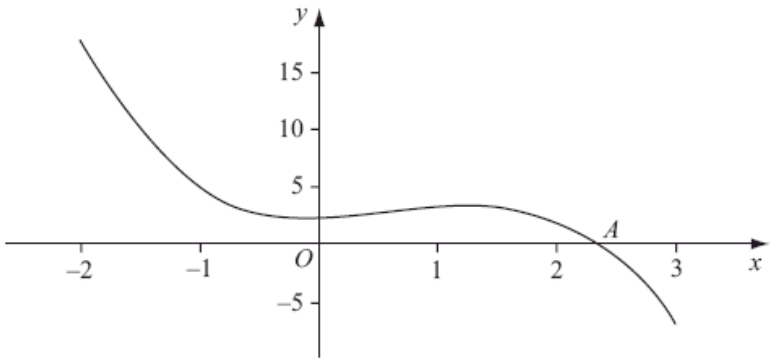
Year 2 Pure Chapter 10: Numerical Methods – Exam Questions (Total Marks 56)

1.



The diagram above shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the x-axis at the point A where $x = \alpha$.

To find an approximation to α, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

(a) Taking $x_0 = 2.5$, find the values of x_1, x_2, x_3 and x_4.
Give your answers to 3 decimal places where appropriate.

(3)

(b) Show that $\alpha = 2.359$ correct to 3 decimal places.

(3)
(Total 6 marks)

2. $f(x) = \ln(x + 2) - x + 1, x > -2, x \in \mathbb{R}$.

(a) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

(2)

(b) Use the iterative formula

$$x_{n+1} = 1n(x_n + 2) + 1, \quad x_0 = 2.5$$

to calculate the values of x_1, x_2 and x_3 giving your answers to 5 decimal places.

(3)

(c) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

(2)

(Total 7 marks)

3. $f(x) = -x^3 + 3x^2 - 1.$

(a) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}.$$

(2)

(b) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of x_2, x_3 and x_4, giving all your answers to 4 decimal places.

(2)

(c) Show that $x = 0.653$ is a root of $f(x) = 0$ correct to 3 decimal places.

(3)

(Total 7 marks)

4.

$$f(x) = x^3 - 2 - \frac{1}{x}, \quad x \neq 0.$$

(a) Show that the equation $f(x) = 0$ has a root between 1 and 2.

(2)

An approximation for this root is found using the iteration formula

$$x_{n+1} = \left(2 + \frac{1}{x_n}\right)^{\frac{1}{3}}, \quad \text{with } x_0 = 1.5.$$

(b) By calculating the values of x_1, x_2, x_3 and x_4, find an approximation to this root, giving your answer to 3 decimal places.

(4)

(c) By considering the change of sign of $f(x)$ in a suitable interval, verify that your answer to part (b) is correct to 3 decimal places.

(2)

(Total 8 marks)

5. $f(x) = x^3 + 2x^2 - 3x - 11$

(a) Show that $f(x) = 0$ can be rearranged as

$$x = \sqrt{\left(\frac{3x+11}{x+2}\right)}, \qquad x \neq -2.$$

The equation $f(x) = 0$ has one positive root α.

(2)

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n+11}{x_n+2}\right)}$ is used to find an approximation to α.

(b) Taking $x_1 = 0$, find, to 3 decimal places, the values of x_2, x_3 and x_4.

(3)

(c) Show that $\alpha = 2.057$ correct to 3 decimal places.

(3)

(Total 8 marks)

6.

$$f(x) = 2x^3 - x - 4.$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}$$

(3)

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)}$$

with $x_0 = 1.35$, to find, to 2 decimal places, the values of x_1, x_2 and x_3.

(3)

The only real root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.

(3)
(Total 9 marks)

7. $f(x) = 3x^3 - 2x - 6$

(a) Show that $f(x) = 0$ has a root, α, between $x = 1.4$ and $x = 1.45$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \ x \neq 0 .$$

(3)

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 4 decimal places.

(3)

(d) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

(3)

(Total 11 marks)