AS and A level Further Mathematics Practice Paper – Series – Mark scheme

Question	Scheme	Marks
1	$\sum_{r=1}^{n} 3(4r^2-4r+1) = 12\sum_{r=1}^{n} r^2 - 12\sum_{r=1}^{n} r + \sum_{r=1}^{n} 3$	M1
	$= \frac{12}{6}n(n+1)(2n+1) - \frac{12}{2}n(n+1), \quad +3n$	A1, B1
	$= n\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$	M1
	$= n\left[4n^2-1\right] = n(2n+1)(2n-1)$	A1cso
		(5 marks)
2(a)	$\sum_{r=1}^{3n} r^2 = \frac{1}{6}3n(3n+1)(6n+1)$ or $\sum_{r=1}^{3n} r^2 = \frac{1}{2}n(3n+1)(6n+1)$ (oe)	B1
		(1)
2(b)	See $\sum_{r=1}^{2n} r^2 = \frac{1}{3}n(2n+1)(4n+1)$ or equivalent	B1
	Attempt to use $\sum_{r=1}^{3n} r^2 - \sum_{r=1}^{2n} r^2 = \frac{n}{6}\{3(3n+1)(6n+1) - 2(2n+1)(4n+1)\}$	M1
	$= \frac{n}{6}\{(54n^2+27n+3) - (16n^2+12n+2)\}$	dM1
	$= \frac{n}{6}\{(38n^2+15n+1)\}$	A1
	$(a = 38, b = 15, c = 1)$	
		(4)
		(5 marks)

Question	Scheme	Marks
3(a)	$\sum_{r=1}^{n}(r+1)(r+4)$	
	$=\sum_{r=1}^{n} r^2+5r+4$	B1
	$=\frac{n}{6}(n+1)(2n+1)+5\frac{n}{2}(n+1)+4n$	M1 A1
	$=\frac{n}{6}\{(n+1)(2n+1)+15(n+1)+24\}$	dM1
	$=\frac{n}{6}\{(2n^2+3n+1)+15n+15+24\}$	
	$=\frac{n}{6}\left(2n^2+18n+40\right)$ or $=\frac{n}{3}\left(n^2+9n+20\right)$	
	$=\dfrac{n}{3}(n+4)(n+5)$ **** given answer****	A1*
		(5)
3(b)	$\sum_{r=n+1}^{2n}(r+1)(r+4) = \dfrac{2n}{3}(2n+4)(2n+5)-\dfrac{n}{3}(n+4)(n+5)$	M1
	$=\dfrac{n}{3}\{8n^2+36n+40-n^2-9n-20\}$	dM1
	$=\dfrac{n}{3}\{7n^2+27n+20\}=\dfrac{n}{3}(n+1)(7n+20)$	A1
	or $a=7$, $b=20$	
		(3)
		(8 marks)

Question	Scheme	Marks
4(a)	$\sum_{r=1}^{n}\left(r^3 + 6r - 3\right)$	
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{2}n(n+1)} - 3n$	M1A1B1
	$= \frac{1}{4}n^2(n+1)^2 + 3n^2$	
	$= \frac{1}{4}n^2\left((n+1)^2 + 12\right)$	dM1
	$= \frac{1}{4}n^2\left(n^2 + 2n + 13\right)$ **(AG)**	A1 *
		(5)
4(b)	$S_n = \sum_{r=16}^{30}\left(r^3 + 6r - 3\right) = S_{30} - S_{15}$	
	$= \frac{1}{4}(30)^2\left(30^2 + 2(30) + 13\right) - \frac{1}{4}(15)^2\left(15^2 + 2(15) + 13\right)$	M1
	$= 203850$	A1 **cao**
		(2)
		(7 marks)

Question	Scheme		Marks
5(a)	$\sum_{r=1}^{n} r(r+1)(r+5)$		
	$= \sum_{r=1}^{n} r^3 + 6r^2 + 5r$	Multiplying out brackets and an attempt to use at least one of the standard formulae correctly.	M1
	$= \underline{\frac{1}{4}n^2(n+1)^2 + 6.\frac{1}{6}n(n+1)(2n+1) + 5.\frac{1}{2}n(n+1)}$	Correct expression.	A1
	$= \frac{1}{4}n^2(n+1)^2 + n(n+1)(2n+1) + \frac{5}{2}n(n+1)$		
	$= \frac{1}{4}n(n+1)\big(n(n+1) + 4(2n+1) + 10\big)$	Factorising out at least $n(n+1)$	dM1
	$= \frac{1}{4}n(n+1)\big(n^2 + n + 8n + 4 + 10\big)$		
	$= \frac{1}{4}n(n+1)\big(n^2 + 9n + 14\big)$	Correct 3 term quadratic factor	A1
	$= \frac{1}{4}n(n+1)(n+2)(n+7)$*	Correct proof. No errors seen.	A1
			(5)
5(b)	$S_n = \sum_{r=20}^{50} r(r+1)(r+5)$		
	$= S_{50} - S_{19}$		
	$= \frac{1}{4}(50)(51)(52)(57) - \frac{1}{4}(19)(20)(21)(26)$	Use of $S_{50} - S_{19}$	M1
	$= 1889550 - 51870$		
	$= 1837680$	1837680	A1
		Correct answer only 2/2	
			(2)
			(7 marks)

Question	Scheme	Marks
6(a)	$r\left(r^2-3\right)=r^3-3r$	B1
	$\sum_{r=1}^{n} r\left(r^2-3\right)=\sum_{r=1}^{n} r^3-3\sum_{r=1}^{n} r$	
	$=\frac{1}{4}n^2\left(n+1\right)^2-\frac{3}{2}n\left(n+1\right)$	M1A1
	$=\frac{1}{4}n\left(n+1\right)\left(n\left(n+1\right)-6\right)$	M1
	$=\frac{1}{4}n\left(n+1\right)\left(n^2+n-6\right)$	
	$=\frac{1}{4}n\left(n+1\right)\left(n+3\right)\left(n-2\right)$	A1
		(5)
6(b)	$\sum_{r=10}^{50} r\left(r^2-3\right)=\mathrm{f}(50)-\mathrm{f}(9\,or\,10)$	M1
	$=\frac{1}{4}(50)(51)(53)(48)-\frac{1}{4}(9)(10)(12)(7)$	A1
	= 1621800 - 1890	
	= 1619910	A1
		(3)
		(8 marks)

Question	Scheme	Marks
7(a)	$((2r-1)^2 =)4r^2-4r+1$	B1
	Proof by induction will usually score no more marks without use of standard results	
	$\sum_{r=1}^{n}(2r-1)^2=\sum_{r=1}^{n}\left(4r^2-4r+1\right)$	
	$=4\sum r^2-4\sum r+\sum 1$	
	$=\underline{4.\frac{1}{6}n(n+1)(2n+1)-4.\frac{1}{2}n(n+1)},+n$	M1A1B1
	$=\frac{1}{3}n\left[4n^2+6n+2-6n-6+3\right]$	M1
	$=\frac{1}{3}n\left[4n^2-1\right]$	A1
		(6)
7(b)	$\sum_{r=2n+1}^{4n}(2r-1)^2=\mathrm{f}(4n)-\mathrm{f}(2n)\text{ or }\mathrm{f}(2n+1)$	M1
	$=\frac{1}{3}4n\left(4.(4n)^2-1\right)-\frac{1}{3}.2n\left(4.(2n)^2-1\right)$	A1
	$=\frac{2}{3}n\left[128n^2-2-16n^2+1\right]$	
	$=\frac{2}{3}n\left[112n^2-1\right]$	A1
		(3)
		(9 marks)

Question	Scheme	Marks
8(a)	$(r+2)(r+3)=r^2+5r+6$	B1
	$\sum(r^2+5r+6)=\frac{1}{6}n(n+1)(2n+1)+5\times\frac{1}{2}n(n+1),+6n$	M1 B1ft
	$=\frac{1}{3}n\left[\frac{1}{2}(n+1)(2n+1)+\frac{15}{2}(n+1)+18\right]$	M1 A1
	$\left(=\frac{1}{3}n\left[n^2+\frac{3}{2}n+\frac{1}{2}+\frac{15}{2}n+\frac{15}{2}+18\right]\right)$	
		A1*cso
	$=\frac{1}{3}n\left[n^2+9n+26\right]$ *	
		(6)
8(b)	$\sum_{r=n+1}^{3n} =\frac{1}{3}3n\left((3n)^2+9(3n)+26\right)-\frac{1}{3}n\left(n^2+9n+26\right)$	M1A1
	3f(***n***) – f(n or n+1) is M0	
	$(=n(9n^2+27n+26)-\frac{1}{3}n(n^2+9n+26))$	
	$=\frac{2}{3}n\left(\frac{27}{2}n^2+\frac{81}{2}n+39-\frac{1}{2}n^2-\frac{9}{2}n-13\right)$	dM1
	$=\frac{2}{3}n(13n^2+36n+26)$	A1
	$(a=13,b=36,c=26)$	
		(4)
		(10 marks)

Question	Scheme		Marks
9(a)	$\{S_n =\} \sum_{r=1}^{n} (2r-1)^2$		
	$= \sum_{r=1}^{n} 4r^2 - 4r + 1$	Multiplying out brackets and an attempt to use at least one of the two standard formulae correctly.	M1
	$= \underline{4.\frac{1}{6}n(n+1)(2n+1) - 4.\frac{1}{2}n(n+1)} + n$	<u>First two terms correct.</u>	A1
		$+ n$	B1
	$= \frac{2}{3}n(n+1)(2n+1) - 2n(n+1) + n$		
		Attempt to factorise out $\frac{1}{3}n$	M1
	$= \frac{1}{3}n\{2(n+1)(2n+1) - 6(n+1) + 3\}$	Correct expression with $\frac{1}{3}n$ factorised out with no errors seen.	A1
	$= \frac{1}{3}n\{2(2n^2+3n+1) - 6(n+1) + 3\}$		
	$= \frac{1}{3}n\{4n^2+6n+2 - 6n - 6 + 3\}$		
	$= \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(2n+1)(2n-1)$	Correct proof. No errors seen.	A1 *
			(6)
9(b)	$\sum_{r=n+1}^{3n} (2r-1)^2 = S_{3n} - S_n$		
		Use of $S_{3n} - S_n$ or $S_{3n} - S_{n+1}$ with the result from (a) used at least once.	M1
	$= \frac{1}{3}.3n(6n+1)(6n-1) - \frac{1}{3}n(2n+1)(2n-1)$	Correct unsimplified expression. E.g. Allow $2(3n)$ for $6n$.	A1
	$= n(36n^2-1) - \frac{1}{3}n(4n^2-1)$		
	$= \frac{1}{3}n(108n^2 - 3 - 4n^2 + 1)$	Factorising out $\frac{1}{3}n$ (or $\frac{2}{3}n$)	dM1
	$= \frac{1}{3}n(104n^2 - 2)$		
	$= \frac{2}{3}n(52n^2-1)$	$\frac{2}{3}n(52n^2-1)$	A1
	$\{a = 52,\ b = -1\}$		
			(4)
			(10 marks)

Question	Scheme		Marks
10(a)	$n=1, \text{LHS}=1^3=1, \text{RHS}=\frac{1}{4}\times1^2\times2^2=1$	Shows **both** LHS = 1 **and** RHS = 1	B1
	Assume true for n = k When n = k + 1 $\sum_{r=1}^{k+1} r^3=\frac{1}{4}k^2(k+1)^2+(k+1)^3$	Adds $(k+1)^3$ to the given result	M1
	$=\frac{1}{4}(k+1)^2[k^2+4(k+1)]$	Attempt to factorise out $\frac{1}{4}(k+1)^2$	dM1
		Correct expression with $\frac{1}{4}(k+1)^2$ factorised out.	A1
	$=\frac{1}{4}(k+1)^2(k+2)^2$ Must see 4 things: true for $n=1$, assumption true for $n=k$, said true for $n=k+1$ and therefore true for all n	Fully complete proof with no errors and comment. All the previous marks must have been scored.	A1cso
			(5)
10(b)	$\sum(r^3-2)=\sum r^3-\sum 2$ $\sum r^3-\sum 2n$ **is M0**	Attempt two sums	M1
	$=\frac{1}{4}n^2(n+1)^2-2n$	Correct expression	A1
	$=\frac{n}{4}(n^3+2n^2+n-8)$ *	Completion to printed answer with no errors seen.	A1
			(3)
10(c)	$\sum_{r=20}^{r=50}(r^3-2)=\frac{50}{4}\times130042-\frac{19}{4}\times7592$ $(=1625525-36062)$	Attempt $S_{50}-S_{20}$ or $S_{50}-S_{19}$ and substitutes into a correct expression at least once.	M1
		Correct numerical expression (unsimplified)	A1
	$=1\ 589\ 463$	cao	A1
			(3)
			(11 marks)

AS and A level Further Mathematics Practice Paper – Series – Mark scheme

	Source paper	Question number	New spec references	Question description	New AOs
1	FP1 Jan 2013	1		Series	1.1b, 2.1
2	FP1 2016	3		Series	1.1b, 2.1
3	FP1 2015	3		Series	1.1b, 2.1
4	FP1 2012	4		Series	1.1b, 2.1
5	FP1 2011	5		Series	1.1b, 2.1,3.1a
6	FP1 2014R	5		Series	1.1b, 2.1
7	FP1 2014	5		Series	1.1b, 2.1
8	FP1 2013	5		Series	1.1b, 2.1
9	FP1 2011	7		Series	1.1b, 3.1a
10	FP1 Jan 2012	6		Series	1.1b, 2.1