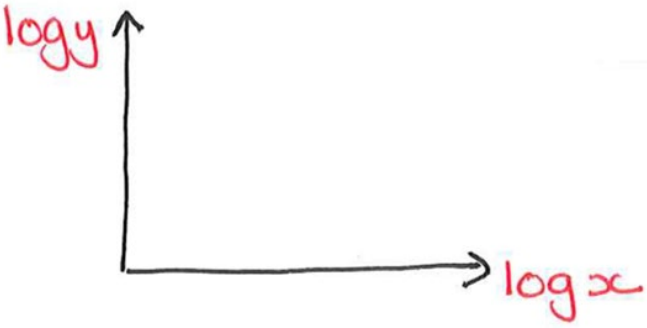
Regression (Non-Linear)

Non-Linear Model

$$y = ax^n$$

Code to Linear Model

$$\log y = \log ax^n$$
$$\log y = \log a + \log x^n$$
$$\log y = \log a + n \log x$$



Compare with y=mx+c

$$\log y = n \log x + \log a$$
$$y = m \quad x \quad + \quad c$$

gradient = n

y-intercept = $\log a$

Example:

Find the exponential equation that can model the data in the form $y = ax^n$
given coding to linear model gives

$y = 0{\cdot}0792x - 0{\cdot}2215$

code to linear model

$$y = ax^n$$
$$\log y = \log a + n \log x$$

y-intercept

$$\log a = -0{\cdot}2215$$
$$\log_{10} a = -0.2215$$
$$10^{-0.2215} = a$$
$$a = 0.600 \text{ (3sf)}$$

gradient

$n = 0.0792$

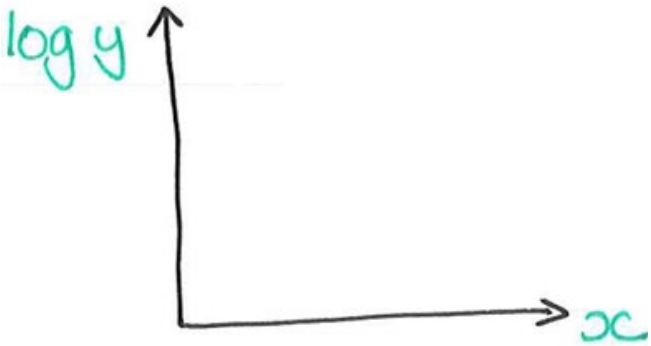
$$y = (0.6)(x)^{0.0792}$$

Non-Linear Model

$$y = ab^x$$

Code to Linear Model

$$\log y = \log ab^x$$
$$\log y = \log a + \log b^x$$
$$\log y = \log a + x \log b$$



Compare with y=mx+c

$$\log y = \log b\, x + \log a$$
$$y = m \quad x \quad + \quad c$$

gradient = log b

y-intercept = log a

Example:

Find the exponential equation that can model the data in the form $g = ab^t$
given coding to linear model gives

$y = 0{\cdot}0792x - 0{\cdot}2215$

code to linear model

$$g = ab^t$$
$$\log g = \log a + t \log b$$

y-intercept

$$\log a = -0{\cdot}2215$$
$$\log_{10} a = -0.2215$$
$$10^{-0.2215} = a$$
$$a = 0.600 \text{ (3sf)}$$

gradient

$$\log b = 0.0792$$
$$\log_{10} b = 0.0792$$
$$10^{0.0792} = b$$
$$b = 1.20 \text{ (3sf)}$$

$$g = (0.6)(1.2)^t$$

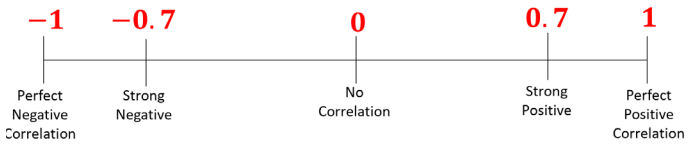
Year 2 Applied Chp 1 – Regression and Correlation

PMCC

Product Moment Correlation Coefficient is the Measurement of Linear Correlation

$$\text{PMCC} = r$$

$$-1 \leq r \leq 1$$



Instructions for **Casio ClassWiz**:

6: Statistics — Press MODE then select 'Statistics'.

$y = a + bx$ — We want to measure **linear** correlation, so select $y = a + bx$

Data Entry

x	y
1	3
2	6
3	5
4	8

PMCC — Press OPTN then choose "Regression Calc" to obtain r.

(You should obtain $r = 0.868$)

Hypothesis Testing for Correlation

Step 1) Write a hypothesis:

H_0: $\rho = 0$ H_1: $\rho > 0$	H_0: $\rho = 0$ H_1: $\rho < 0$	H_0: $\rho = 0$ H_1: $\rho \neq 0$
H_0: *No correlaton* H_1: *Positve correlation*	H_0: *No correlaton* H_1: *Negative correlation*	H_0: *No correlaton* H_1: *There is correlation*

r denotes the PMCC for a sample,
ρ denoted the PMCC for a whole population,
It is the Greek letter rho.

Step 2) Use a PMCC table to identify the minimum value needed to make the correlation statistically significant:

Product Moment Coefficient					
Level					Sample size, n
0.10	0.05	0.025	0.01	0.005	
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.5509	0.6694	0.7545	0.8329	0.8745	7
0.5067	0.6215	0.7067	0.7887	0.8343	8
0.4716	0.5822	0.6664	0.7498	0.7977	9
0.4428	0.5494	0.6319	0.7155	0.7646	10
0.4187	0.5214	0.6021	0.6851	0.7348	11
0.3981	0.4973	0.5760	0.6581	0.7079	12
0.3802	0.4762	0.5529	0.6339	0.6835	13
0.3646	0.4575	0.5324	0.6120	0.6614	14
0.3507	0.4409	0.5140	0.5923	0.6411	15
0.3383	0.4259	0.4973	0.5742	0.6226	16
0.3271	0.4124	0.4821	0.5577	0.6055	17
0.3170	0.4000	0.4683	0.5425	0.5897	18
0.3077	0.3887	0.4555	0.5285	0.5751	19
0.2992	0.3783	0.4438	0.5155	0.5614	20
0.2914	0.3687	0.4329	0.5034	0.5487	21
0.2841	0.3598	0.4227	0.4921	0.5368	22
0.2774	0.3515	0.4133	0.4815	0.5256	23
0.2711	0.3438	0.4044	0.4716	0.5151	24
0.2653	0.3365	0.3961	0.4622	0.5052	25
0.2598	0.3297	0.3882	0.4534	0.4958	26
0.2546	0.3233	0.3809	0.4451	0.4869	27
0.2497	0.3172	0.3739	0.4372	0.4785	28
0.2451	0.3115	0.3673	0.4297	0.4705	29
0.2407	0.3061	0.3610	0.4226	0.4629	30
0.2070	0.2638	0.3120	0.3665	0.4026	40
0.1843	0.2353	0.2787	0.3281	0.3610	50
0.1678	0.2144	0.2542	0.2997	0.3301	60
0.1550	0.1982	0.2352	0.2776	0.3060	70
0.1448	0.1852	0.2199	0.2597	0.2864	80
0.1364	0.1745	0.2072	0.2449	0.2702	90
0.1292	0.1654	0.1966	0.2324	0.2565	100

Step 3) Accept or Reject

$H_0: \rho = 0$ $H_1: \rho > 0$	Table PMCC: **0.35** Sample PMCC: **0.25**	Accept H_0
$H_0: \rho = 0$ $H_1: \rho < 0$	Table PMCC: **-0.5** Sample PMCC: **-0.7**	Reject H_0 Accept H_1
$H_0: \rho = 0$ $H_1: \rho \neq 0$	Table PMCC: **0.4** Sample PMCC: **0.5**	Reject H_0 Accept H_1

Example:
A scientist takes 30 observations of the masses of two reactants in an experiment.

She calculates a product moment correlation coefficient of r=−0.45.

The scientist believes there is no correlation between the masses of the two reactants.

Test at the 10% level of significance, the scientist's claim, stating your hypotheses clearly.

Two-tailed so 5% at each tail.

Product Moment Coefficient					
	Level				Sample
0.10	0.05	0.025	0.01	0.005	size, n
0.8000	0.9000	0.9500	0.9800	0.9900	4
0.6870	0.8054	0.8783	0.9343	0.9587	5
0.6084	0.7293	0.8114	0.8822	0.9172	6
0.2992	0.3783	0.4438	0.5155	0.5614	20
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0.2070	0.2638	0.3120	0.3665	0.4026	40
0.1843	0.2353	0.2787	0.3281	0.3610	50
0.1678	0.2144	0.2542	0.2997	0.3301	60

$$H_0: \rho = 0$$
$$H_1: \rho \neq 0$$

Sample size $= 30$

Critical value at 5% significance: 0.306

$-0.45 < -0.3061$

Accept H_1

There is evidence, that there is a correlation between the masses of the two reactants.