Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

AS and A level Further Mathematics
Core Pure Mathematics

Practice Paper
Proof

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 6 questions in this question paper. The total mark for this paper is 53.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (i) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^n - 1) & 5^n \end{pmatrix}$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} (2r-1)^2 = \frac{1}{3} n(4n^2 - 1)$$

(6)

(Total 12 marks)

---

2. (i) A sequence of numbers is defined by

$$u_1 = 6, \qquad u_2 = 27$$
$$u_{n+2} = 6u_{n+1} - 9u_n \qquad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 3^n(n + 1)$$

(6)

(ii) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\mathrm{f}(n) = 3^{3n-2} + 2^{3n+1} \text{ is divisible by 19}$$

(6)

(Total 12 marks)

---

3. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\mathrm{f}(n) = 8^n - 2^n$$

is divisible by 6.

(Total 6 marks)

---

4. Prove by induction, that for $n \in \mathbb{Z}^+$,

(*a*) $\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$

(6)

(*b*) $f(n) = 7^{2n-1} + 5$ is divisible by 12.

(6)

(Total 12 marks)

---

5. A sequence of numbers $u_1, u_2, u_3, u_4, \ldots$, is defined by

$$u_{n+1} = 4u_n + 2, \quad u_1 = 2.$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$u_n = \frac{2}{3}(4^n - 1).$$

(5)

(Total 5 marks)

---

6. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1}$$

is divisible by 5.

(Total 6 marks)

TOTAL FOR PAPER: 53 MARKS