Year 2 Pure Chapter 7: Addition Formula - Exam Questions (Answers)

1. (a) $\tan(45° + \theta) = \dfrac{\tan 45° + \tan\theta}{1 - \tan 45° \tan\theta} = \dfrac{1 + \tan\theta}{1 - \tan\theta}$

(b) Put $\theta = 60°$: $\tan 105° = \dfrac{1+\sqrt{3}}{1-\sqrt{3}}$

$$= \frac{(1+\sqrt{3})^2}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{1+2\sqrt{3}+3}{-2}$$

$$= -2 - \sqrt{3}$$

2. (a) $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ …….(i)

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ …….(ii)

add the two equations (i) & (ii) together

$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2\sin\alpha\cos\beta$

3.

(a) $$\sin\left(x+\frac{\pi}{3}\right)=\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}$$

$$2\sin\left(x+\frac{\pi}{3}\right)=\sin x+\sqrt{3}\cos x$$

(b) $$\cos\left(x+\frac{\pi}{6}\right)=\cos x\cos\frac{\pi}{6}-\sin x\sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x$$

$$\sin x + \frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x+\sqrt{3}\cos x=0$$

Dividing by cos x gives $3\tan x + \sqrt{3}=0$

(c) $\tan x = -\frac{1}{\sqrt{3}}$

hence $x = \frac{5\pi}{6}$ or $x = \frac{11\pi}{6}$

4.

(a) $\cos x \cos\frac{5\pi}{6} - \sin x \sin\frac{5\pi}{6} = \sin x$

$\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$ and $\sin\frac{5\pi}{6} = \frac{1}{2}$

$-\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \sin x$

$\sqrt{3}\cos x + 3\sin x = 0 \Rightarrow \sqrt{3}\sin x + \cos x = 0$

(b) $\tan x = -\frac{1}{\sqrt{3}}$

$x = \frac{5\pi}{6}, \frac{11\pi}{6}$

5.

Solve $2\sin(\theta + 60) = \cos(\theta - 45)$

$\therefore 2(\sin\theta\cos 60^\circ + \cos\theta\sin 60^\circ) = \cos\theta\cos 45^\circ + \sin\theta\sin 45^\circ$

$\therefore \overset{1}{\cancel{2}}\left(\frac{1}{\cancel{2}_{1}}\sin\theta + \frac{\sqrt{3}}{\cancel{2}_{1}}\cos\theta\right) = \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta$

$\therefore \sqrt{2}\sin\theta + \sqrt{6}\cos\theta = \cos\theta + \sin\theta$

$\div \cos\theta$

$\therefore \sqrt{2}\tan\theta + \sqrt{6} = 1 + \tan\theta$

$\therefore (\sqrt{2} - 1)\tan\theta = 1 - \sqrt{6}$

$\therefore \tan\theta = \dfrac{1-\sqrt{6}}{\sqrt{2}-1}$

$\therefore \theta = \tan^{-1}\dfrac{1-\sqrt{6}}{\sqrt{2}-1}$

Ө = 105.94 and 285.94

6.

Proof:

$$\cot(A+B) \equiv \frac{\cos(A+B)}{\sin(A+B)}$$

$$\equiv \frac{\cos A\cos B - \sin A\sin B}{\sin A\cos B + \sin B\cos A}$$

$$\equiv \frac{\dfrac{\cos A\cos B}{\sin A\sin B} - \dfrac{\cancel{\sin A\sin B}}{\cancel{\sin A\sin B}}}{\dfrac{\cancel{\sin A}\cos B}{\cancel{\sin A}\sin B} + \dfrac{\cancel{\sin B}\cos A}{\sin A\,\cancel{\sin B}}}$$

$$\equiv \frac{\cot A\cot B - 1}{\cot B + \cot A}$$

$$\equiv \frac{\cot A\cot B - 1}{\cot A + \cot B}$$