Year 1 Pure Exam Questions – Binomial Expansion (Answers)

Q1.

Question Number	Scheme	Marks
 (a) (b)	$(2+3x)^4$ - Mark (a) and (b) together $2^4 + {}^4C_1 2^3(3x) + {}^4C_2 2^2(3x)^2 + {}^4C_3 2^1(3x)^3 + (3x)^4$ First term of 16 $\left({}^4C_1 \times ... \times x\right) + \left({}^4C_2 \times ... \times x^2\right) + \left({}^4C_3 \times ... \times x^3\right) + \left({}^4C_4 \times ... \times x^4\right)$ $=(16+)\ 96x + 216x^2 + 216x^3 + 81x^4$ Must use Binomial – otherwise A0, A0 $(2-3x)^4 = 16 - 96x + 216x^2 - 216x^3 + 81x^4$	 B1 M1 A1 A1 (4) B1ft (1) 5
Alternative method (a)	$(2+3x)^4 = 2^4(1+\frac{3x}{2})^4$ $2^4(1 + {}^4C_1(\frac{3x}{2}) + {}^4C_2(\frac{3x}{2})^2 + {}^4C_3(\frac{3x}{2})^3 + (\frac{3x}{2})^4)$ Scheme is applied exactly as before	
	Notes for Question	
(a) (b)	B1: The constant term should be 16 in their expansion M1: Two binomial coefficients must be correct and must be with the correct power of x. Accept ${}^4C_1$ or $\binom{4}{1}$ or 4 **as a coefficient, and** ${}^4C_2$ or $\binom{4}{2}$ or 6 **as another**…….. Pascal's triangle may be used to establish coefficients. A1: Any two of the final four terms correct (i.e. two of $96x + 216x^2 + 216x^3 + 81x^4$) in expansion following Binomial Method. A1: All four of the final four terms correct in expansion. (Accept answers without + signs, can be listed with commas or appear on separate lines) B1ft: Award for correct answer as printed above or **ft their previous answer** provided it has **five** terms ft and must be **subtracting the** x **and** x^3 **terms** Allow terms in (b) to be in descending order and allow +-96x and +-216x^3 in the series. (Accept answers without + signs, can be listed with commas or appear on separate lines)	
	e.g. The common error $2^4 + {}^4C_1 2^3 3x + {}^4C_2 2^2 3x^2 + {}^4C_3 2^1 3x^3 + 3x^4 = (16) + 96x + 72x^2 + 24x^3 + 3x^4$ would earn B1, M1, A0, A0, and if followed by $=(16) - 96x + 72x^2 - 24x^3 + 3x^4$ gets B1ft so 3/5 **Fully correct answer with no working can score B1 in part (a) and B1 in part (b).** The question stated use the Binomial theorem and if there is no evidence of its use then M mark and hence A marks cannot be earned. **Squaring the bracket and squaring again** may also earn **B1 M0 A0 A0 B1 if correct** **Omitting the final term** but otherwise correct is B1 M1 A1 A0 B0ft so 3/5 If the series is divided through by 2 or a power of 2 at the final stage after an error or omission resulting in all even coefficients then apply scheme to series before this division and ignore subsequent work (isw)	

Q2.

Question Number	Scheme		Marks
	$\left(1+\frac{3x}{2}\right)^8$		
	$1+12x$	Both terms correct as printed (allow $12x^1$ but not 1^8)	B1
	$\ldots+\frac{8(7)}{2!}\left(\frac{3x}{2}\right)^2+\frac{8(7)(6)}{3!}\left(\frac{3x}{2}\right)^3+\ldots$ $\ldots+{}^8C_2\left(\frac{3x}{2}\right)^2+{}^8C_3\left(\frac{3x}{2}\right)^3+\ldots$	$\left(\frac{8(7)}{2!}\times\ldots\times x^2\right)$ or $\left(\frac{8(7)(6)}{3!}\times\ldots\times x^3\right)$ or $\left({}^8C_2\times\ldots\times x^2\right)$ or $\left({}^8C_3\times\ldots\times x^3\right)$ M1: For <u>either</u> the x^2 term <u>or</u> the x^3 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or 3 or signs) may be wrong or missing.	M1
	<u>Special Case:</u> Allow this M1 <u>only</u> for an attempt at a descending expansion provided the equivalent conditions are met for any term <u>other than the first</u> e.g. $\ldots+8\left(\frac{3x}{2}\right)^7(1)+\frac{8(7)}{2!}\left(\frac{3x}{2}\right)^6(1)^2+\ldots$ $\ldots+{}^8C_1\left(\frac{3x}{2}\right)^7+{}^8C_2\left(\frac{3x}{2}\right)^6+\ldots$		
	$\ldots+63x^2+189x^3+\ldots$	A1: **Either** $63x^2$ **or** $189x^3$ A1: **Both** $63x^2$ **and** $189x^3$	A1A1
	Terms may be listed but must be positive		
			[4]
			Total 4
	Note it is common not to square the 2 in the denominator of $\left(\frac{3x}{2}\right)$ and this gives $1+12x+126x^2+756x^3$. This could score B1M1A0A0.		
	Note $\ldots+{}^8C_2\left(1^4+\frac{3x}{2}\right)^2+{}^8C_3\left(1^3+\frac{3x}{2}\right)^3+\ldots$ would score M0 unless a correct method was implied by later work		

Q3.

Question Number	Scheme	Marks
	$\left(3-\frac{1}{3}x\right)^5$ - $3^5 + {}^5C_1 3^4(-\frac{1}{3}x) + {}^5C_2 3^3(-\frac{1}{3}x)^2 + {}^5C_3 3^2(-\frac{1}{3}x)^3 ...$ First term of 243 $\left({}^5C_1 \times ... \times x\right) + \left({}^5C_2 \times ... \times x^2\right) + \left({}^5C_3 \times ... \times x^3\right)...$ $=(243 ..)\ -\frac{405}{3}x + \frac{270}{9}x^2 - \frac{90}{27}x^3 ...$ $=(243 ..) - 135x + 30x^2 - \frac{10}{3}x^3 ..$	 B1 M1 A1 A1 (4) **[4]**
Alternative method	$\left(3-\frac{1}{3}x\right)^5 = 3^5(1-\frac{x}{9})^5$ $3^5(1 + {}^5C_1(-\frac{1}{9}x) + {}^5C_2(-\frac{1}{9}x)^2 + {}^5C_3(-\frac{1}{9}x)^3\ ...)$ Scheme is applied exactly as before	
	Notes B1: The constant term should be 243 in their expansion M1: Two of the three binomial coefficients must be correct and must be with the correct power of x. Accept ${}^5C_1$ or $\binom{5}{1}$ or 5 **as a coefficient, and** ${}^5C_2$ or $\binom{5}{2}$ or 10 **as another and** ${}^5C_3$ or $\binom{5}{3}$ or 10 **as another……..** Pascal's triangle may be used to establish coefficients. NB: If they only include the first two of these terms then the M1 may be awarded. A1: Two of the final three terms correct – may be unsimplified i.e. two of $-135x + 30x^2 - \frac{10}{3}x^3$ correct, or two of $-\frac{405}{3}x + \frac{270}{9}x^2 - \frac{90}{27}x^3$ (may be just two terms) A1: All three final terms correct and simplified. (Can be listed with commas or appear on separate lines. Accept in reverse order.) Accept correct alternatives to $-\frac{10}{3}$ e.g. $-3\frac{1}{3}$ or $-3.\dot{3}$ the recurring must be clear. 3.3 is not acceptable. Allow e.g. $+-135x$	
	e.g. The common error $3^5 + {}^5C_1 3^4(-\frac{1}{3})x + {}^5C_2 3^3(-\frac{1}{3})x^2 + {}^5C_3 3^2(-\frac{1}{3})x^3 = (243) - 135x - 90x^2 - 30x^3$ would earn B1, M1, A0, A0, so 2/4 If extra terms are given then isw No negative signs in answer also earns B1, M1, A0, A0 If the series is divided through by 3 at the final stage after an error or omission resulting in all multiple of three coefficients then apply scheme to series before this division and ignore subsequent work (isw) **Special Case**: Only gives first three terms $=(243 ..) - 135x + 30x^2$ ... or $243 - \frac{405}{3}x + \frac{270}{9}x^2$ Follow the scheme to give B1 M1 A1 A0 special case. (Do not treat as misread.) Answers such as $243\ + 405 - \frac{1}{3}x + 270 - \frac{1}{9}x^2 + 90 - \frac{1}{27}x^3..$ gain no credit as the binomial coefficients are not linked to the x terms.	

Q4.

Question Number	Scheme		Marks	
(a)	$\left\{(3+bx)^5\right\} = (3)^5 + \underline{{}^5C_1}(3)^4(b\underline{x}) + \underline{{}^5C_2}(3)^3(b\underline{x})^2 + \dots$	243 as a constant term seen.	B1	
	$= 243 + 405bx + 270b^2x^2 + \dots$	$405bx$	B1	
		$\left({}^5C_1 \times \dots \times x\right)$ or $\left({}^5C_2 \times \dots \times x^2\right)$	<u>M1</u>	
		$270b^2x^2$ or $270(bx)^2$	A1	[4]
(b)	$\left\{2(\text{coeff } x) = \text{coeff } x^2\right\} \Rightarrow 2(405b) = 270b^2$	Establishes an equation from their coefficients. Condone 2 on the wrong side of the equation.	M1	
	So, $\left\{b = \dfrac{810}{270} \Rightarrow\right\} b = 3$	$b = 3$ (Ignore $b = 0$, if seen.)	A1	
				[2]
				6

Q5.

Question number	Scheme	Marks
(a).	$(1+\frac{x}{4})^8 = 1+2x+..,\ \ ......$	B1
	$+\dfrac{8\times 7}{2}(\frac{x}{4})^2 + \dfrac{8\times 7\times 6}{2\times 3}(\frac{x}{4})^3,$	M1 A1
	$= \quad +\frac{7}{4}x^2+\frac{7}{8}x^3$ or $= \quad +1.75x^2+0.875x^3$	A1 (4)
(b)	States or implies that $x = 0.1$	B1
	Substitutes their value of x (provided it is <1) into series obtained in (a)	M1
	i.e. $1 + 0.2 + 0.0175 + 0.000875,\ = 1.2184$	A1 cao (3) 7
Alternative for (b) Special case	Starts again and expands $(1+0.025)^8$ to $1+\ 8\times 0.025\ +\dfrac{8\times 7}{2}(0.025)^2+\dfrac{8\times 7\times 6}{2\times 3}(0.025)^3,\ = 1.2184$ (Or 1 + 1/5 + 7/400 + 7/8000 = 1.2184)	B1,M1,A1
Notes	**(a) B1** must be simplified The **method** mark **(M1)** is awarded for an attempt at Binomial to get the third and/or fourth term – need correct binomial coefficient combined with correct power of x. Ignore bracket errors or errors in powers of 4. Accept any notation for ${}^8C_2$ and ${}^8C_3$, e.g. $\binom{8}{2}$ and $\binom{8}{3}$ (unsimplified) or 28 and 56 from Pascal's triangle. (The terms may be listed without + signs) First **A1** is for two completely correct unsimplified terms **A1** needs the fully simplified $\frac{7}{4}x^2$ *and* $\frac{7}{8}x^3$. **(b) B1** – states or uses x =0.1 or $\frac{x}{4}=\frac{1}{40}$ **M1** for substituting their value of x ($0 < \mathbf{x} < 1$) into expansion (e.g. 0.1 (correct) or 0.01, 0.00625 or even 0.025 but **not** 1 nor 1.025 which would earn **M0**) **A1** Should be answer printed cao (not answers which round to) and should follow correct work. **Answer with no working at all** is B0, M0, A0 States 0.1 then just writes down answer is B1 M0A0	

Q6.

Question Number	Scheme	Marks
Q (a)	$(7\times\ldots\times x)$ or $(21\times\ldots\times x^2)$ The 7 or 21 can be in 'unsimplified' form.	M1
	$(2+kx)^7 = 2^7 + 2^6\times 7\times kx + 2^5\times\binom{7}{2}k^2x^2$	
	$= 128;\quad +448kx,\quad +672k^2x^2$ [or $672(kx)^2$]	B1; A1, A1
	(If $672kx^2$ follows $672(kx)^2$, isw and allow A1)	(4)
(b)	$6\times 448k = 672k^2$	M1
	$k = 4$ (Ignore $k=0$, if seen)	A1 (2) **[6]**
(a)	The terms can be 'listed' rather than added. Ignore any extra terms. M1 for <u>either</u> the x term <u>or</u> the x^2 term. Requires <u>correct</u> binomial coefficient in any form <u>with the correct power of x</u>, but the other part of the coefficient (perhaps including powers of 2 and/or k) may be wrong or missing. <u>Allow</u> binomial coefficients such as $\binom{7}{1}, \left(\frac{7}{1}\right), \binom{7}{2}, {}^7C_1, {}^7C_2$. However, $448+kx$ or similar is M0. B1, A1, A1 for the <u>simplified</u> versions seen above. <u>Alternative:</u> Note that a factor 2^7 can be taken out first: $2^7\left(1+\frac{kx}{2}\right)^7$, but the mark scheme still applies. <u>Ignoring subsequent working (isw):</u> Isw if necessary after correct working: e.g. $128+448kx+672k^2x^2$ M1 B1 A1 A1 $= 4+14kx+21k^2x^2$ isw (Full marks are still available in part (b)).	