Year 2 Pure Chapter 9: Trig Functions, Exponentials and Logs Exam Questions (Answers)

Q1.

(i)	$\frac{dx}{dy} = 4\sec^2 2y \tan 2y$	B1
	Use $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$	M1
	Uses $\tan^2 2y = \sec^2 2y - 1$ and $\sec 2y = \sqrt{x}$ to get $\frac{dx}{dy}$ or $\frac{dy}{dx}$ in terms of just x	M1
	$\frac{dy}{dx} = \frac{1}{4x(x-1)^{\frac{1}{2}}}$ (conclusion stated with no errors previously)	A1* (4)
(ii)	$\frac{dy}{dx} = (x^2 + x^3) \times \frac{2}{2x} + (2x + 3x^2)\ln 2x$	M1 A1 A1
	When $x = \frac{e}{2}$, $\frac{dy}{dx} = 3(\frac{e}{2}) + 4(\frac{e}{2})^2 = 3(\frac{e}{2}) + e^2$	dM1 A1 (5)
(iii)	$f'(x) = \frac{(x+1)^{\frac{1}{2}}(-3\sin x) - 3\cos x(\frac{1}{2}(x+1)^{-\frac{1}{2}})}{(x+1)^{\frac{1}{2}}}$	M1 A1
	$f'(x) = \frac{-3(x+1)(\sin x) - \cos x}{(x+1)^{\frac{1}{2}}}$	A1 (3)

Q2.

(a)	$\frac{d}{dx}(\cos 2x) = -2\sin 2x$	B1	
	Applies $\frac{vu'-uv'}{v^2}$ to $\frac{\cos 2x}{\sqrt{x}}$ $= \frac{\sqrt{x}\times -2\sin 2x - \cos 2x \times \frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2}$	M1A1	
	$= \frac{-2\sqrt{x}\sin 2x - \frac{1}{2}x^{-\frac{1}{2}}\cos 2x}{x}$		**(3)**
(b)	$\frac{d}{dx}(\sec^2 3x) = 2\sec 3x \times 3\sec 3x \tan 3x \,(= 6\sec^2 3x \tan 3x)$	M1	
	$= 6(1+\tan^2 3x)\tan 3x$	dM1	
	$= 6(\tan 3x + \tan^3 3x)$	A1	**(3)**
(c)	$x = 2\sin\left(\frac{y}{3}\right) \Rightarrow \frac{dx}{dy} = \frac{2}{3}\cos\left(\frac{y}{3}\right)$	M1A1	
	$\frac{dy}{dx} = \frac{1}{\frac{2}{3}\cos\left(\frac{y}{3}\right)} = \frac{1}{\frac{2}{3}\sqrt{\left(1-\sin^2\left(\frac{y}{3}\right)\right)}} = \frac{1}{\frac{2}{3}\sqrt{1-\left(\frac{x}{2}\right)^2}}$	dM1	
	$\frac{dy}{dx} = \frac{3}{\sqrt{4-x^2}}$ cao	A1	**(4)**

Q3.

(i)(a)	$\frac{dy}{dx} = 3x^2 \times \ln 2x + x^3 \times \frac{1}{2x} \times 2$ $= 3x^2 \ln 2x + x^2$	M1A1A1 (3)
(i)(b)	$\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$	B1M1A1 (3)
(ii)	$\frac{dx}{dy} = -\text{cosec}^2 y$	M1A1
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y}$	M1
Uses $\text{cosec}^2 y = 1 + \cot^2 y$ and $x = \cot y$ in $\frac{dy}{dx}$ or $\frac{dx}{dy}$ to get an expression in x		
	$\frac{dy}{dx} = -\frac{1}{\text{cosec}^2 y} = -\frac{1}{1+\cot^2 y} = -\frac{1}{1+x^2}$ cso	M1, A1* (5)

Q4.

	$\frac{dI}{dt} = -16\ln(0.5)0.5^t$	M1 A1
At $t = 3$	$\frac{dI}{dt} = -16\ln(0.5)0.5^3$	M1
	$= -2\ln 0.5 = \ln 4$	M1 A1
		[5]

Q5.

(a)		
	$\frac{d}{dx}(\ln(3x)) \rightarrow \frac{B}{x}$ for any constant B	M1
Applying vu'+uv' ,	$\ln(3x) \times 2x + x$	M1, A1 A1 (4)
(b)		
Applying $\frac{vu'-uv'}{v^2}$		
	$\frac{x^3 \times 4\cos(4x) - \sin(4x) \times 3x^2}{x^6}$	M1 $\frac{A1+A1}{A1}$
	$= \frac{4x\cos(4x) - 3\sin(4x)}{x^4}$	A1 (5)
		(9 MARKS)

Q6.

(a)(i)	$\frac{d}{dx}\left(e^{3x}(\sin x + 2\cos x)\right) = 3e^{3x}(\sin x + 2\cos x) + e^{3x}(\cos x - 2\sin x)$	M1 A1 A1 (3)
	$\left(= e^{3x}(\sin x + 7\cos x)\right)$	
(ii)	$\frac{d}{dx}\left(x^3 \ln(5x+2)\right) = 3x^2 \ln(5x+2) + \frac{5x^3}{5x+2}$	M1 A1 A1 (3)
(b)	$\frac{dy}{dx} = \frac{(x+1)^2(6x+6) - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$	M1 $\frac{A1}{A1}$
	$= \frac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$	M1
	$= \frac{20}{(x+1)^3}$ *	A1 cso (5)