

AS and A level Further Mathematics Practice Paper – Discrete probability distributions
Mark scheme

Question	Scheme						Marks
1(a)	x		-1	0	1	2	M1
	$P(X=x)$		$4k$	k	0	k	A1
	$4k + k + (0) + k = 1$ (Allow verify approach)						A1cso
	$6k = 1 \Rightarrow k = \frac{1}{6} \quad (*)$						(3)
1(b)	$[E(X)] = -4k \text{ (+ 0 + 0) + 2k} \quad \underline{\text{or}} \quad -2k \quad \underline{\text{or}} \quad -1 \times \frac{4}{6} + 2 \times \frac{1}{6}$						M1
	$= -\frac{1}{3} \text{ (or } -0.\dot{3})$						A1
							(2)
1(c)	$\left[E\left(X^2\right)\right]=(-1)^2 \times 4 k+(0+0)+2^2 k \quad \underline{\text{or}} \quad 4 k+4 k \quad \underline{\text{or}} \quad(-1)^2 \times \frac{4}{6}+2^2 \times \frac{1}{6}$						M1
	(o.e.) <						

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2(a)	$p + q + 0.2 + 0.3 + p = 1$ or $2p + q = 0.5$ (oe)	B1 (1)												
2(b)	$[E(X) =] - 2p - q + \frac{1}{2} \times 0.2 + \frac{3}{2} \times 0.3 + 2p [= 0.4]$ or $- q + 0.1 + 0.45 [= 0.4]$ $q = \underline{\underline{0.15}}$	M1A1 A1 (3)												
2(c)	$2p + \text{“}0.15\text{”} = 0.5$ (oe) $p = \underline{\underline{0.175}}$	M1 A1 (2)												
2(d)	$[\text{Var}(X) =] 2.275 - (0.4)^2$ $= \underline{\underline{2.115}}$	M1 A1 (2)												
2(e)	<table border="1"><tr><td>r</td><td>$-\frac{1}{2}$</td><td>-1</td><td>2</td><td>$\frac{2}{3}$</td><td>$\frac{1}{2}$</td></tr><tr><td>$P(R = r)$</td><td>p</td><td>q</td><td>0.2</td><td>0.3</td><td>p</td></tr></table> $E(R) = -\frac{1}{2} p - q + 0.4 + 0.2 + \frac{1}{2} p$ $= 0.6 - q = \underline{\underline{0.45}}$ (or $\frac{9}{20}$)	r	$-\frac{1}{2}$	-1	2	$\frac{2}{3}$	$\frac{1}{2}$	$P(R = r)$	p	q	0.2	0.3	p	M1 dM1 A1ft (3)
r	$-\frac{1}{2}$	-1	2	$\frac{2}{3}$	$\frac{1}{2}$									
$P(R = r)$	p	q	0.2	0.3	p									
2f(i)	$S > R$ when $x = 1.5$ and 2 $P(\text{Sarah wins}) = 0.3 + p = \underline{\underline{0.475}}$ (or $\frac{19}{40}$)	M1 A1ft												
2f(ii)	$R > S$ when $x = -2$ and $\frac{1}{2}$ $P(\text{Rebecca wins}) = 0.2 + p = \underline{\underline{0.375}}$ (or $\frac{15}{40}$)	M1 A1ft (3)												
		(15 marks)												

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3(a)	To score 15 points, 2 correct and 1 not correct $[0.6 \times 0.6 \times 0.4] + [0.6 \times 0.4 \times 0.6] + [0.4 \times 0.6 \times 0.6]$ or $3 \times (0.6 \times 0.6 \times 0.4)$ $= 0.432$ (*)	M1 A1cso (2)										
3(b)	$1 - (0.216 + 0.432 + 0.064) = \underline{\underline{0.288}}$ or $3 \times 0.6 \times (0.4)^2$	B1 (1)										
3(c)	$[(30, 0), (0, 30) \text{ or } (15, 15)]$ $0.216 \times 0.288 + 0.288 \times 0.216 + 0.432 \times 0.432$ awrt <u>0.311</u>	M1 A1ft A1 (3)										
3(d)	$E(X) = [30 \times 0.216] + [15 \times 0.432] + [0 \times 0.288] + [(-15) \times 0.064]$ $E(X) = 12$ <u>12</u> (only)	M1 A1 (2)										
3(e)	$E(X^2) = 30^2 \times 0.216 + 15^2 \times 0.432 + 0^2 \times 0.288 + (-15)^2 \times 0.064 (= 306)$ $\text{Var}(X) = E(X^2) - [E(X)]^2 = 306 - 12^2 =$ <u>162</u>	M1 M1 A1 (3)										
3(f)	Let Y = number of points scored in bonus round <table><tr><td>$[y]$</td><td>60</td><td>35</td><td>10</td><td>-15</td></tr><tr><td>$[P(Y = y)]$</td><td>0.216</td><td>0.432</td><td>0.288</td><td>0.064</td></tr></table> $E(Y) = 60 \times 0.216 + 35 \times 0.432 + 10 \times 0.288 + (-15) \times 0.064$ $= \underline{\underline{30}}$	$[y]$	60	35	10	-15	$[P(Y = y)]$	0.216	0.432	0.288	0.064	M1 dM1 A1 (3)
$[y]$	60	35	10	-15								
$[P(Y = y)]$	0.216	0.432	0.288	0.064								
		(14 marks)										

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4(a)	$k + 2k + 3k + 4k = 1$ or $10k = 1$ $k = 0.1$ (*) [allow verification with a comment e.g. “so $k = 0.1$ ”]	B1cso (1)																
4(b)	$E(X) = 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.3 + 4 \times 0.4 = 3$	M1A1 (2)																
4(c)	$E(X^2) = 1 \times 0.1 + 4 \times 0.2 + 9 \times 0.3 + 16 \times 0.4 = 10$	M1A1 (2)																
4(d)	$\text{Var}(X) = 10 - 9 (= 1)$ $\text{Var}(2 - 5X) = 5^2 \text{Var}(X) = 25$	M M1A1 (3)																
4(e)	$P(1, 3) + P(2, 2) = 2 \times 0.1 \times 0.3 + 0.2 \times 0.2 = 0.1$ (*)	M1A1cso (2)																
4(f)	<table border="1"><tr><td>$X_1 + X_2$</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>p</td><td>0.01</td><td>0.04</td><td>0.1</td><td>0.2</td><td>0.25</td><td>0.24</td><td>0.16</td></tr></table>	$X_1 + X_2$	2	3	4	5	6	7	8	p	0.01	0.04	0.1	0.2	0.25	0.24	0.16	B1B1 (2)
$X_1 + X_2$	2	3	4	5	6	7	8											
p	0.01	0.04	0.1	0.2	0.25	0.24	0.16											
4(g)	$P(2) + P(3) = 0.05$	M1A1 (2)																
		(14 marks)																

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5(a)	b	1	3	5	Also allow b values 1,1,3,3,5,5 and probabilities all $\frac{1}{6}$	B1 B1 (2)
	$P(B = b)$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		
5(b)	Discrete Uniform {distribution}					B1 (1)
5(c)	[E(B) =] 3 (by symmetry)					B1 (1)
5(d)	$[E(R) =] \quad 2 \times \frac{2}{3} + 4 \times \frac{1}{6} + 6 \times \frac{1}{6}$ $\qquad \qquad \qquad = \underline{3}$					M1 A1 (2)
5(e)	$[E(R^2) =] \quad 2^2 \times \frac{2}{3} + 4^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6} \quad \left[\quad = \frac{34}{3} \quad \right]$ $[\text{Var}(R) =] \frac{34}{3} - 3^2 = \frac{7}{3}$ (or any exact equivalent. NB 2.33 is A0)					M1 dM1A1 (3)
5(f)	Coin lands on 2 , choose blue die; coin lands on 5 choose red die $P(\text{Avisha wins}) = \frac{1}{2} \times \left(\frac{1}{3} + \frac{1}{3} \right) + \frac{1}{2} \times \frac{1}{6}$ $\qquad \qquad \qquad = \frac{5}{12} \quad (\text{allow awrt } 0.417)$					B2/1/0 M1 A1 (4)
						(13 marks)

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6(a)	$E(S) = 0 + 1 \times 0.2 + 2 \times 0.1 + 4 \times 0.3 + 5 \times 0.2 = [0.2 + 0.2 + 1.2 + 1.0]$ <u>2.6</u>	M1 A1 (2)
6(b)	$E(S^2) = 0 + 1 \times 0.2 + 2^2 \times 0.1 + 4^2 \times 0.3 + 5^2 \times 0.2$ <u>or</u> $0.2 + 0.4 + 4.8 + 5$ <u>10.4</u> (*)	M1 A1cso (2)
6(c)	$\text{Var}(S) = 10.4 - ("2.6")^2$ <u>3.64</u> or $\frac{91}{25}$ (o.e.)	M1 A1 (2)
6(d)(i)	$5E(S) - 3 = 5 \times "2.6" - 3, \quad = \underline{\underline{10}}$	M1A1
6(d)(ii)	$5^2 \text{Var}(S) = 25 \times 3.64, \quad = \underline{\underline{91}}$	M1A1 (4)
6(e)	$5S - 3 > S + 3 \Rightarrow 4S > 6$ or $S > 1.5$, so need $P(S \geq 2)$ $P(S \geq 2) = \underline{\underline{0.6}}$	M1A1 A1 (3)
6(f)	$P(S_1 = 1) \times P(S_2 \leq 4), = 0.2 \times 0.8 = 0.16$ (*)	M1A1cso (2)
6(g)	$P(S_1 = 2) \times P(S_2 \leq 2) = 0.1 \times 0.5 \quad = 0.05$ $P(S_1 = 4) \times P(S_2 \leq 1) = 0.3 \times 0.4 \quad = 0.12$ $P(S_1 = 5) \times P(S_2 = 0) = 0.2 \times 0.2 \quad = 0.04$ $P(S_1 = 0) \times P(S_2 = \text{any value}) = 0.2 \times 1 = 0.20$ Full method – all cases listed all correct products <u>0.57</u>	M1 A1 A1 (3)
		(18 marks)

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7(a)	$1 = p + (0.25 + 0.25 + 0.2 + 0.2), \Rightarrow p = \underline{\frac{1}{10} \text{ or } 0.1}$	M1A1 (2)
7(b)	$E(S) = \frac{1}{4} + 2 \times \frac{1}{4} + 4 \times \frac{1}{5} + 5 \times \frac{1}{5}, \text{ (or equiv. in decimals)} = \underline{2.55}$	M1A1 (2)
7(c)	$E(S^2) = \frac{1}{4} + \frac{2^2}{4} + \frac{4^2}{5} + \frac{5^2}{5} \text{ or } 0.25 + 1 + 3.2 + 5 = \underline{9.45} (*)$	M1A1cso (2)
7(d)	$\text{Var}(S) = 9.45 - (E(S))^2, = \underline{2.9475 \text{ or } \frac{1179}{400}} \text{ (accept awrt 2.95)}$	M1A1 (2)
7(e)	$P(5 \text{ and } 5) = \left(\frac{1}{5}\right)^2, = \underline{\frac{1}{25} \text{ or } 0.04}$	M1A1 (2)
7(f)	$P(4, 4, 2) = \left(\frac{1}{5}\right)^2 \times \frac{1}{4} \times 3 \quad (= 0.03 \text{ or } \frac{3}{100})$	M1M1
	$P(4, 4, 4) = \left(\frac{1}{5}\right)^3 \quad (= 0.008 \text{ or } \frac{1}{125})$ $P(\text{Tom wins in 3 spins}) = \underline{0.038}$	B1 A1 (4)
7(g)	$P(\bar{5} \cap 5 \cap 5) + P(5 \cap \bar{5} \cap 5) = \frac{4}{5} \times \left(\frac{1}{5}\right)^2 \times 2 = \underline{0.064 \text{ or } \frac{8}{125}}$	M1M1A1 (3)
		(17 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	S1 2012	1	FS1 1.1	Discrete random variables	1.1b, 2.1, 3.1a, 3.4
2	S1 2016	2	FS1 1.1	Discrete random variables	1.1b, 3.1b, 3.4
3	S1 2015	5	FS1 1.1	Discrete random variables	1.1b, 2.1, 3.3, 3.4
4	S1 Jan 2011	6	FS1 1.1	Discrete random variables	1.1b, 2.1, 3.1a
5	S1 Jan 2013	6	FS1 1.1	Discrete probability distributions	1.1b, 1.2, 2.4, 3.1b, 3.4
6	S1 2013R	7	FS1 1.1	Discrete probability distributions	1.1b, 1.2, 2.1, 3.1a, 3.4
7	S1 2011	8	FS1 1.1	Discrete probability distributions	1.1b, 1.2, 2.1, 3.4