

Topic Test Summer 2022

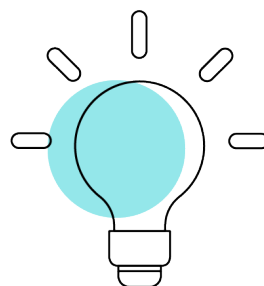
Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 8: Integration

Contents

<u>General guidance to Topic Tests</u>	<u>3</u>
<u>Revise Revision Guide content coverage</u>	<u>4</u>
<u>Questions</u>	<u>5</u>
<u>Mark Scheme</u>	<u>66</u>



Question T8_Q2

10. The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H \cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

- (a) show that $H = 5e^{0.1 \sin(0.25t)}$ (5)
- (b) State the maximum height of the passenger above the ground. (1)

(Total for Question 10 is 8 marks)

Question T8_Q3

13. Show that

$$\int_0^2 2x\sqrt{x+2} \, dx = \frac{32}{15}(2 + \sqrt{2})$$

(7)



Question T8_Q6

8.

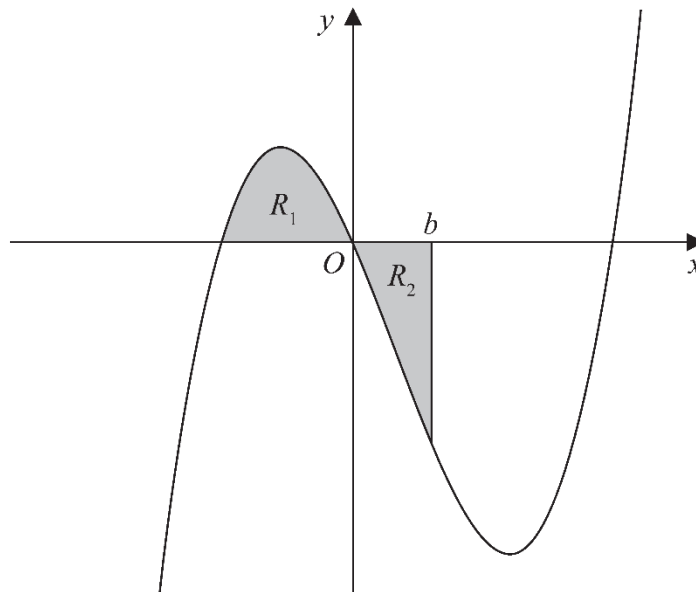


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x -axis.

- (a) Show that the exact area of R_1 is $\frac{20}{3}$ (4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x -axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

- (b) verify that b satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0 \quad (4)$$

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places.
The value of b is therefore 1.225 to 3 decimal places.

- (c) Explain, with the aid of a diagram, the significance of the root 5.442 (2)

Question 8 continued

Question T8_Q9

14. (a) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{dh}{4 - \sqrt{h}} = -8 \ln|4 - \sqrt{h}| - 2\sqrt{h} + k$$

where k is a constant

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{dh}{dt} = \frac{t^{0.25}(4 - \sqrt{h})}{20}$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

(b) Find, according to the model, the range in heights of trees in this species.

(2)

One of these trees is one metre high when it is first planted.

According to the model,

(c) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

(7)

Question T8_Q14

12.

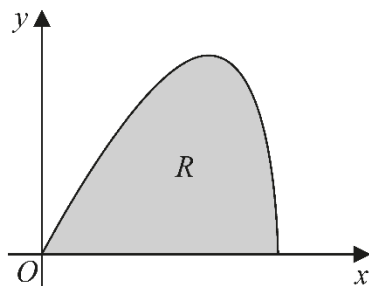


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6 \sin t \quad y = 5 \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region R , shown shaded in Figure 3, is bounded by the curve and the x -axis.

(a) (i) Show that the area of R is given by $\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt$

(3)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20

(3)

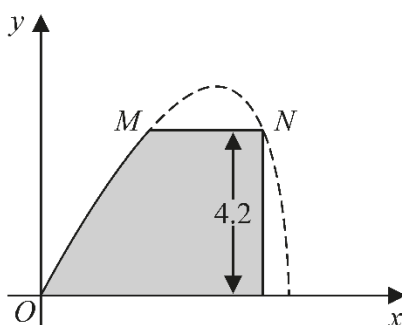


Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4.

Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

(b) calculate the width of the walkway.

(5)

Question T8_Q16

14.

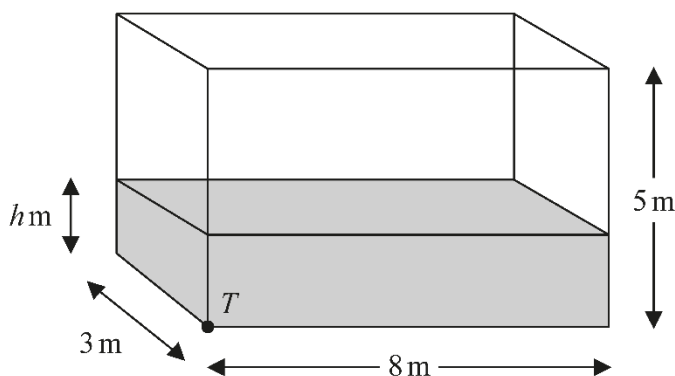


Figure 5

Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point T at the bottom of the tank, as shown in Figure 5.

At time t minutes after the tap has been opened

- the depth of water in the tank is h metres
- water is flowing into the tank at a constant rate of 0.48 m^3 per minute
- water is modelled as leaving the tank through the tap at a rate of $0.1h \text{ m}^3$ per minute

(a) Show that, according to the model,

$$1200 \frac{dh}{dt} = 24 - 5h \quad (4)$$

Given that when the tap was opened, the depth of water in the tank was 2 m,

(b) show that, according to the model,

$$h = A + Be^{-kt}$$

where A , B and k are constants to be found.

(6)

Given that the tap remains open,

(c) determine, according to the model, whether the tank will ever become full, giving a reason for your answer.

(2)
