldots$; $S = 53.638666\ldots$)		
	Solve their 3 equations for F and R **OR** X and Y **OR** H and S	M1	1.1b
	$\lvert\text{Force}\rvert = \sqrt{R^2 + F^2}$ Main scheme **OR** $= \sqrt{X^2 + Y^2}$ Alternative 1 **OR** $= \sqrt{(H^2 + S^2 - 2HS\cos(90^\circ - \theta)}$ Alternative 2	M1	3.1b
	Magnitude = 67 or 66.5 (N)	A1	2.2a
		(9)	
(c)	Magnitude of the normal reaction (at C) will **decrease**.	B1	3.5a
		(1)	
		(11)	

Marks		Notes
4a	B1	Ignore any extra incorrect comments.
		Generally 3 independent equations required so **at least one moments equation.**: M1A1M1A1M1A1. More than 3 equations, give marks for the best 3. For each: M1 All terms required. Must be dimensionally correct so if a length is missing from a moments equation it's M0 Condone sin/cos confusion. A1 For a correct equation (trig ratios do not need to be substituted and allow e.g. cos(24/25) if they recover Enter marks on ePEN in order in which equations appear. **N.B.** If reaction at C is not perpendicular to the ramp, can only score marks for M(C) Allow use of (μR) for F
4b	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required, dim correct, condone sin/cos confusion
	A1	Correct unsimplified equation
		N.B. They can find F and R using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 1	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
		N.B. They can find X and Y using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 2	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.

	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct.
	A1	Correct unsimplified equation
		N.B. They can find H and S using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
	M1	Substitute for trig and solve for their two cpts. This is an independent mark but must use 3 equations (unless it's the special case when 2 is sufficient)
	M1	Use Pythagoras to find magnitude (this is an independent M mark but must have found a value for F (or X) and a value for R (or Y)) **OR** a complete method to find magnitude e.g. cosine rule but must have found a value for H and a value for S
	A1	Correct answer only
	B1	Ignore reasons

Question T5_Q4

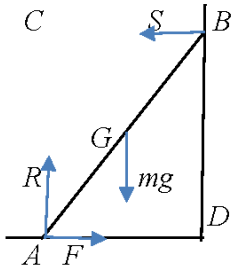
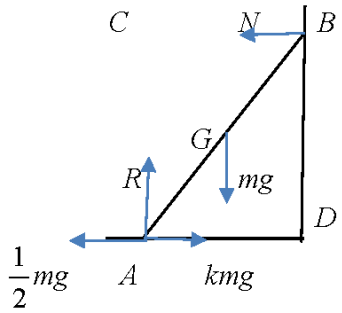
Question	Scheme	Marks	AOs
4(a)	Take moments about A	M1	3.3
	$N \times \frac{4a}{\sin\alpha} = Mg \times 3a\cos\alpha$	A1	1.1b
	$\frac{9Mg}{25}$ *	A1*	1.1b
		(3)	
4(b)	Resolve horizontally	M1	3.4
	$(\rightarrow)\ F = \frac{9Mg}{25}\sin\alpha$	A1	1.1b
	Resolve vertically	M1	3.4
	$(\uparrow) R + \frac{9Mg}{25}\cos\alpha = Mg$	A1	1.1b
	Other possible equations: $(\nwarrow), R\cos\alpha + \frac{9Mg}{25} = Mg\cos\alpha + F\sin\alpha$ $(\nearrow), Mg\sin\alpha = F\cos\alpha + R\sin\alpha$ $\text{M}(C), Mg.2a\cos\alpha + F.5a\sin\alpha = R.5a\cos\alpha$ $\text{M}(G), \frac{9Mg}{25}.2a + F.3a\sin\alpha = R.3a\cos\alpha$ $\text{M}(B), Mg.3a\cos\alpha + F.6a\sin\alpha = R.6a\cos\alpha + \frac{9Mg}{25}a$ $(F = \frac{36Mg}{125}, R = \frac{98Mg}{125})$		
	$F = \mu R$ used	M1	3.4
	Eliminate R and F and solve for μ	M1	3.1b
	Alternative equations if they have at A: X horizontally and Y perpendicular to the rod. $(\nwarrow), Y + \frac{9Mg}{25} = Mg\cos\alpha + X\sin\alpha$ $(\nearrow), Mg\sin\alpha = X\cos\alpha$ $(\uparrow), \frac{9Mg}{25}\cos\alpha + Y\cos\alpha = Mg$ $(\rightarrow), Y\sin\alpha + \frac{9Mg}{25}\sin\alpha = X$		

	$M(C), Mg.2a\cos\alpha + X.5a\sin\alpha = Y.5a$ $M(G), \frac{9Mg}{25}.2a + X.3a\sin\alpha = Y.3a$ M1A1 M1A1 $M(B), Mg.3a\cos\alpha + X.6a\sin\alpha = Y.6a + \frac{9Mg}{25}a$ $(X = \frac{4Mg}{3}, Y = \frac{98Mg}{75})$ Then $F = \mu R$ becomes: $X - Y\sin\alpha = \mu Y\cos\alpha$ M1 Eliminate X and Y and solve for μ M1		
	$\mu = \frac{18}{49}$ (0.3673…..accept 0.37 or better)	A1	2.2a
		(7)	
			(10 marks)

Notes:

4a	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors for an equation in N and Mg only. For perp distance allow any of : $\frac{4a}{\sin\alpha}, \frac{4a}{\cos\alpha}, 5a$ but use of any of : $6a, 5a\sin\alpha, 4a\cos\alpha, \ldots$ or anything involving $\tan\alpha$ is M0 Also M0 if no a's in their first equation.
	A1	Correct equation, trig does not need to be substituted
	A1*	Given answer correctly obtained.
4b	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted but N does.
	M1	Correct no. of terms, dim correct, condone sin/cos confusion and sign errors
	A1	Correct equation, trig does not need to be substituted but N does.
		N.B. The above 4 marks are for any two equations, either resolutions or moments or one of each. Mark best two equations. Equations may appear in part (a) but must be used in (b) to earn marks.
	M1	Must be used, e.g. seen on the diagram. i.e. M0 if merely quoting it. (M0 if $F = \mu \times \frac{9Mg}{25}$ used)
	M1	Must have 3 equations (<u>and all 3 previous M marks</u>)
	A1	Accept 0.37 or better

Question T5_Q5

Question	Scheme	Marks	AOs
	Part (a) is a 'Show that..' so equations need to be given in full to earn A marks		
3(a)			
	Moments equation: (M1A0 for a moments inequality)	M1	3.3
	M(A), $mga\cos\theta = 2Sa\sin\theta$ M(B), $mga\cos\theta + 2Fa\sin\theta = 2Ra\cos\theta$ M(C), $F\times 2a\sin\theta = mga\cos\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + 2Sa\sin\theta$ M(G), $Ra\cos\theta = Fa\sin\theta + Sa\sin\theta$.	A1	1.1b
	($\updownarrow$) $R = mg$ **OR** ($\leftrightarrow$) $F = S$	B1	3.4
	Use their equations (they must have enough) and $F \le \mu R$ to give an inequality in μ and θ **only** (allow DM1 for use of $F = \mu R$ to give an *equation* in μ and θ only)	DM1	2.1
	$\mu \ge \frac{1}{2}\cot\theta$ *	A1*	2.2a
		(5)	
3(b)			
	Moments equation:	M1	3.4
	M(A), $mga\cos\theta = 2Na\sin\theta$ M(B), $mga\cos\theta + 2kmga\sin\theta = 2Ra\cos\theta + \frac{1}{2}mg2a\sin\theta$ M(D), $2Ra\cos\theta = mga\cos\theta + N2a\sin\theta$ M(G), $kmga\sin\theta + Na\sin\theta = \frac{1}{2}mga\sin\theta + Ra\cos\theta$	A1	1.1b

	S.C. M(C), $mga\cos\theta+\frac{1}{2}mg2a\sin\theta=kmg2a\sin\theta$ **M1A1B1** $1+\frac{5}{4}=\frac{5k}{2}$ **M1** $k=0.9$ **A1**		
	$N=kmg-F$ **OR** $R=mg$	B1	3.3
	Use their equations (they must have enough) to solve for k (numerical)	DM1	3.1b
	$k=0.9$ oe	A1	1.1b
		(5)	
			(10 marks)

Notes:

3a	M1	Any moments equation with correct terms, condone sign errors and sin/cos confusion
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) and $F\le\mu R$ to give an inequality in μ and θ only (allow M1 for use of $F=\mu R$ to give an equation in μ and θ only)
	A1*	Given answer correctly obtained with no wrong working seen (e.g. if they use $F=\mu R$ anywhere, A0)
3b	M1	Any moments equation with correct terms, condone sign errors
	A1	Correct equation
	B1	Correct equation
	DM1	Dependent on M1, for using their equations (they must have enough) with trig substituted, to solve for k, which must be numerical.
	A1	cao