Year 2 Pure Chapter 8: Parametric Equations - Exam Questions (Answers)

1. (a)

$y = t(9 - t^2)$

$y = 0$

$0 = t(9 - t^2)$

$t = 0$ or $t = -3$ or $t = 3$

$x = 5t^2 - 4$

when $t = 0$,

$x = -4$ or 41 or 41

2. (a)

$x = 2 \cos 2t$ $y = 6 \sin t$

$x/2 = \cos 2t$ $y/6 = \sin t$

$\cos 2t = 1 - 2 \sin^2 t$

$x/2 = 1 - 2 (y/6)^2$

$x/2 = 1 - y^2/18$

$9x = 18 - y^2$

y = square root of $18 - 9x$

k = 2 because cant square root a negative

(b) $0 \leq f(x) \leq 6$ Either $0 \leq f(x)$ or $f(x) \leq 6$ B1

Accept $0 \leq y \leq 6$, [0, 6] B1

3. (a) $y = 2 \sin t \cos t$ M1

$y = 2 \sqrt{1-\cos^2 t} \cos t$ M1

$y = 2x \sqrt{1-x^2}$ A1

(b) $y = -2x\sqrt{1-x^2}$ B1

4. (a) Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$, or equivalent, to eliminate t M1

$1 + \left(\frac{x}{2}\right)^2 = \frac{2}{y}$ A1

$y = \frac{8}{4 + x^2}$ cao A1

The domain is $x > 0$ B1

5. (a) $x = \ln(t + 2)$,

$$y = \frac{1}{t+1}$$

$$e^x = t + 2$$

$$t = e^x - 2$$

$$y = \frac{1}{e^x - 2 + 1} \Rightarrow y = \frac{1}{e^x - 1}$$

Aliter
Way 2

$$t + 1 = \frac{1}{y} \Rightarrow t = \frac{1}{y} - 1 \text{ or } t = \frac{1-y}{y}$$

$$y(t + 1) = 1 \Rightarrow yt + y = 1 \Rightarrow yt = 1 - y \Rightarrow t = \frac{1-y}{y}$$

$$x = \ln\left(\frac{1}{y} - 1 + 2\right) \text{ or } x = \ln\left(\frac{1-y}{y} + 2\right)$$

$$x = \ln\left(\frac{1}{y} + 1\right)$$

$$e^x = \frac{1}{y} + 1$$

$$e^x - 1 = \frac{1}{y}$$

$$y = \frac{1}{e^x - 1}$$

Aliter
Way 3

$$e^x = t + 2$$

$$t + 1 = e^x - 1$$

$$y = \frac{1}{t+1} \Rightarrow y = \frac{1}{e^x - 1}$$

(b) Domain: $\underline{x > 0}$

$\underline{x > 0}$ or just > 0 B1

6. (a) **Either** obtain t in terms of x and y i,e,

$t = \frac{1}{x} - 1$

or

$t = 1 - \frac{1}{x}$ (or both) M1

Then substitute into other expression

$y = f(x)$ or $x = g(y)$ and rearrange M1

(or put $\frac{1}{x} - 1 = 1 - \frac{1}{y}$ and rearrange)

To obtain $y = \frac{x}{2x-1}$ A1

Or

Substitute into $\frac{x}{2x-1} = \frac{\frac{1}{(1+t)}}{\frac{2}{1+t} - 1}$ M1

$= \frac{1}{2-(1+t)} = \frac{1}{1-t}$ A1

$= y$ M1