

Topic Test

Summer 2022

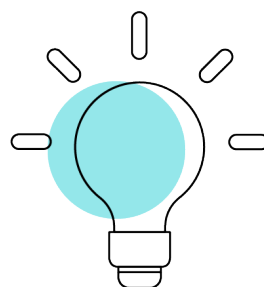
Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 3: Coordinate geometry in the (x,y) plane

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Mark Scheme

Question T3_Q1

Question	Scheme	Marks	AOs
6 (a)	Deduces that gradient of PA is $-\frac{1}{2}$	M1	2.2a
	Finding the equation of a line with gradient " $-\frac{1}{2}$ " and point $(7, 5)$ $y - 5 = -\frac{1}{2}(x - 7)$	M1	1.1b
	Completes proof $2y + x = 17$ *	A1*	1.1b
		(3)	
(b)	Solves $2y + x = 17$ and $y = 2x + 1$ simultaneously	M1	2.1
	$P = (3, 7)$	A1	1.1b
	Length $PA = \sqrt{(3-7)^2 + (7-5)^2} = (\sqrt{20})$	M1	1.1b
	Equation of C is $(x-7)^2 + (y-5)^2 = 20$	A1	1.1b
		(4)	
(c)	Attempts to find where $y = 2x + k$ meets C using $\overrightarrow{OA} + \overrightarrow{PA}$	M1	3.1a
	Substitutes their $(11, 3)$ in $y = 2x + k$ to find k	M1	2.1
	$k = -19$	A1	1.1b
		(3)	
(10 marks)			
(c)	Attempts to find where $y = 2x + k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $5x^2 + (4k - 34)x + k^2 - 10k + 54 = 0$	M1	3.1a
	Uses $b^2 - 4ac = 0$ oe and proceeds to $k = \dots$	M1	2.1
	$k = -19$	A1	1.1b
		(3)	
Notes: (a) M1: Uses the idea of perpendicular gradients to deduce that gradient of PA is $-\frac{1}{2}$. Condone $-\frac{1}{2}x$ if followed by correct work. You may well see the perpendicular line set up as $y = -\frac{1}{2}x + c$ which scored this mark M1: Award for the method of finding the equation of a line with a changed gradient and the point $(7, 5)$ So sight of $y - 5 = \frac{1}{2}(x - 7)$ would score this mark If the form $y = mx + c$ is used expect the candidates to proceed as far as $c = \dots$ to score this mark.			

A1*: Completes proof with no errors or omissions $2y + x = 17$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point P . ie for an attempt at solving $2y + x = 17$ and $y = 2x + 1$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates. Do not allow where they start $17 - x = 2x + 1$ as they have set $2y = y$ but condone bracketing errors, eg $2 \times 2x + 1 + x = 17$

A1: $P = (3, 7)$

M1: Uses Pythagoras' Theorem to find the radius or radius ² using their $P = (3, 7)$ and $(7, 5)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: $(x-7)^2 + (y-5)^2 = 20$. Do not accept $(x-7)^2 + (y-5)^2 = (\sqrt{20})^2$

(c)

M1: Attempts to find where $y = 2x + k$ meets C .

Awarded for using $\overrightarrow{OA} + \overrightarrow{PA}$. $(11, 3)$ or one correct coordinate of $(11, 3)$ is evidence of this award.

M1: For a full method leading to k . Scored for either substituting their $(11, 3)$ in $y = 2x + k$

or, **in the alternative**, for solving their $(4k - 34)^2 - 4 \times 5 \times (k^2 - 10k + 54) = 0 \Rightarrow k = \dots$ Allow use of a calculator here to find roots. Award if you see use of correct formula but it would be implied by $\pm$ correct roots

A1: $k = -19$ only

.....

Alternative I

M1: For solving $y = 2x + k$ with their $(x-7)^2 + (y-5)^2 = 20$ and creating a quadratic eqn of the form $ax^2 + bx + c = 0$ where both b and c are dependent upon k . The terms in x^2 and x must be collected together or implied to have been collected by their correct use in " $b^2 - 4ac$ "

FYI the correct quadratic is $5x^2 + (4k - 34)x + k^2 - 10k + 54 = 0$

M1: For using the discriminant condition $b^2 - 4ac = 0$ to find k . It is not dependent upon the previous M and may be awarded from only one term in k .

$(4k - 34)^2 - 4 \times 5 \times (k^2 - 10k + 54) = 0 \Rightarrow k = \dots$ Allow use of a calculator here to find roots.

Award if you see use of correct formula but it would be implied by $\pm$ correct roots

A1: $k = -19$ only

.....

Alternative II

M1: For solving $2y + x = 17$ with their $(x-7)^2 + (y-5)^2 = 20$, creating a 3TQ and solving.

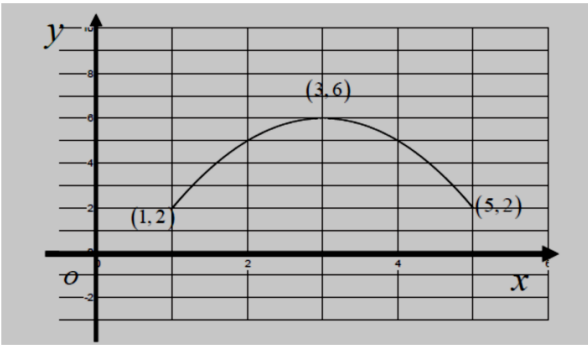
M1: For substituting their $(11, 3)$ into $y = 2x + k$ and finding k

A1: $k = -19$ only

.....

Other method are possible using trigonometry.

Question T3_Q2

Question	Scheme	Marks	AOs
14(a)	Attempts to use $\cos 2t = 1 - 2\sin^2 t \Rightarrow \frac{y-4}{2} = 1 - 2\left(\frac{x-3}{2}\right)^2$	M1	2.1
	$\Rightarrow y - 4 = 2 - 4 \times \frac{(x-3)^2}{4} \Rightarrow y = 6 - (x-3)^2 *$	A1*	1.1b
		(2)	
(b)	 shaped parabola Fully correct with 'ends' at (1,2) & (5,2) Suitable reason : Eg states as $x = 3 + 2\sin t, 1 \leq x \leq 5$	M1	1.1b
		A1	1.1b
		B1	2.4
		(3)	
(c)	Either finds the lower value for $k = 7$ or deduces that $k < \frac{37}{4}$	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ $\Rightarrow k - x = 6 - (x-3)^2$ and proceeds to 3TQ in x or y	M1	3.1a
	Correct 3TQ in x $x^2 - 7x + (k+3) = 0$ Or y $y^2 + (7-2k)y + (k^2 - 6k + 3) = 0$	A1	1.1b
	Uses $b^2 - 4ac = 0 \Rightarrow 49 - 4 \times 1 \times (k+3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$ or $(7-2k)^2 - 4 \times 1 \times (k^2 - 6k + 3) = 0 \Rightarrow k = \left(\frac{37}{4}\right)$	M1	2.1
	Range of values for $k = \left\{k : 7 \leq k < \frac{37}{4}\right\}$	A1	2.5
		(5)	
(10 marks)			
(a)			
M1: Uses $\cos 2t = 1 - 2\sin^2 t$ in an attempt to eliminate t			

A1*: Proceeds to $y = 6 - (x-3)^2$ without any errors

Allow a proof where they start with $y = 6 - (x-3)^2$ and substitute the parametric coordinates. M1 is scored for a correct $\cos 2t = 1 - 2\sin^2 t$ but A1 is only scored when both sides are seen to be the same AND a comment is made, hence proven, or similar.

(b)

M1: For sketching a $\cap$ parabola with a maximum in quadrant one. It does not need to be symmetrical

A1: For sketching a $\cap$ parabola with a maximum in quadrant one and with end coordinates of $(1, 2)$ and $(5, 2)$

B1: Any suitable explanation as to why C does not include all points of $y = 6 - (x-3)^2$, $x \in \mathbb{R}$

This should include a reference to **the limits on sin or cos** with a **link to a restriction on x or y**. For example

'As $-1 \leq \sin t \leq 1$ then $1 \leq x \leq 5$ ' Condone in words 'x lies between 1 and 5' and strict inequalities

'As $\sin t \leq 1$ then $x \leq 5$ ' Condone in words 'x is less than 5'

'As $-1 \leq \cos(2t) \leq 1$ then $2 \leq y \leq 6$ ' Condone in words 'y lies between 2 and 6'

Withhold if the statement is incorrect Eg "because the domain is $2 \leq x \leq 5$ "

Do not allow a statement on the top limit of y as this is the same for both curves

(c)

B1: Deduces either

- the correct that the lower value of $k = 7$ This can be found by substituting into $(5, 2)$
 $x + y = k \Rightarrow k = 7$ or substituting $x = 5$ into $x^2 - 7x + (k+3) = 0 \Rightarrow 25 - 35 + k + 3 = 0$
 $\Rightarrow k = 7$
- or deduces that $k < \frac{37}{4}$ This may be awarded from later work

M1: For an attempt at the upper value for k .

Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ once by using an appropriate method.

Eg. Sets $k - x = 6 - (x-3)^2$ and proceeds to a 3TQ

A1: Correct 3TQ $x^2 - 7x + (k+3) = 0$ The $= 0$ may be implied by subsequent work

M1: Uses the "discriminant" condition. Accept use of $b^2 = 4ac$ or $b^2 \leq 4ac$ where ... is any inequality leading to a critical value for k . Eg. one root $\Rightarrow 49 - 4 \times 1 \times (k+3) = 0 \Rightarrow k = \frac{37}{4}$

A1: Range of values for $k = \left\{ k : 7 \leq k < \frac{37}{4} \right\}$ Accept $k \in \left[7, \frac{37}{4} \right)$ or exact equivalent

ALT	As above	B1	2.2a
	Finds where $x + y = k$ meets $y = 6 - (x-3)^2$ once by using an appropriate method. Eg. Sets gradient of $y = 6 - (x-3)^2$ equal to -1	M1	3.1a
	$-2x + 6 = -1 \Rightarrow x = 3.5$	A1	1.1b
	Finds point of intersection and uses this to find upper value of k . $y = 6 - (3.5 - 3)^2 = 5.75$ Hence using $k = 3.5 + 5.75 = 9.25$	M1	2.1
	Range of values for $k = \{k : 7 \leq k < 9.25\}$	A1	2.5

Question T3_Q3

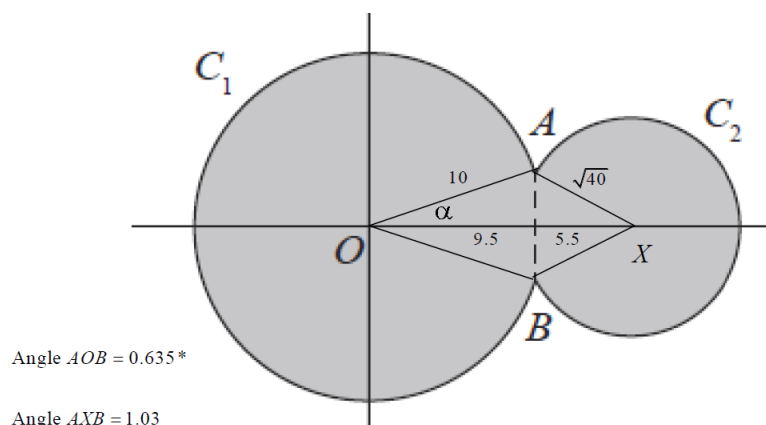
Question	Scheme	Marks	AOs
7	£y is the total cost of making x bars of soap Bars of soap are sold for £2 each		
(a)	$y = kx + c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	(1)	
(b) Way 1	Either $x = 800 \Rightarrow y = 2(800) - 500 \Rightarrow 1100 \Rightarrow (x, y) = (800, 1100)$ $x = 300 \Rightarrow y = 2(300) + 80 \Rightarrow 680 \Rightarrow (x, y) = (300, 680)$	M1	3.1b
	Applies (800, their 1100) and (300, their 680) to give two equations $1100 = 800k + c$ and $680 = 300k + c \Rightarrow k, c = \dots$	dM1	1.1b
	Solves correctly to find $k = 0.84, c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	(3)	
(b) Way 2	Either $x = 800 \Rightarrow y = 2(800) - 500 \Rightarrow 1100 \Rightarrow (x, y) = (800, 1100)$ $x = 300 \Rightarrow y = 2(300) + 80 \Rightarrow 680 \Rightarrow (x, y) = (300, 680)$	M1	3.1b
	Complete method for finding both $k = \dots$ and $c = \dots$ e.g. $k = \frac{1100 - 680}{800 - 300} \Rightarrow 0.84$ $(800, 1100) \Rightarrow 1100 = 800(0.84) + c \Rightarrow c = \dots$	dM1	1.1b
	Solves to find $k = 0.84, c = 428$ and states $y = 0.84x + 428$ *	A1*	2.1
	Note: the answer $y = 0.84x + 428$ must be stated in (b)	(3)	
(b) Way 3	Either $x = 800 \Rightarrow y = 2(800) - 500 \Rightarrow 1100 \Rightarrow (x, y) = (800, 1100)$ $x = 300 \Rightarrow y = 2(300) + 80 \Rightarrow 680 \Rightarrow (x, y) = (300, 680)$	M1	3.1b
	$\{y = 0.84x + 428 \Rightarrow\}$ $x = 800 \Rightarrow y = (0.84)(800) + 428 = 1100$ $x = 300 \Rightarrow y = (0.84)(300) + 428 = 680$	dM1	1.1b
	Hence $y = 0.84x + 428$ *	A1*	2.1
		(3)	
(c)	Allow any of {0.84, in £s} represents - the <i>cost</i> of {making} each extra bar {of soap} - the direct <i>cost</i> of {making} a bar {of soap} - the marginal <i>cost</i> of {making} a bar {of soap} - the <i>cost</i> of {making} a bar {of soap} (Condone this answer) Note: Do not allow {0.84, in £s} is the profit per bar {of soap} {0.84, in £s} is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) Way 1	{Let n be the least number of bars required to make a profit}		
	$2n = 0.84n + 428 \Rightarrow n = \dots$ (Condone $2x = 0.84x + 428 \Rightarrow x = \dots$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) Way 2	- Trial 1: $n = 368 \Rightarrow y = (0.84)(368) + 428 \Rightarrow y = 737.12$ {revenue = $2(368) = 736$ or loss = 1.12} - Trial 2: $n = 369 \Rightarrow y = (0.84)(369) + 428 \Rightarrow y = 737.96$ {revenue = $2(369) = 738$ or profit = 0.04}	M1	3.4
	leading to an answer of 369 {bars}	A1	3.2a
		(2)	
(7 marks)			

Notes for Question 7	
(a)	
B1:	Obtains a correct form of the equation. E.g. $y = kx + c$; $k \neq 0, c \neq 0$. Note: Must be seen in (a)
Note:	Ignore how the constants are labelled – as long as they appear to be constants. e.g. k, c, m etc.
(b)	Way 1
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	dependent on the previous M mark See scheme
A1:	See scheme – no errors in their working
Note	Allow 1 st M1 for any of $1600 - y = 500$ $600 - y = -80$
(b)	Way 2
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	dependent on the previous M mark See scheme
A1:	See scheme – no error in their working
(b)	Way 3
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	dependent on the previous M mark Uses the model to test both points (800, their 1100) and (300, their 680)
A1:	Confirms $y = 0.84x + 428$ is true for both (800, 1100) and (300, 680) and gives a conclusion
Note:	Conclusion could be “ $y = 0.84x + 428$ ” or “QED” or “proved”
Note:	Give 1 st M0 for $500 = 800k + c, 80 = 300k + c \Rightarrow k = \frac{500 - 80}{800 - 300} = 0.84$
(c)	
B1:	see scheme
Note:	Also condone B1 for “rate of change of cost”, “cost of {making} a bar”, “constant of proportionality for cost per bar of soap” or “rate of increase in cost”,
Note:	Do not allow reasons such as “price increase or decrease”, “rate of change of the bar of soap” or “decrease in cost”
Note:	Give B0 for incorrect use of units. E.g. Give B0 for “the cost of making each extra bar of soap is £84” Condone the use of £0.84p

Notes for Question 7 Continued	
(d)	Way 1
M1:	Using the model and constructing an argument leading to a critical value for the number of bars of soap sold. See scheme.
A1:	369 only. Do not accept decimal answers.
(d)	Way 2
M1:	Uses either 368 or 369 to find the cost $y = \dots$
A1:	Attempts both trial 1 and trial 2 to find both the cost $y = \dots$ and arrives at an answer of 369 only. Do not accept decimal answers.
Note:	You can ignore inequality symbols for the method mark in part (d)
Note:	Give M1 A1 for no working leading to 369 {bars}
Note:	Give final A0 for $x > 369$ or $x > 368$ or $x \geq 369$ without $x = 369$ or 369 stated as their final answer
Note:	Condone final A1 for in words “at least 369 bars must be made/sold”
Note:	<u>Special Case:</u> Assuming a profit of £1 is required and achieving $x = 370$ scores special case M1A0

Question T3_Q4

Question	Scheme	Marks	AOs
11 (a)	Solves $x^2 + y^2 = 100$ and $(x-15)^2 + y^2 = 40$ simultaneously to find x or y E.g. $(x-15)^2 + 100 - x^2 = 40 \Rightarrow x = \dots$	M1	3.1a
	Either $\Rightarrow -30x + 325 = 40 \Rightarrow x = 9.5$ Or $y = \frac{\sqrt{39}}{2} = \text{awrt } \pm 3.12$	A1	1.1b
	Attempts to find the angle AOB in circle C_1 Eg Attempts $\cos \alpha = \frac{9.5}{10}$ to find α then $\times 2$	M1	3.1a
	Angle $AOB = 2 \times \arccos\left(\frac{9.5}{10}\right) = 0.635 \text{ rads (3sf)}$ *	A1*	2.1
		(4)	
(b)	Attempts $10 \times (2\pi - 0.635) = 56.48$	M1	1.1b
	Attempts to find angle AXB or AXO in circle C_2 (see diagram) E.g. $\cos \beta = \frac{15 - 9.5}{\sqrt{40}} \Rightarrow \beta = \dots$ (Note $AXB = 1.03 \text{ rads}$)	M1	3.1a
	Attempts $10 \times (2\pi - 0.635) + \sqrt{40} \times (2\pi - 2\beta)$	dM1	2.1
	$= 89.7$	A1	1.1b
		(4)	
(8 marks)			
Notes:			



(a)

M1: For the key step in an attempt to find either coordinate for where the two circles meet.

Look for an attempt to set up an equation in a single variable leading to a value for x or y .

A1: $x = 9.5$ (or $y = \frac{\sqrt{39}}{2} = \text{awrt } \pm 3.12$)

M1: Uses the radius of the circle and correct trigonometry in an attempt to find angle AOB in circle C_1

E.g. Attempts $\cos \alpha = \frac{9.5}{10}$ to find α then $\times 2$

Alternatives include $\tan \alpha = \frac{\sqrt{100 - 9.5^2}}{9.5} = (0.3286\dots)$ to find α then $\times 2$

$$\text{And } \cos AOB = \frac{10^2 + 10^2 - (\sqrt{39})^2}{2 \times 10 \times 10} = \frac{161}{200}$$

A1*: Correct and careful work in proceeding to the given answer. Condone an answer with greater accuracy.
Condone a solution where the intermediate value has been truncated, provided the trig equation is correct.

E.g. $\sin \alpha = \frac{\sqrt{39}}{20} \Rightarrow \alpha = 0.317 \Rightarrow AOB = 2\alpha = 0.635$

Condone a solution written down from awrt 36.4° (without the need to shown any calculation.)
E

(b)

M1: Attempts to use the formula $s = r\theta$ with $r = 10$ and $\theta = 2\pi - 0.635$

The formula may be embedded. You may see $\underline{2\pi 10 + 2\pi \sqrt{40} - 10 \times 0.635\dots}$ which is fine for this M1

M1: Attempts to use a correct method in order to find angle AXB or AXO in circle C_2

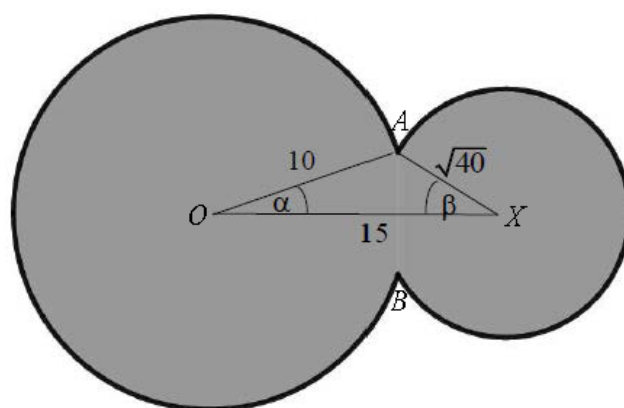
Amongst many other methods are $\tan \beta = \frac{3.12}{15 - 9.5}$ and $\cos AXB = \frac{40 + 40 - (\sqrt{39})^2}{2 \times \sqrt{40} \times \sqrt{40}} = \frac{41}{80}$

Note that many candidates believe this to be 0.635. This scores M0 dM0 A0

dM1: A full and complete attempt to find the perimeter of the region.

It is dependent upon having scored both M's.

A1: awrt 89.7



(a)

M1: For the key step in attempting to find all lengths in triangle OAX , condoning slips

A1: All three lengths correct

M1: Attempts cosine rule to find α then $\times 2$

A1*: Correct and careful work in proceeding to the given answer

Question T3_Q5

Question	Scheme	Marks	AOs
14 (a)	C is $(x-r)^2 + (y-r)^2 = r^2$ or $x^2 + y^2 - 2rx - 2ry + r^2 = 0$	B1	2.2a
	$y = 12 - 2x, x^2 + y^2 - 2rx - 2ry + r^2 = 0$ $\Rightarrow x^2 + (12 - 2x)^2 - 2rx - 2r(12 - 2x) + r^2 = 0$ or	M1	1.1b

	$y = 12 - 2x, (x-r)^2 + (y-r)^2 = r^2$ $\Rightarrow (x-r)^2 + (12 - 2x - r)^2 = r^2$		
	$x^2 + 144 - 48x + 4x^2 - 2rx - 24r + 4rx + r^2 = 0$ $\Rightarrow 5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$ *	A1*	2.1
		(3)	
(b)	$b^2 - 4ac = 0 \Rightarrow (2r - 48)^2 - 4 \times 5 \times (r^2 - 24r + 144) = 0$	M1	3.1a
	$r^2 - 18r + 36 = 0$ or any multiple of this equation	A1	1.1b
	$\Rightarrow (r - 9)^2 - 81 + 36 = 0 \Rightarrow r = \dots$	dM1	1.1b
	$r = 9 \pm 3\sqrt{5}$	A1	1.1b
		(4)	
(7 marks)			

Notes:

- (a)
- B1:** Deduces the correct equation of the circle
- M1:** Attempts to form an equation with terms of the form x^2 , x , r^2 , and xr only using $y = 12 \pm 2x$ and their circle equation which must be of an appropriate form. I.e. includes or implies an x^2 , y^2 , r^2 such as $x^2 + y^2 = r^2$
- If their circle equation starts off as e.g. $(x \pm a)^2 + (y \pm b)^2 = r^2$ then the B mark and the M mark can be awarded when the “a” and “b” are replaced by r or $-r$ as appropriate for their circle equation.
- A1*:** Uses correct and accurate algebra leading to the given solution.
- (b)
- M1:** Attempts to use $b^2 - 4ac \dots 0$ o.e. with $a = 5, b = 2r - 48, c = r^2 - 24r + 144$ and where ... is “=” or any inequality
- Allow minor slips when copying the a, b and c provided it does not make the work easier and allow their a, b and c if they are similar expressions.
- FYI $(2r - 48)^2 - 4 \times 5 \times (r^2 - 24r + 144) = 4r^2 - 192r + 2304 - 20r^2 + 480r - 2880 = -16r^2 + 288r - 576$
- A1:** Correct quadratic equation in r (or inequality). Terms need not be all one side but must be collected.
- E.g. allow $r^2 - 18r = -36$ and allow any multiple of this equation (or inequality).
- dM1:** Correct attempt to solve their 3TQ in r . Dependent upon previous M
- A1:** Careful and accurate work leading to both answers in the required form (must be simplified surds)

Question T3_Q6

Question	Scheme	Marks	AOs
7(a)(i)	$(x-5)^2 + (y+2)^2 = \dots$	M1	1.1b
	$(5, -2)$	A1	1.1b
(ii)	$r = \sqrt{{}^n5^{{}^n2} + {}^n-2^{{}^n2} - 11}$	M1	1.1b
	$r = 3\sqrt{2}$	A1	1.1b
		(4)	
(b)	$y = 3x + k \Rightarrow x^2 + (3x+k)^2 - 10x + 4(3x+k) + 11 = 0$ $\Rightarrow x^2 + 9x^2 + 6kx + k^2 - 10x + 12x + 4k + 11 = 0$	M1	2.1
	$\Rightarrow 10x^2 + (6k+2)x + k^2 + 4k + 11 = 0$	A1	1.1b
	$b^2 - 4ac = 0 \Rightarrow (6k+2)^2 - 4 \times 10 \times (k^2 + 4k + 11) = 0$	M1	3.1a
	$\Rightarrow 4k^2 + 136k + 436 = 0 \Rightarrow k = \dots$	M1	1.1b
	$k = -17 \pm 6\sqrt{5}$	A1	2.2a
		(5)	
(9 marks)			
Notes			

(a)(i)

M1: Attempts to complete the square on by halving both x and y terms.

Award for sight of $(x \pm 5)^2, (y \pm 2)^2 = \dots$. This mark can be implied by a centre of $(\pm 5, \pm 2)$.

A1: Correct coordinates. (Allow $x = 5, y = -2$)

(a)(ii)

M1: Correct strategy for the radius or radius². For example award for $r = \sqrt{{}^n\pm 5^{{}^n2} + {}^n\pm 2^{{}^n2} - 11}$

or an attempt such as $(x-a)^2 - a^2 + (y-b)^2 - b^2 + 11 = 0 \Rightarrow (x-a)^2 + (y-b)^2 = k \Rightarrow r^2 = k$

A1: $r = 3\sqrt{2}$. Do not accept for the A1 either $r = \pm 3\sqrt{2}$ or $\sqrt{18}$

The A1 can be awarded following sign slips on $(5, -2)$ so following $r^2 = {}^n\pm 5^{{}^n2} + {}^n\pm 2^{{}^n2} - 11$

(b) Main method seen

M1: Substitutes $y = 3x + k$ into the given equation (or their factorised version) and makes progress by attempting to expand the brackets. Condone lack of $= 0$

A1: Correct 3 term quadratic equation.

The terms must be collected but this can be implied by correct a, b and c

M1: Recognises the requirement to use $b^2 - 4ac = 0$ (or equivalent) where both b and c are expressions in k . It is dependent upon having attempted to substitute $y = 3x + k$ into the given equation

M1: Solves 3TQ in k . See General Principles.

The 3TQ in k must have been found as a result of attempt at $b^2 - 4ac \dots 0$

A1: Correct simplified values

Look carefully at the method used. It is possible to attempt this using gradients

(b) Alt 1	$x^2 + y^2 - 10x + 4y + 11 = 0 \Rightarrow 2x + 2y \frac{dy}{dx} - 10 + 4 \frac{dy}{dx} = 0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx} = 3 \Rightarrow x + 3y + 1 = 0$ and combines with equation for C $\Rightarrow 5x^2 - 50x + 44 = 0$ or $5y^2 + 20y + 11 = 0$ $\Rightarrow x = \dots$ or $y = \dots$	M1	3.1a
	$x = \frac{25 \pm 9\sqrt{5}}{5}, y = \frac{-10 \pm 3\sqrt{5}}{5}, k = y - 3x \Rightarrow k = \dots$	M1	1.1b
	$k = -17 \pm 6\sqrt{5}$	A1	2.2a

M1: Differentiates implicitly condoning slips but must have two $\frac{dy}{dx}$'s coming from correct terms

A1: Correct differentiation.

M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into C and attempts to solve the resulting quadratic in x or y .

M1: Uses at least one pair of coordinates and l to find at least one value for k . It is dependent upon having attempted both M's

A1: Correct simplified values

(b) Alt 2	$x^2 + y^2 - 10x + 4y + 11 = 0 \Rightarrow 2x + 2y \frac{dy}{dx} - 10 + 4 \frac{dy}{dx} = 0$	M1 A1	2.1 1.1b
	Sets $\frac{dy}{dx} = 3 \Rightarrow x + 3y + 1 = 0$ and combines with equation for l $y = 3x + k, x + 3y = 1$ $\Rightarrow x = \dots$ and $y = \dots$ in terms of k	M1	3.1a
	$x = \frac{-3k-1}{10}, y = \frac{k-3}{10}, x^2 + y^2 - 10x + 4y + 11 = 0 \Rightarrow k = \dots$	M1	1.1b
	$k = -17 \pm 6\sqrt{5}$	A1	2.2a

Very similar except it uses equation for l instead of C in mark 3

M1 A1: Correct differentiation (See alt 1)

M1: Sets $\frac{dy}{dx} = 3$, makes y or x the subject, substitutes back into l to obtain x and y in terms of k

M1: Substitutes for x and y into C and solves resulting 3TQ in k

A1: Correct simplified values

(b) Alt 3	$y = 3x + k \Rightarrow m = 3 \Rightarrow m_r = -\frac{1}{3}$	M1	
	$y + 2 = -\frac{1}{3}(x - 5)$	A1	
	$(x - 5)^2 + (y + 2)^2 = 18, y + 2 = -\frac{1}{3}(x - 5)$ $\Rightarrow \frac{10}{9}(x - 5)^2 = 18 \Rightarrow x = \dots$ or $\Rightarrow 10(y + 2)^2 = 18 \Rightarrow y = \dots$	M1	
	$x = \frac{25 \pm 9\sqrt{5}}{5}, y = \frac{-10 \pm 3\sqrt{5}}{5}, k = y - 3x \Rightarrow k = \dots$	M1	
	$k = -17 \pm 6\sqrt{5}$	A1	

M1: Applies negative reciprocal rule to obtain gradient of radius

A1: Correct equation of radial line passing through the centre of C

M1: Solves simultaneously to find x or y

Alternatively solves " $y = -\frac{1}{3}x - \frac{1}{3}$ " and $y = 3x + k$ to get x in terms of k which they substitute in

$x^2 + (3x + k)^2 - 10x + 4(3x + k) + 11 = 0$ to form an equation in k .

M1: Applies $k = y - 3x$ with at least one pair of values to find k

A1: Correct simplified values