

Year 2 Pure Chapter 11: Integration - Exam Questions Integration (Total Marks 52)

1. (a) Use integration by parts to find $\int xe^x \, dx$. (3)

(b) Hence find $\int x^2 e^x \, dx$.

(3)
(Total 6 marks)

2. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{(\cos x) + 1} \sin x \, dx = e(e - 1)$$

(Total 6 marks)

3. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x + 1)^2} dx.$$

(Total 6 marks)

4. (a) Find $\int \frac{9x+6}{x} dx, x > 0$.

(2)

(b) Given that $y = 8$ at $x = 1$, solve the differential equation

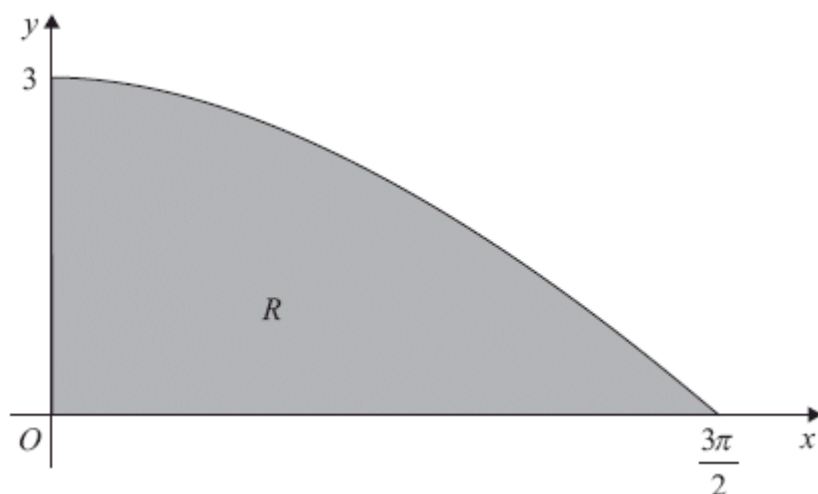
$$\frac{dy}{dx} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = g(x)$.

(6)

(Total 8 marks)

5.



The diagram above shows the finite region R bounded by the x -axis, the y -axis and the curve with equation $y = 3 \cos\left(\frac{x}{3}\right), 0 \leq x \leq \frac{3\pi}{2}$.

The table shows corresponding values of x and y for $y = 3 \cos\left(\frac{x}{3}\right)$.

x	0	$\frac{3\pi}{8}$	$\frac{3\pi}{4}$	$\frac{9\pi}{8}$	$\frac{3\pi}{2}$
y	3	2.77164	2.12132		0

(a) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(b) Using the trapezium rule, with all the values of y from the completed table, find an approximation for the area of R , giving your answer to 3 decimal places.

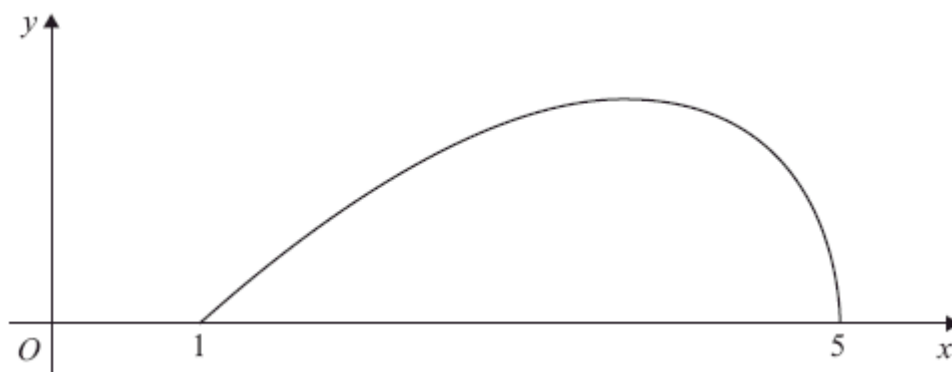
(4)

(c) Use integration to find the exact area of R .

(3)

(Total 8 marks)

6. (a) Find $\int \sqrt{5-x} \, dx$



(2)

The diagram above shows a sketch of the curve with equation

$$y = (x-1)\sqrt{5-x}, \quad 1 \leq x \leq 5$$

- (b) (i) Using integration by parts, or otherwise, find

$$\int (x-1)\sqrt{5-x} \, dx$$

(4)

- (ii) Hence find $\int_1^5 (x-1)\sqrt{5-x} \, dx$.

(2)

(Total 8 marks)

7.

$$f(x) = \frac{4-2x}{(2x+1)(x+1)(x+3)} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{x+3}$$

- (a) Find the values of the constants A , B and C .

(4)

- (b) (i) Hence find $\int f(x) dx$.

(3)

- (ii) Find $\int_0^2 f(x) dx$ in the form $\ln k$, where k is a constant.

(3)

(Total 10 marks)