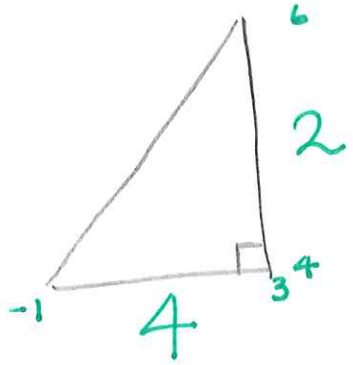
chapter 6 COORDINATES GEOMETRY

(1)

centre $\left(\frac{-1+3}{2} , \frac{4+6}{2} \right)$

$(1 , 5)$

radius



$\sqrt{4^2 + 2^2}$

$= \sqrt{20}$

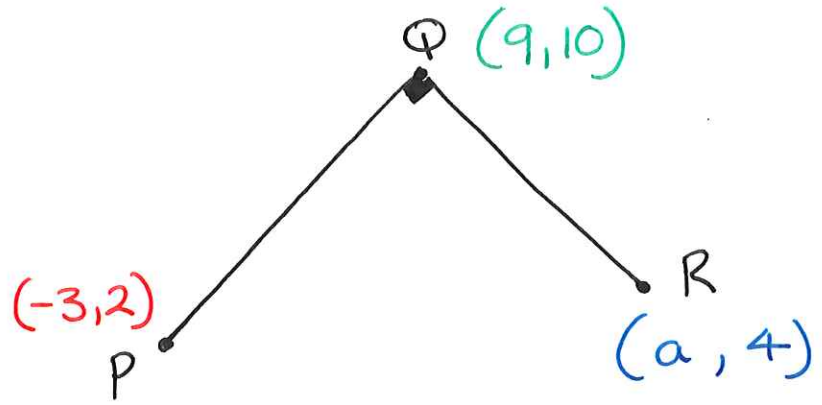
hence radius $= \frac{\sqrt{20}}{2}$

$$(x-1)^2 + (y-5)^2 = \left(\frac{\sqrt{20}}{2}\right)^2$$

$$(x-1)^2 + (y-5)^2 = 5$$

(2)

a)



Gradient of PQ $= \frac{10-2}{9--3} = \frac{8}{12} = \frac{2}{3}$

$\therefore$ Gradient of QR $= -\frac{3}{2}$

OR

Equation of QR $y = -\frac{3}{2}x + c$

then using (9,10) $10 = -\frac{3}{2}(9) + c$

$c = 23.5$

$y = -\frac{3}{2}x + 23.5$

then using (a,4) $4 = -\frac{3}{2}(a) + 23.5$

$a = 13$

$-\frac{3}{2} = \frac{10-4}{9-a}$

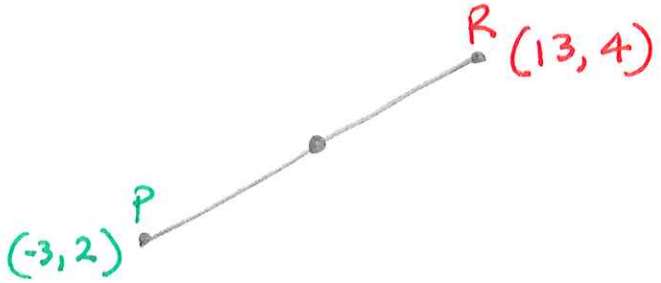
$-27 + 3a = 12$

$3a = 39$

$a = 13$

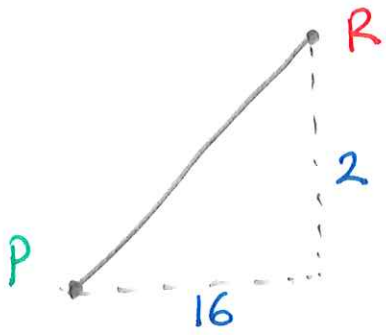
(2)

b)



$$\left(\frac{-3+13}{2} , \frac{2+4}{2} \right)$$

$$(5 , 3)$$



Diameter $= \sqrt{16^2 + 2^2}$

$= \sqrt{260}$

$\therefore$ Radius $= \frac{\sqrt{260}}{2}$

$$(x-5)^2 + (y-3)^2 = \left(\frac{\sqrt{260}}{2}\right)^2$$

$$(x-5)^2 + (y-3)^2 = 65$$

(3)

a) $x^2 + y^2 - 10x + 9 = 0$

$x^2 - 10x + y^2 + 9 = 0$

$(x-5)^2 - 25 + (y+0)^2 + 9 = 0$

$(x-5)^2 + (y+0)^2 = 25 - 9$

$(x-5)^2 + (y+0)^2 = 16$

centre $(5, 0)$

b) Radius $= \sqrt{16} = 4$

c) $y = 0$ $\quad (x-5)^2 + (0+0)^2 = 16$

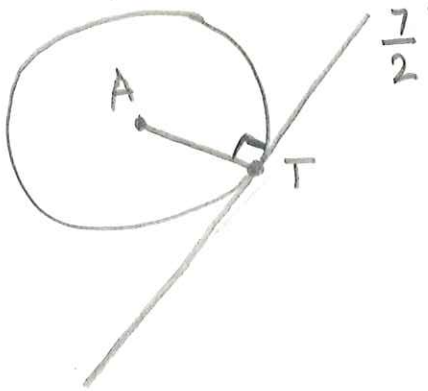
$(x-5)^2 = 16$

$x - 5 = \pm 4$

$x = 9$ or 1

$(9, 0)$ $(1, 0)$

d)



Gradient AT $= -\frac{2}{7}$

$$y = mx + c$$

$$y = -\frac{2}{7}x + c$$

$$(5, 0)$$

$$0 = -\frac{2}{7}(5) + c$$

$$c = \frac{10}{7}$$

Equation of AT

$$y = -\frac{2}{7}x + \frac{10}{7}$$

(4)

a) $x^2 + y^2 - 6x + 4y = 12$

$x^2 - 6x + y^2 + 4y = 12$

$(x-3)^2 - 9 + (y+2)^2 - 4 = 12$

$(x-3)^2 + (y+2)^2 = 25$

Centre $(3, -2)$

Radius $= \sqrt{25} = 5$

b) $\left(\frac{-1+7}{2}, \frac{-5+1}{2}\right)$

$(3, -2)$ passes through the centre

$\sqrt{(7--1)^2 + (-5-1)^2} = \sqrt{8^2 + 6^2} = \sqrt{100}$

$\therefore$ diameter $= 10$

radius $= 5$

hence diameter

(4)

c) $R(0, y)$

$$x^2 + y^2 - 6x + 4y = 12$$

$$x = 0$$

$$0^2 + y^2 - 6(0) + 4y = 12$$

$$y^2 + 4y = 12$$

$$y^2 + 4y - 12 = 0$$

$$(y + 6)(y - 2) = 0$$

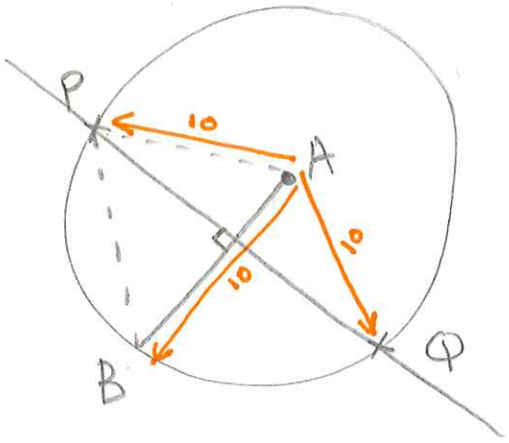
$$y = -6 \quad y = 2$$

R lies on positive y axis

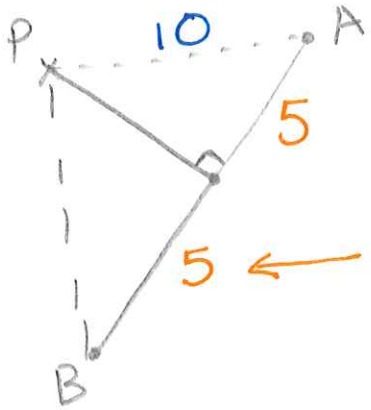
$$\therefore \quad (0, 2)$$

(5)

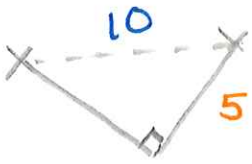
c)



NOT TO SCALE



← half radius



$\sqrt{10^2 - 5^2} = \sqrt{75}$

$\sqrt{75} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

$\therefore PQ = 2 \times 5\sqrt{3}$

$= 10\sqrt{3}$

(5)

a) $(x-2)^2 + (y-1)^2 = r^2$

$r = \sqrt{8^2+6^2} = \sqrt{100} = 10$

$(x-2)^2 + (y-1)^2 = 100$

b) gradient AB $= \frac{7-1}{10-2} = \frac{6}{8} = \frac{3}{4}$

$\therefore$ gradient $L = -\frac{4}{3}$

$y = mx + c$

$y = -\frac{4}{3}x + c$

$(10, 7)$

$7 = -\frac{4}{3}(10) + c$

$c = \frac{61}{3}$

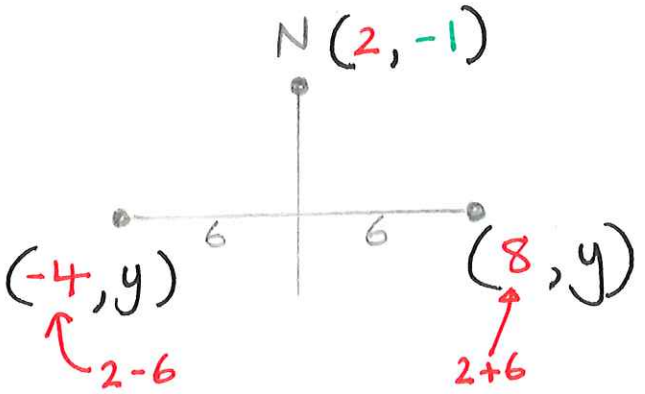
$y = -\frac{4}{3}x + \frac{61}{3}$

(6)

a) $(2, -1)$

b) $\sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5$

c)



$(x-2)^2 + (y+1)^2 = \frac{169}{4}$

when $x = -4$ $\quad (-6)^2 + (y+1)^2 = \frac{169}{4}$

$(y+1)^2 = 6.25$

$y+1 = \pm 2.5$

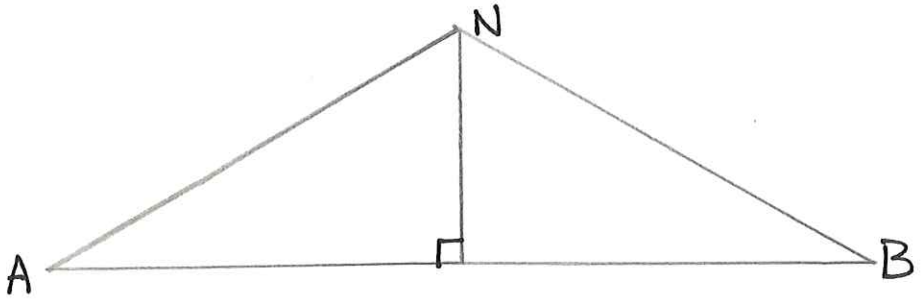
$y = -3.5$ or 1.5

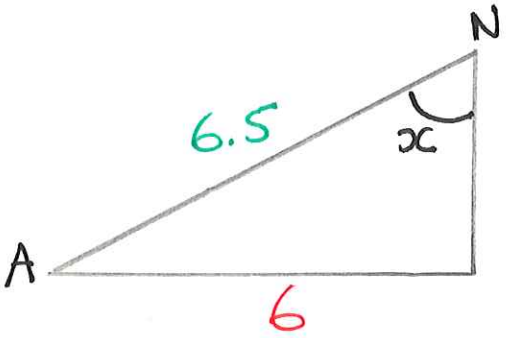
hence B $(8, -3.5)$

A $(-4, -3.5)$

(6)

d)



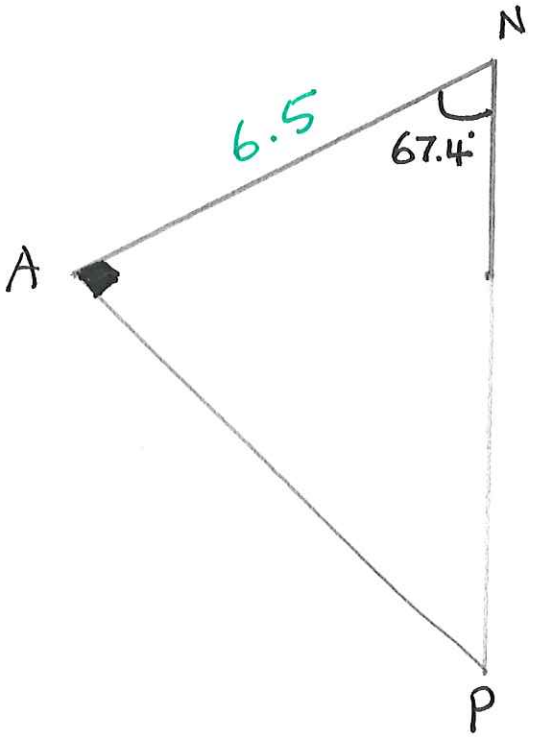


$$\sin x = \frac{6}{6.5}$$

$$x = 67.4^\circ$$

$$\therefore \underset{2x}{ANB} = \underline{\underline{134.8^\circ}}$$

c)



$$\tan 67.4 = \frac{AP}{6.5}$$

$$AP = \underline{\underline{15.6}}$$