Year 2 Pure Chapter 9: Differentiation – Exam Questions (Answers)

Q1.

(a)	$\dfrac{1}{(x^2+3x+5)} \times \ldots\ldots = \dfrac{2x+3}{(x^2+3x+5)}$	M1,A1	
			(2)
(b)	Applying $\dfrac{vu'-uv'}{v^2}$	M1,	
	$\dfrac{x^2\times -\sin x - \cos x \times 2x}{(x^2)^2} = \dfrac{-x^2\sin x - 2x\cos x}{x^4} = \dfrac{-x\sin x - 2\cos x}{x^3}$ oe	A2,1,0	
			(3)
			5 Marks

Q2.

(a)	From question, $V=\dfrac{4}{3}\pi r^3$, $S=4\pi r^2$, $\dfrac{dV}{dt}=3$		
	$\left\{V=\dfrac{4}{3}\pi r^3 \Rightarrow\right\} \dfrac{dV}{dr}=4\pi r^2$	$\dfrac{dV}{dr}=4\pi r^2$ (Can be implied)	B1 oe
	$\left\{\dfrac{dV}{dr}\times\dfrac{dr}{dt}=\dfrac{dV}{dt}\Rightarrow\right\}$ $(4\pi r^2)\dfrac{dr}{dt}=3$ $\left\{\dfrac{dr}{dt}=\dfrac{dV}{dt}\div\dfrac{dV}{dr}\Rightarrow\right\}$ $\dfrac{dr}{dt}=(3)\dfrac{1}{4\pi r^2}$; $\left\{=\dfrac{3}{4\pi r^2}\right\}$	$\left(\text{Candidate's }\dfrac{dV}{dr}\right)\times\dfrac{dr}{dt}=3$ or $3\div$ Candidate's $\dfrac{dV}{dr}$;	M1 oe
	When $r=4\text{cm}$, $\dfrac{dr}{dt}=\dfrac{3}{4\pi(4)^2}$ $\left\{=\dfrac{3}{64\pi}\right\}$	**dependent on previous M1. see notes**	dM1
	Hence, $\dfrac{dr}{dt}=0.01492077591\ldots(\text{cm}^2\,\text{s}^{-1})$	anything that rounds to 0.0149	A1
			[4]
(b)	$\left\{\dfrac{dS}{dt}=\dfrac{dS}{dr}\times\dfrac{dr}{dt}=\right\}\Rightarrow \dfrac{dS}{dt}=8\pi r\times\dfrac{3}{4\pi r^2}$ $\left\{\text{or }\dfrac{6}{r}\text{ or }8\pi r\times 0.0149\ldots\right\}$	$8\pi r\times$ Candidate's $\dfrac{dr}{dt}$	M1; oe
	When $r=4\text{cm}$, $\dfrac{dr}{dt}=8\pi(4)\times\dfrac{3}{4\pi(4)^2}$ or $\dfrac{6}{4}$ or $8\pi(4)\times 0.0149\ldots$		
	Hence, $\dfrac{dS}{dt}=1.5\ (\text{cm}^2\,\text{s}^{-1})$	anything that rounds to 1.5	A1 cso
			[2] **6**

Q3.

Question Number	Scheme		Marks	
	(a) $y = 0 \Rightarrow t(9-t^2) = t(3-t)(3+t) = 0$			
	$t = 0, 3, -3$	Any one correct value	B1	
	At $t = 0$, $x = 5(0)^2 - 4 = -4$	Method for finding one value of x	M1	
	At $t = 3$, $x = 5(3)^2 - 4 = 41$			
	$\left(\text{At } t = -3,\ x = 5(-3)^2 - 4 = 41\right)$			
	At A, $x = -4$; at B, $x = 41$	Both	A1	(3)
	(b) $\frac{dx}{dt} = 10t$	Seen or implied	B1	
	$\int y\,dx = \int y\frac{dx}{dt}\,dt = \int t(9-t^2)10t\,dt$		M1 A1	
	$= \int (90t^2 - 10t^4)\,dt$			
	$= \frac{90t^3}{3} - \frac{10t^5}{5}\ (+C) \quad \left(= 30t^3 - 2t^5\ (+C)\right)$		A1	
	$\left[\frac{90t^3}{3} - \frac{10t^5}{5}\right]_0^3 = 30\times3^3 - 2\times3^5 \quad (= 324)$		M1	
	$A = 2\int y\,dx = 648 \quad (\text{units}^2)$		A1	(6)
				[9]

Q4.

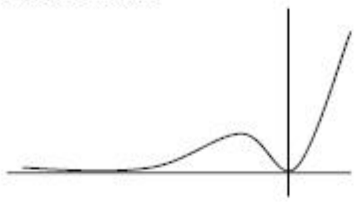
Question Number	Scheme		Marks
(a)	$x^3 - 4y^2 = 12xy$ (eqn *)		
	$x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$ $-512 - 4y^2 = -96y$	Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in y. Condone the loss of $= 0$.	M1
	$4y^2 - 96y + 512 = 0$ $y^2 - 24y + 128 = 0$		
	$(y-16)(y-8) = 0$ $y = \frac{24 \pm \sqrt{576 - 4(128)}}{2}$	An attempt to solve the quadratic in y by either factorising or by the formula or by ***completing the square***.	dM1
	$y = 16$ or $y = 8.$	Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$.	A1
			[3]
(b)	$\left\{ \frac{\cancel{dy}}{\cancel{dx}} \times \right\}$ $3x^2 - 8y\frac{dy}{dx}; = \left(\underline{12y + 12x\frac{dy}{dx}}\right)$	Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = ...$ Correct LHS equation; <u>Correct application of product rule</u>	M1 A1; (B1)
	$\left\{\frac{dy}{dx} = \frac{3x^2 - 12y}{12x + 8y}\right\}$	*not necessarily required.*	
	@ $(-8, 8)$, $\frac{dy}{dx} = \frac{3(64) - 12(8)}{12(-8) + 8(8)} = \frac{96}{-32} = \underline{-3}$, @ $(-8, 16)$, $\frac{dy}{dx} = \frac{3(64) - 12(16)}{12(-8) + 8(16)} = \frac{0}{32} = \underline{0}$.	Substitutes $x = -8$ and *at least one* of their y-values to attempt to find any one of $\frac{dy}{dx}$. One gradient found. Both gradients of $\underline{-3}$ and $\underline{0}$ ***correctly*** found.	dM1 A1 A1 cso
			[6]
			9 marks

Q5.

(a)	$\frac{dx}{dt} = 3, \quad \frac{dy}{dt} = 6t^{-2}$
	$\frac{dy}{dx} = \frac{6t^{-2}}{3} \left\{= \frac{6}{3t^2} = 2t^{-2} = \frac{2}{t^2}\right\}$

(b)	$\left\{t = \frac{1}{2} \Rightarrow\right\} P\left(-\frac{5}{2}, -7\right)$	$x = -\frac{5}{2}, y = -7$ **or** $P\left(-\frac{5}{2}, -7\right)$ seen or implied.	B1
	$\frac{dy}{dx} = \frac{2}{\left(\frac{1}{2}\right)^2}$ **and** either • $y - "-7" = "8"\left(x - "-\frac{5}{2}"\right)$ • $"-7" = ("8")("-\frac{5}{2}") + c$ So, $y = (\text{their } m_T)x + "c"$	**Some** attempt to substitute $t = 0.5$ into their $\frac{dy}{dx}$ which contains t in order to find m_T **and either** applies $y - (\text{their } y_P) = (\text{their } m_T)(x - \text{their } x_P)$ **or** finds c from $(\text{their } y_P) = (\text{their } m_T)(\text{their } x_P) + c$ and uses their numerical c in $y = (\text{their } m_T)x + c$	M1
	T: $y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 cso
	Note: their x_P, their y_P and their m_T must be numerical values in order to award M1		**[3]**

Q6.

Question Number	Scheme	Marks
(*a*)	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2\mathrm{e}^x + 2x\mathrm{e}^x$	M1,A1,A1 (3)
(*b*)	If $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $\mathrm{e}^x(x^2 + 2x) = 0$ setting $(a) = 0$ $[\mathrm{e}^x \neq 0]$ $x(x+2) = 0$ $(x = 0)$ $x = -2$ $x = 0, y = 0$ **and** $x = -2, y = 4\mathrm{e}^{-2}$ $(= 0.54\ldots)$	M1 A1 A1 √ (3)
(*c*)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x^2\mathrm{e}^x + 2x\mathrm{e}^x + 2x\mathrm{e}^x + 2\mathrm{e}^x$ $\left[= (x^2 + 4x + 2)\mathrm{e}^x\right]$	M1, A1 (2)
(*d*)	$x = 0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ $(=2)$ $x = -2$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$ $[= -2\mathrm{e}^{-2}$ $(= -0.270\ldots)]$ M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b) $\therefore$minimum $\therefore$maximum	M1 A1 (cso) (2)
Alt.(*d*)	For M1: Evaluate, or state sign of, $\frac{\mathrm{d}y}{\mathrm{d}x}$ at two appropriate values – on either side of at least one of their answers from (b) or Evaluate y at two appropriate values – on either side of at least one of their answers from (b) or Sketch curve 	**(10 marks)**

Q7.

Question Number	Scheme		Marks	
	(a) $\dfrac{dx}{dt} = 2\sin t\cos t,\ \dfrac{dy}{dt} = 2\sec^2 t$		B1 B1	
	$\dfrac{dy}{dx} = \dfrac{\sec^2 t}{\sin t\cos t}\left(= \dfrac{1}{\sin t\cos^3 t}\right)$	or equivalent	M1 A1	**(4)**
	(b) At $t = \dfrac{\pi}{3},\ x = \dfrac{3}{4},\ y = 2\sqrt{3}$		B1	
	$\dfrac{dy}{dx} = \dfrac{\sec^2 \frac{\pi}{3}}{\sin\frac{\pi}{3}\cos\frac{\pi}{3}} = \dfrac{16}{\sqrt{3}}$		M1 A1	
	$y - 2\sqrt{3} = \dfrac{16}{\sqrt{3}}\left(x - \dfrac{3}{4}\right)$		M1	
	$y = 0 \quad \Rightarrow \quad x = \dfrac{3}{8}$		M1 A1	**(6)**
				[10]

Q8.

Question Number	Scheme		Marks
	$x^2 + y^2 + 10x + 2y - 4xy = 10$		
(a)	$\left\{\frac{\cancel{dy}}{\cancel{dx}}\times\right\}$ $\underline{2x + 2y\frac{dy}{dx} + 10 + 2\frac{dy}{dx}} - \left(\underline{\underline{4y + 4x\frac{dy}{dx}}}\right) = \underline{0}$	**See notes**	M1 <u>A1</u> <u>M1</u>
	$2x + 10 - 4y + (2y + 2 - 4x)\frac{dy}{dx} = 0$	**Dependent on the first M1 mark.**	dM1
	$\frac{dy}{dx} = \frac{2x + 10 - 4y}{4x - 2y - 2}$		
	Simplifying gives $\frac{dy}{dx} = \frac{x + 5 - 2y}{2x - y - 1}$ $\left\{= \frac{-x - 5 + 2y}{-2x + y + 1}\right\}$		A1 cso oe
			[5]
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}$ $x + 5 - 2y = 0$		M1
	So $x = 2y - 5$,		
	$(2y - 5)^2 + y^2 + 10(2y - 5) + 2y - 4(2y - 5)y = 10$		M1
	$4y^2 - 20y + 25 + y^2 + 20y - 50 + 2y - 8y^2 + 20y = 10$		
	gives $-3y^2 + 22y - 35 = 0$ or $3y^2 - 22y + 35 = 0$	$3y^2 - 22y + 35\ \{= 0\}$ **see notes**	A1 oe
	$(3y - 7)(y - 5) = 0$ and $y = ...$	Method mark for solving a quadratic equation.	ddM1
	7	7	