

Year 1 Pure Chapter 14: Differentiation and Modelling
Exam Questions (Answers)

1. (a) $e^{2x+1} = 2$

$$2x + 1 = \ln 2 \quad \text{M1}$$

$$x = \frac{1}{2}(\ln 2 - 1) \quad \text{A1}$$

(b) $\frac{dy}{dx} = 8e^{2x+1} \quad \text{B1}$

$$x = \frac{1}{2}(\ln 2 - 1) \Rightarrow \frac{dy}{dx} = 16 \quad \text{B1}$$

$$y - 8 = 16\left(x - \frac{1}{2}(\ln 2 - 1)\right) \quad \text{M1}$$

$$y = 16x + 16 - 8 \ln 2 \quad \text{A1}$$

2. (a) 425 °C B1 1

(b) $300 = 400e^{-0.05 t} + 25$

$400e^{-0.05 t} = 275$ M1

$e^{-0.05t} = \frac{275}{400}$ A1

M1 correct application of logs M1

$t = 7.49$ A1

(c) $\frac{dT}{dt} = -20 e^{-0.05t}$ (M1 for $ke^{-0.05t}$) M1 A1

At $t = 50$, rate of decrease = $(\pm) 1.64$ °C / min A1

(d) $T > 25$, (since $e^{-0.05t} \rightarrow 0$ as $t \rightarrow \infty$) B1

[9]

3. (a) $P = 80e^{\frac{t}{5}}$

$$t = 0$$

$$P = 80 e^{\frac{0}{5}} = 80(1) = \underline{80} \quad \text{B1}$$

(b) $P = 1000$

$$1000 = 80e^{\frac{t}{5}}$$

$$\frac{1000}{80} = e^{\frac{t}{5}} \quad \text{Substitutes } P = 1000 \text{ and}$$

rearranges equation to make $e^{\frac{t}{5}}$ the subject. M1

$$t = 5 \ln\left(\frac{1000}{80}\right)$$

$$t = 12.6286... \quad \text{awrt 12.6 or 13 years} \quad \text{A1}$$

(c) $\frac{dP}{dt} = 16e^{\frac{t}{5}} \quad ke^{\frac{t}{5}} \text{ and } k \neq 80. \text{M1}$

$$16 e^{\frac{1}{5}t} \quad \text{A1}$$

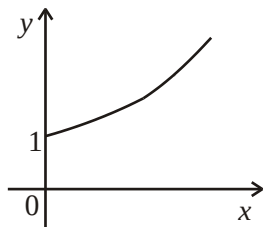
(d) $50 = 16e^{\frac{t}{5}}$

$$t = 5 \ln\left(\frac{50}{16}\right) \quad \{ 5.69717... \} \quad \text{M1}$$

$$P = 80 e^{\frac{1}{5}\left(5 \ln\left(\frac{50}{16}\right)\right)} \quad \text{or} \quad P = 80e^{\frac{1}{5}(5.69717...)} \quad \text{dM1}$$

$$P = \frac{80(50)}{16} = \underline{250} \quad \underline{250} \text{ or awrt 250} \quad \text{A1}$$

4. (a)



Shape

B1

domain, intercept

B1

(b) $£800 \times 1.04^{10} \approx £1184$

M1 A1

(c) $1.04^x = 2$

M1

$$x = \frac{\ln 2}{\ln 1.04} \approx 18 \text{ (years)}$$

M1 A1

months

accept 17.7, 17 years 8

[7]

5. (a) $D = 10, t = 5,$

$$x = 10e^{-\frac{1}{8} \times 5} \quad \text{M1}$$

$$x = 5.353 \quad \text{awrt} \quad \text{A1}$$

(b) $D = 10 + 10e^{-\frac{5}{8}},$

$$t = 1, x = 15.3526... \times e^{-\frac{1}{8}} \quad \text{M1}$$

$$x = 13.549 (*) \quad \text{A1}$$

$$\text{Alt. (b)} \quad x = 10e^{-\frac{1}{8} \times 6} + 10e^{-\frac{1}{8} \times 1} \quad \text{M1}$$

$$\text{or} \quad (10 + 10e^{-\frac{5}{8}})e^{-\frac{1}{8}}$$

$$x = 13.549 (*) \quad \text{A1cso}$$

$$(c) \quad 15.3526... e^{-\frac{1}{8}T} = 3 \quad \text{M1}$$

$$e^{-\frac{1}{8}T} = \frac{3}{15.3526...} = 0.1954...$$

$$-\frac{1}{8}T = \ln 0.1954... \quad \text{M1}$$

$$T = 13.06... \text{ or } 13.1 \text{ or } 13 \quad \text{A1}$$