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**Pearson Edexcel GCE**

| Centre Number | Candidate Number |
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**A level Mathematics**

**Practice Paper**
**Mechanics – Kinematics of a particle**

**You must have:**
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

**Instructions**

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

**Information**

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 12 questions in this question paper. The total mark for this paper is 154.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

**Advice**

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**1.** [*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle $P$ moves in such a way that its velocity $\mathbf{v}$ m s$^{-1}$ at time $t$ seconds is given by

$$\mathbf{v} = (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}.$$

(*a*) Find the magnitude of the acceleration of $P$ when $t = 1$.

**(5)**

Given that, when $t = 0$, the position vector of $P$ is **i** metres,

(*b*) find the position vector of $P$ when $t = 3$.

**(5)**

**(Total 10 marks)**

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**2.** At time $t$ seconds, where $t \geq 0$, a particle $P$ is moving on a horizontal plane with acceleration $[(3t^2 - 4t)\mathbf{i} + (6t - 5)\mathbf{j}]$ m s$^{-2}$.

When $t = 3$ the velocity of $P$ is $(11\mathbf{i} + 10\mathbf{j})$ m s$^{-1}$.

Find

(*a*) the velocity of $P$ at time $t$ seconds,

**(5)**

(*b*) the speed of $P$ when it is moving parallel to the vector **i**.

**(4)**

**(Total 9 marks)**

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**3.** A particle $P$ is moving in a plane. At time $t$ seconds, $P$ is moving with velocity $\mathbf{v}$ m s$^{-1}$, where $\mathbf{v} = 2t\mathbf{i} - 3t^2\,\mathbf{j}$.

Find

(*a*) the speed of $P$ when $t = 4$,

**(2)**

(*b*) the acceleration of $P$ when $t = 4$.

**(3)**

Given that $P$ is at the point with position vector $(-4\mathbf{i} + \mathbf{j})$ m when $t = 1$,

(*c*) find the position vector of $P$ when $t = 4$.

**(5)**

**(Total 10 marks)**

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**4.** Two particles $P$ and $Q$, of masses $2m$ and $m$ respectively, are on a smooth horizontal table. Particle $Q$ is at rest and particle $P$ collides directly with it when moving with speed $u$. After the collision the total kinetic energy of the two particles is $\frac{3}{4}mu^2$. Find

($a$) the speed of $Q$ immediately after the collision,

**(10)**

($b$) the coefficient of restitution between the particles.

**(3)**

**(Total 13 marks)**

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**5.**

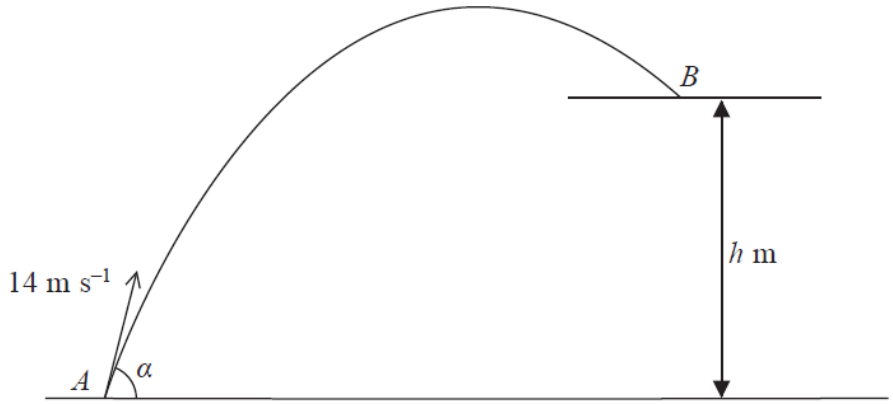


**Figure 1**

A small ball is projected with speed 14 m $s^{-1}$ from a point $A$ on horizontal ground. The angle of projection is $\alpha$ above the horizontal. A horizontal platform is at height $h$ metres above the ground. The ball moves freely under gravity until it hits the platform at the point $B$, as shown in Figure 1. The speed of the ball immediately before it hits the platform at $B$ is 10 m $s^{-1}$.

($a$) Find the value of $h$.

**(4)**

Given that $\sin \alpha = 0.85$,

($b$) find the horizontal distance from $A$ to $B$.

**(8)**

**(Total 12 marks)**

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**6.**

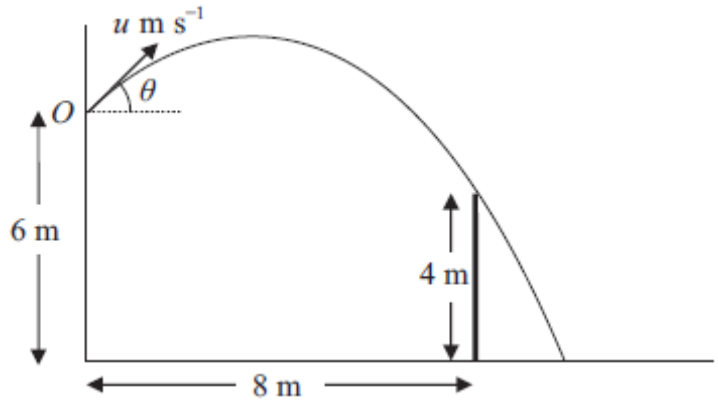


**Figure 2**

A ball is thrown from a point $O$, which is 6 m above horizontal ground. The ball is projected with speed $u$ m s$^{-1}$ at an angle $\theta$ above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through $O$, as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.

($a$) Show that $\tan\theta = 2.2$.

**(5)**

($b$) Find the value of $u$.

**(2)**

The ball hits the ground $T$ seconds after projection.

($c$) Find the value of $T$.

**(3)**

Immediately before the ball hits the ground the direction of motion of the ball makes an angle $\alpha$ with the horizontal.

($d$) Find $\alpha$.

**(5)**

**(Total 15 marks)**

**7.** [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

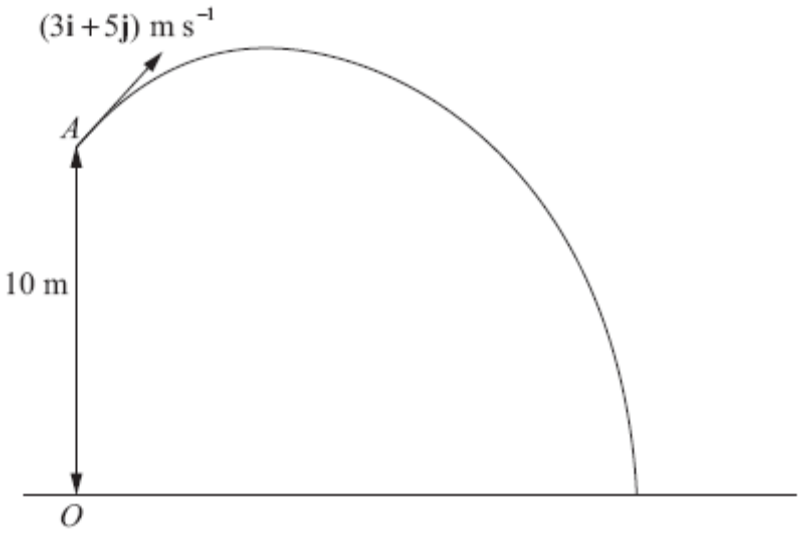


**Figure 3**

At time $t = 0$, a particle $P$ is projected from the point $A$ which has position vector $10\mathbf{j}$ metres with respect to a fixed origin $O$ at ground level. The ground is horizontal. The velocity of projection of $P$ is $(3\mathbf{i} + 5\mathbf{j})$ m s$^{-1}$, as shown in Figure 3. The particle moves freely under gravity and reaches the ground after $T$ seconds.

(*a*) For $0 \leq t \leq T$, show that, with respect to $O$, the position vector, **r** metres, of $P$ at time $t$ seconds is given by

$$\mathbf{r} = 3t\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$$

**(3)**

(*b*) Find the value of $T$.

**(3)**

(*c*) Find the velocity of $P$ at time $t$ seconds $(0 \leq t \leq T)$.

**(2)**

When $P$ is at the point $B$, the direction of motion of $P$ is 45° below the horizontal.

(*d*) Find the time taken for $P$ to move from $A$ to $B$.

**(2)**

(*e*) Find the speed of $P$ as it passes through $B$.

**(2)**

**(Total 12 marks)**

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**8.** [*In this question, the unit vectors* **i** *and* **j** *are horizontal and vertical respectively.*]

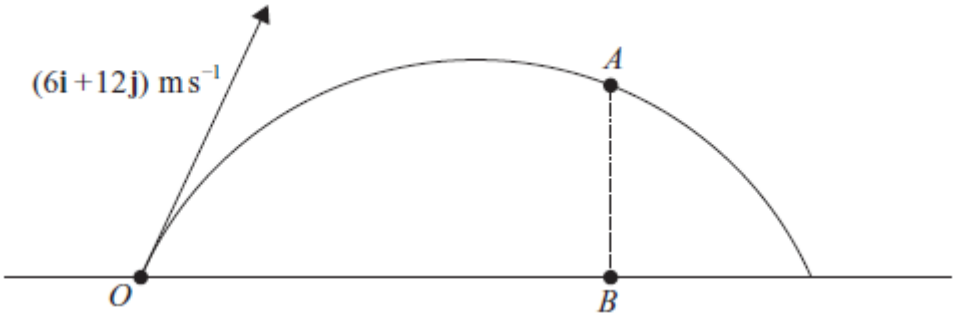


**Figure 4**

The point $O$ is a fixed point on a horizontal plane. A ball is projected from $O$ with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s$^{-1}$, and passes through the point $A$ at time $t$ seconds after projection. The point $B$ is on the horizontal plane vertically below $A$, as shown in Figure 4. It is given that $OB = 2AB$.

Find

($a$) the value of $t$,

**(7)**

($b$) the speed, $V$ m s$^{-1}$, of the ball at the instant when it passes through $A$.

**(5)**

At another point $C$ on the path the speed of the ball is also $V$ m s$^{-1}$.

($c$) Find the time taken for the ball to travel from $O$ to $C$.

**(3)**

**(Total 15 marks)**

**9.**

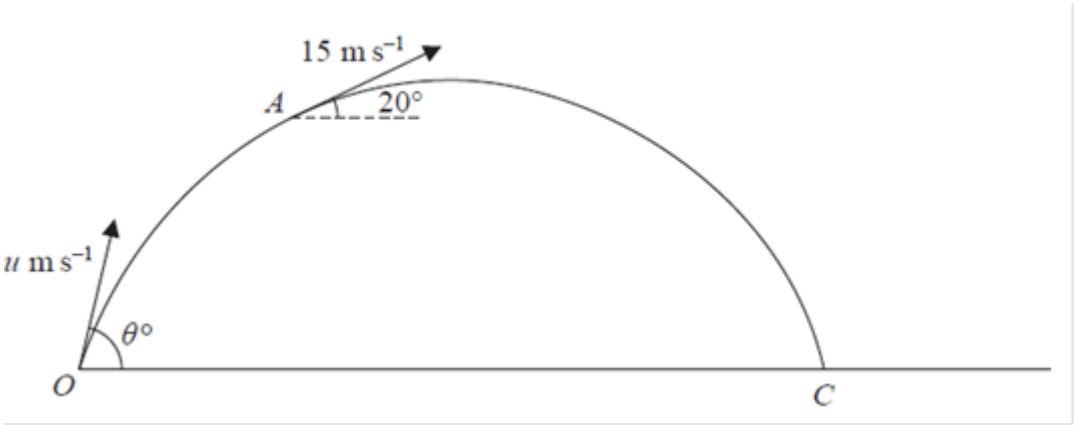


**Figure 5**

At time $t = 0$, a particle is projected from a fixed point $O$ on horizontal ground with speed $u$ m s$^{-1}$ at an angle $\theta°$ to the horizontal. The particle moves freely under gravity and passes through the point $A$ when $t = 4$ s. As it passes through $A$, the particle is moving upwards at 20° to the horizontal with speed 15 m s$^{-1}$, as shown in Figure 5.

($a$) Find the value of $u$ and the value of $\theta$.

**(7)**

At the point $B$ on its path the particle is moving downwards at 20° to the horizontal with speed 15 m s$^{-1}$.

($b$) Find the time taken for the particle to move from $A$ to $B$.

**(2)**

The particle reaches the ground at the point $C$.

($c$) Find the distance $OC$.

**(3)**

**(Total 12 marks)**

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**10.**

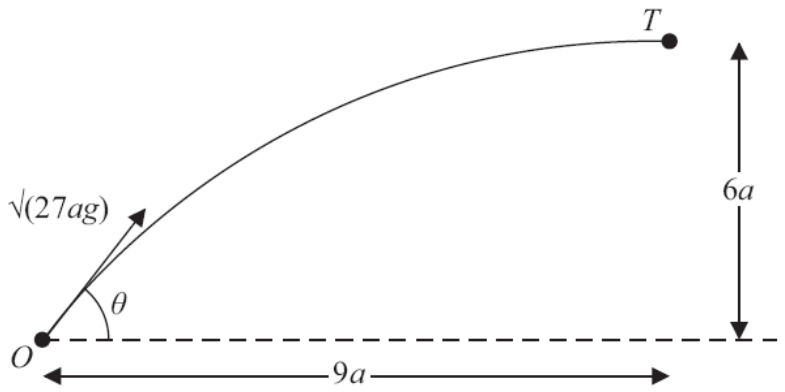


**Figure 6**

A small ball is projected from a fixed point $O$ so as to hit a target $T$ which is at a horizontal distance $9a$ from $O$ and at a height $6a$ above the level of $O$. The ball is projected with speed $\sqrt{(27ag)}$ at an angle $\theta$ to the horizontal, as shown in Figure 6. The ball is modelled as a particle moving freely under gravity.

($a$) Show that $\tan^2\theta - 6\tan\theta + 5 = 0$.

**(7)**

The two possible angles of projection are $\theta_1$ and $\theta_2$, where $\theta_1 > \theta_2$.

($b$) Find $\tan\theta_1$ and $\tan\theta_2$.

**(3)**

The particle is projected at the larger angle $\theta_1$.

($c$) Show that the time of flight from $O$ to $T$ is $\sqrt{\left(\frac{78a}{g}\right)}$.

**(3)**

($d$) Find the speed of the particle immediately before it hits $T$.

**(3)**

**(Total 16 marks)**

**11.**

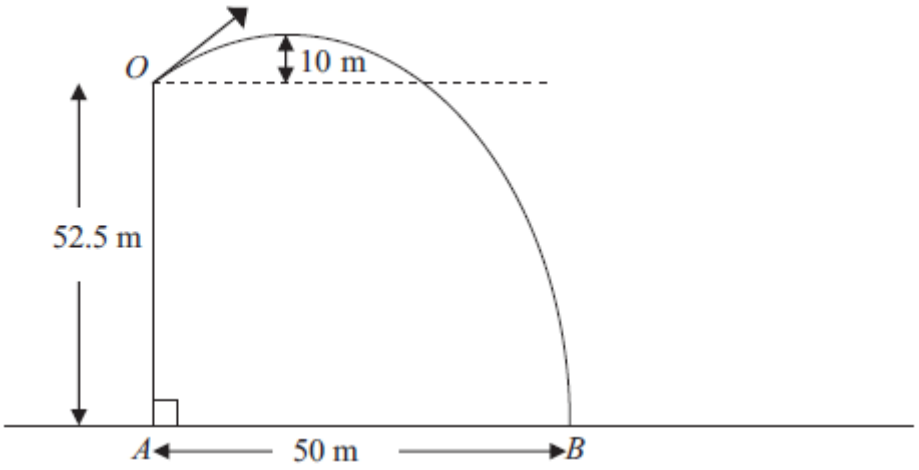


**Figure 7**

A small stone is projected from a point $O$ at the top of a vertical cliff $OA$. The point $O$ is 52.5 m above the sea. The stone rises to a maximum height of 10 m above the level of $O$ before hitting the sea at the point $B$, where $AB = 50$ m, as shown in Figure 7. The stone is modelled as a particle moving freely under gravity.

($a$) Show that the vertical component of the velocity of projection of the stone is 14 m $s^{-1}$.

**(3)**

($b$) Find the speed of projection.

**(9)**

($c$) Find the time after projection when the stone is moving parallel to $OB$.

**(5)**

**(Total 17 marks)**

**12.** A particle is projected from a point $O$ with speed $u$ at an angle of elevation $\alpha$ above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance $x$, its height above $O$ is $y$.

($a$) Show that

$$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}.$$

**(4)**

A girl throws a ball from a point $A$ at the top of a cliff. The point $A$ is 8 m above a horizontal beach. The ball is projected with speed 7 m $s^{-1}$ at an angle of elevation of 45°. By modelling the ball as a particle moving freely under gravity,

($b$) find the horizontal distance of the ball from $A$ when the ball is 1 m above the beach.

**(5)**

A boy is standing on the beach at the point $B$ vertically below $A$. He starts to run in a straight line with speed $v$ m $s^{-1}$, leaving $B$ 0.4 seconds after the ball is thrown.

He catches the ball when it is 1 m above the beach.

($c$) Find the value of $v$.

**(4)**

**(Total 13 marks)**

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**TOTAL FOR PAPER: 154 MARKS**