Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number

Candidate Number

AS and A level Further Mathematics
Core Pure Mathematics

Practice Paper
Complex numbers (part 1)

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. $$f(x) = 9x^3 - 33x^2 - 55x - 25.$$

Given that $x = 5$ is a solution of the equation $f(x) = 0$, use an algebraic method to solve $f(x) = 0$ completely.

(Total 5 marks)

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2. $$f(x) = 2x^3 - 6x^2 - 7x - 4.$$

(*a*) Show that $f(4) = 0$.

(1)

(*b*) Use algebra to solve $f(x) = 0$ completely.

(4)

(Total 5 marks)

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3. The roots of the equation

$$2z^3 - 3z^2 + 8z + 5 = 0$$

are z_1, z_2 and z_3.

Given that $z_1 = 1 + 2i$, find z_2 and z_3.

(5)

(Total 5 marks)

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4. The complex numbers z_1 and z_2 are given by

$$z_1 = p + 2\text{i} \text{ and } z_2 = 1 - 2\text{i}$$

where p is an integer.

(*a*) Find $\dfrac{z_1}{z_2}$ in the form $a + b\text{i}$ where a and b are real. Give your answer in its simplest form in terms of p.

(4)

Given that $\left|\dfrac{z_1}{z_2}\right| = 13$,

(*b*) find the possible values of p.

(4)

(Total 8 marks)

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5. The complex numbers z and w are given by

$$z = 8 + 3\text{i}, \qquad w = -2\text{i}$$

Express in the form $a + b\text{i}$, where a and b are real constants,

(*a*) $z - w$,

(1)

(*b*) zw.

(2)

(Total 3 marks)

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6. Given that $z_1 = 1 - \text{i}$,

(*a*) find arg (z_1).

(2)

Given also that $z_2 = 3 + 4\text{i}$, find, in the form $a + \text{i}b$, $a, b \in \mathbb{R}$,

(*b*) $z_1 z_2$,

(2)

(*c*) $\dfrac{z_2}{z_1}$.

(3)

In part (*b*) and part (*c*) you must show all your working clearly.

(Total 7 marks)

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7. $$z = 5 - 3\text{i}, \quad w = 2 + 2\text{i}$$

Express in the form $a + b\text{i}$, where a and b are real constants,

(*a*) z^2,

(2)

(*b*) $\dfrac{z}{w}$.

(3)

(Total 5 marks)

Turn over

8. $$z_1 = -2 + \mathrm{i}$$

(*a*) Find the modulus of z_1.

(1)

(*b*) Find, in radians, the argument of z_1, giving your answer to 2 decimal places.

(2)

The solutions to the quadratic equation

$$z^2 - 10z + 28 = 0$$

are z_2 and z_3.

(*c*) Find z_2 and z_3, giving your answers in the form $p \pm \mathrm{i}\sqrt{q}$, where p and q are integers.

(3)

(*d*) Show, on an Argand diagram, the points representing your complex numbers z_1, z_2 and z_3 .

(2)

(Total 8 marks)

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9. $$z = \frac{50}{3 + 4\mathrm{i}}.$$

Find, in the form a + ib where $a, b \in \mathbb{R}$,

(*a*) z,

(2)

(*b*) z^2.

(2)

Find

(*c*) $|z|$,

(2)

(*d*) $\arg z^2$, giving your answer in degrees to 1 decimal place.

(2)

(Total 8 marks)

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10. Given that 2 and $1 - 5\text{i}$ are roots of the equation

$$x^3 + px^2 + 30x + q = 0, \qquad p, q \in \mathbb{R}$$

(*a*) write down the third root of the equation.

(1)

(*b*) Find the value of p and the value of q.

(5)

(*c*) Show the three roots of this equation on a single Argand diagram.

(2)

Total 8 marks)

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11. Given that $x = \dfrac{1}{2}$ is a root of the equation

$$2x^3 - 9x^2 + kx - 13 = 0, \qquad k \in \mathbb{R}$$

find

(*a*) the value of k,

(3)

(*b*) the other 2 roots of the equation.

(4)

(Total 7 marks)

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12. (i) The complex number w is given by

$$w = \frac{p - 4\text{i}}{2 - 3\text{i}}$$

where p is a real constant.

(a) Express w in the form $a + b\text{i}$, where a and b are real constants.
Give your answer in its simplest form in terms of p.

(3)

Given that $\arg w = \frac{\pi}{4}$

(b) find the value of p.

(2)

(ii) The complex number z is given by

$$z = (1 - \lambda\,\text{i})(4 + 3\text{i})$$

where λ is a real constant.

Given that

$$|z| = 45$$

find the possible values of λ.
Give your answers as exact values in their simplest form.

(3)

(Total 8 marks)

13.

$$z_1 = 3\text{i} \text{ and } z_2 = \frac{6}{1 + \text{i}\sqrt{3}}.$$

(a) Express z_2 in the form $a + \text{i}b$, where a and b are real numbers.

(2)

(b) Find the modulus and the argument of z_2, giving the argument in radians in terms of π.

(4)

(c) Show the three points representing z_1, z_2 and $(z_1 + z_2)$ respectively, on a single Argand diagram.

(2)

(Total 8 marks)

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14. The complex number z is given by

$$z = \frac{p + 2\mathrm{i}}{3 + p\mathrm{i}}$$

where p is an integer.

(a) Express z in the form $a + b\mathrm{i}$ where a and b are real. Give your answer in its simplest form in terms of p.

(4)

(b) Given that $\arg(z) = \theta$, where $\tan\theta = 1$ find the possible values of p.

(5)

(Total 9 marks)

15.

$$\mathrm{f}(x) = (4x^2 + 9)(x^2 - 2x + 5)$$

(a) Find the four roots of $\mathrm{f}(x) = 0$.

(4)

(b) Show the four roots of $\mathrm{f}(x) = 0$ on a single Argand diagram.

(2)

(Total 6 marks)

TOTAL FOR PAPER: 100 MARKS