AS and A level Further Mathematics Practice Paper – Complex numbers (part 1) – Mark scheme

Question	Scheme	Marks
1	$(x-5)$ is a factor of f(x) so f(x) = $(x-5)(9x^2 \ldots\ )$	M1
	f(x) = $(x-5)(9x^2+12x+5)$	A1
	Solve $(9x^2+12x+5)=0$ to give x =	M1
	$(x=)-\frac{2}{3}-\frac{1}{3}\text{i},\ -\frac{2}{3}+\frac{1}{3}\text{i}$ or $-\frac{2}{3}\pm\frac{1}{3}\text{i}$ or $\frac{-2\pm\text{i}}{3}$ oe (and 5)	A1cao A1ft
		(5 marks)
2(a)	f$(x) = 2x^3 - 6x^2 - 7x - 4$	
	f$(4) = \underline{128 - 96 - 28 - 4 = 0}$	B1
		(1)
2(b)	f$(4) = 0 \Rightarrow (x-4)$ is a factor.	
	f$(x) = (x-4)(2x^2 + 2x + 1)$	M1 A1
	So, $x = \frac{-2 \pm \sqrt{4 - 4(2)(1)}}{2(2)}$	M1
	$(2)\left(x^2 + x + \frac{1}{2}\right) = 0 \Rightarrow (2)\left(\left(x \pm \frac{1}{2}\right)^2 \pm k \pm \frac{1}{2}\right)\ k \neq 0 \Rightarrow x =$	
	$\Rightarrow x = 4,\ \frac{-2 \pm 2\text{i}}{4}$	A1
		(4)
		(5 marks)
3	$\text{f}(z) = 2z^3 - 3z^2 + 8z + 5$	
	$1-2\text{i}$ (is also a root)	B1
	$(z-(1+2\text{i}))(z-(1-2\text{i})) = z^2 - 2z + 5$	M1A1
	$\text{f}(z) = \left(z^2 - 2z + 5\right)\left(2z + 1\right)$	M1
	$\left(z_3\right) = -\frac{1}{2}$	A1
		(5 marks)

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Question	Scheme	Marks
4(a)	$\frac{z_1}{z_2} = \frac{p+2i}{1-2i}.\frac{1+2i}{1+2i}$	M1
	$= \frac{p+2pi+2i-4}{5}$	M1
	$= \frac{p-4}{5}, \quad + \frac{2p+2}{5}\text{i}$	A1A1
		(4)
4(b)	$\left\|\frac{z_1}{z_2}\right\|^2 = \left(\frac{p-4}{5}\right)^2 + \left(\frac{2p+2}{5}\right)^2$,	M1
	$\left(\frac{p-4}{5}\right)^2 + \left(\frac{2p+2}{5}\right)^2 = 13^2$	dM1
	or $\sqrt{\left(\frac{p-4}{5}\right)^2 + \left(\frac{2p+2}{5}\right)^2} = 13$	
	$\frac{p^2-8p+16}{25} + \frac{4p^2+8p+4}{25} = 169$ *or* 13^2	
	$5p^2 + 20 = 4225$	
	$p^2 = 841 \Rightarrow p = \pm 29$	dM1A1
	OR	
	$\frac{\lvert z_1\rvert}{\lvert z_2\rvert} = \frac{\sqrt{p^2+4}}{\sqrt{5}}$	M1
	$\frac{\sqrt{p^2+4}}{\sqrt{5}} = 13$ oe	dM1
	$\frac{p^2+4}{5} = 169$ *or* 13^2oe	
	$p^2 = 841 \Rightarrow p = \pm 29$	dM1A1
		(4)
		(8 marks)

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Question	Scheme	Marks
5(a)	$z = 8 + 3\text{i}, \quad w = -2\text{i}$ $z - w \{= (8 + 3\text{i}) - (-2\text{i})\} = 8 + 5\text{i}$	B1 **(1)**
5(b)	$zw \{= (8 + 3\text{i})(-2\text{i})\} = 6 - 16\text{i}$	M1 A1 **(2)**
		(3 marks)
6(a)	$\arg z_1 = -\arctan(1)$ $-\arctan(1)$ or $\arctan(1)$or $\arctan(-1)$ $= -\frac{\pi}{4}$ or -45 or awrt -0.785 (oe e.g $\frac{7\pi}{4}$) **Correct answer only 2/2**	M1 A1 **(2)**
6(b)	$z_1 z_2 = (1 - i)(3 + 4i) = 3 - 3i + 4i - 4i^2$ At least 3 correct terms (Unsimplified) $= 7 + \text{i}$ cao	M1 A1 **(2)**
6(c)	$\frac{z_2}{z_1} = \frac{(3 + 4\text{i})}{(1 - \text{i})} = \frac{(3 + 4\text{i}).(1 + \text{i})}{(1 - \text{i}).(1 + \text{i})}$ Multiply top and bottom by (1 + i) $= \frac{(3 + 4\text{i}).(1 + \text{i})}{2}$ $(1 + \text{i})(1 - \text{i}) = 2$ $= -\frac{1}{2} + \frac{7}{2}\text{i}$ or $\frac{-1 + 7\text{i}}{2}$ **Correct answers only in (b) and (c) scores no marks**	M1 A1 A1 **(3)**
		(7 marks)

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Question	Scheme		Marks
7(a)	$z = 5 - 3\mathrm{i},\ w = 2 + 2\mathrm{i}$ $z^2 = (5 - 3\mathrm{i})(5 - 3\mathrm{i})$ $= 25 - 15\mathrm{i} - 15\mathrm{i} + 9\mathrm{i}^2$ $= 25 - 15\mathrm{i} - 15\mathrm{i} - 9$ $= 16 - 30\mathrm{i}$	An attempt to multiply out the brackets to give four terms (or four terms implied). zw is M0 $16 - 30\mathrm{i}$ Answer only 2/2	M1 A1 **(2)**
7(b)	$\frac{z}{w} = \frac{(5 - 3\mathrm{i})}{(2 + 2\mathrm{i})}$ $= \frac{(5 - 3\mathrm{i})}{(2 + 2\mathrm{i})} \times \frac{(2 - 2\mathrm{i})}{(2 - 2\mathrm{i})}$ $= \frac{10 - 10\mathrm{i} - 6\mathrm{i} - 6}{4 + 4}$ $= \frac{4 - 16\mathrm{i}}{8}$ $= \frac{1}{2} - 2\mathrm{i}$	Multiplies $\frac{z}{w}$ by $\frac{(2 - 2\mathrm{i})}{(2 - 2\mathrm{i})}$ Simplifies realising that a real number is needed on the denominator and applies $\mathrm{i}^2 = -1$ on their numerator expression and denominator expression. $\frac{1}{2} - 2\mathrm{i}$ or $a = \frac{1}{2}$ and $b = -2$ or equivalent Answer as a single fraction A0	M1 M1 A1 **(3)**
			(5 marks)

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Question	Scheme		Marks
8(a)	$\lvert z_1\rvert = \sqrt{(-2)^2 + 1^2} = \sqrt{5} = 2.236...$	$\sqrt{5}$ or awrt 2.24	B1 **(1)**
8(b)	$\arg z = \pi - \tan^{-1}\left(\frac{1}{2}\right)$	$\tan^{-1}\left(\frac{1}{2}\right)$ or $\tan^{-1}\left(\frac{2}{1}\right)$ or $\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ or $\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ or $\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ or $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$	M1
	$= 2.677945045... = 2.68$ (2 dp)	awrt 2.68	A1 oe
	$\arg z = \tan^{-1}\left(\frac{1}{-2}\right)$ = -0.46 on its own is M0 but $\pi + \tan^{-1}\left(\frac{1}{-2}\right)$ = 2.68 scores M1A1 $\pi - \tan^{-1}\left(\frac{1}{-2}\right)$ = is M0 as is $\pi - \tan\left(\frac{1}{2}\right)$ (2.60)		**(2)**
8(c)	$z^2 - 10z + 28 = 0$		
	$z = \frac{10 \pm \sqrt{100 - 4(1)(28)}}{2(1)}$	An attempt to use the quadratic formula (usual rules)	M1
	$= \frac{10 \pm \sqrt{100 - 112}}{2}$		
	$= \frac{10 \pm \sqrt{-12}}{2}$		
	$= \frac{10 \pm 2\sqrt{3}\,i}{2}$	Attempt to simplify their $\sqrt{-12}$ in terms of i,. e.g. i $\sqrt{12}$ or i $\sqrt{3\times 4}$	M1
	So, $z = 5 \pm \sqrt{3}\,i$. $\{p = 5,\ q = 3\}$	$5 \pm \sqrt{3}\,i$	A1 oe **(3)**

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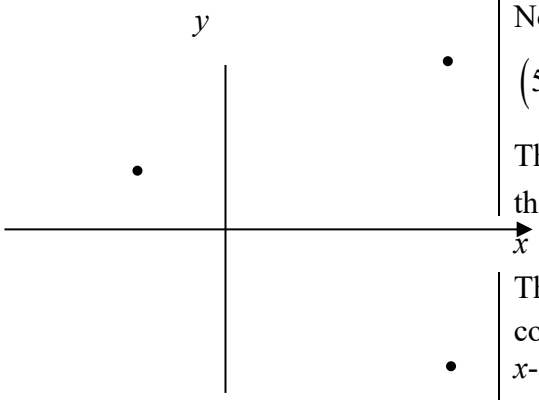
Question	Scheme		Marks
8(d)		Note that the points are $(-2, 1)$, $(5, \sqrt{3})$ and $(5, -\sqrt{3})$. The point $(-2, 1)$ plotted correctly on the Argand diagram with/without label. The distinct points z_2 and z_3 plotted correctly and symmetrically about the *x*-axis on the Argand diagram with/without label.	B1 B1 **(2)**
			(8 marks)
9(a)	$\frac{50}{3+4i} = \frac{50(3-4i)}{(3+4i)(3-4i)} = \frac{50(3-4i)}{25} = 6-8i$		M1 A1cao **(2)**
9(b)	$z^2 = (6-8i)^2 = 36-64-96\text{i} = -28-96\text{i}$		M1 A1 **(2)**
9(c)	$\lvert z\rvert = \sqrt{6^2+(-8)^2} = 10$		M1 A1ft **(2)**
9(d)	$\tan\alpha = \frac{-96}{-28}$ so $\alpha = -106.3^\circ$ or 253.7°		M1 A1 cao **(2)**
			(8 marks)

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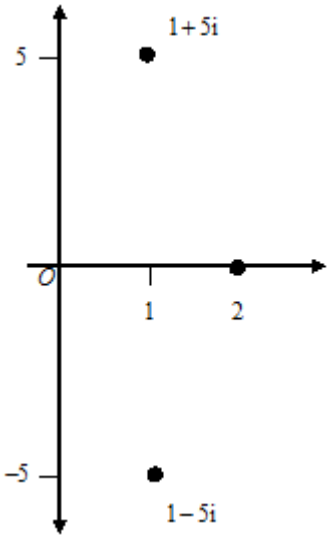
Question	Scheme	Marks
10(a)	$x^3 + px^2 + 30x + q = 0$ $1+5i$	 B1 **(1)**
10(b)	$\left(\left(x-\left(1+5i\right)\right)\left(x-\left(1-5i\right)\right)\right) = x^2 - 2x + 26$ $\left((x-2)\left(x-\left(1\pm 5i\right)\right)\right) = x^2 - (3\pm 5i)x + 2(1\pm 5i)$ $\left(x^2 - 2x + 26\right)\left(x-2\right) = x^3 + px^2 + 30x + q$ $p = -4, \quad q = -52$ f(1+5i)=0 or f(1-5i)=0 $q - 24p - 44 = 0$ and $10p + 40 = 0$ $p = -4, \quad q = -52$	M1A1 M1 A1A1 M1 A1 M1 A1A1 **(5)**
10(c)		B1 B1 **(2)**
		(8 marks)

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Question	Scheme	Marks
11(a)	**Ignore part labels and mark part (a) and part (b) together.**	
	$f\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3-9\left(\frac{1}{2}\right)^2+k\left(\frac{1}{2}\right)-13$	M1
	$\left(\frac{1}{4}\right)-\left(\frac{9}{4}\right)+\left(\frac{k}{2}\right)-13=0\Rightarrow k=.....$	dM1
	k = 30	A1 cao
	Alternative using long division:	
	$2x^3-9x^2+kx-13\div(2x-1)$	
	$=x^2-4x+\frac{1}{2}k-2\,(\text{Quotient})$	M1
	$\text{Re mainder}\,\frac{1}{2}k-15$	
	$\frac{1}{2}k-15=0$	dM1
	$k=30$	A1
	Alternative by inspection:	
	$(2x-1)(x^2-4x+13)=2x^3-9x^2+30x-13$	M1dM1
	k = 30	A1
		(3)
11(b)	$f(x)=(2x-1)(x^2-4x+13)$	
	$or\left(x-\frac{1}{2}\right)(2x^2-8x+26)$	M1
	$x^2-4x+13\text{or}\;\;2x^2-8x+26$	A1
	$x=\frac{4\pm\sqrt{4^2-4\times13}}{2}$ or equivalent	M1
	$x=\frac{4\pm6i}{2}=2\pm3i$	A1 oe
		(4)
		(7 marks)

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Question	Scheme	Marks
12(a)	$w = \dfrac{p-4\mathrm{i}}{2-3\mathrm{i}} \qquad \arg w = \dfrac{\pi}{4}$	
	$w = \dfrac{(p-4\mathrm{i})}{(2-3\mathrm{i})} \times \dfrac{(2+3\mathrm{i})}{(2+3\mathrm{i})}$	M1
	$= \left(\dfrac{2p+12}{13}\right) + \left(\dfrac{3p-8}{13}\right)\mathrm{i}$	A1 A1
		(3)
12(b)	$\left\{\arg w = \dfrac{\pi}{4} \Rightarrow\right\} \quad 2p+12 = 3p-8$ o.e. seen anywhere.	M1
	$\Rightarrow p = 20$	A1
		(2)
12(c)	$z = (1-\lambda\mathrm{i})(4+3\mathrm{i})$ and $\lvert z\rvert = 45$	
	$\sqrt{1+\lambda^2}\ \sqrt{4^2+3^2}$	M1
	$\sqrt{1+\lambda^2}\ \sqrt{4^2+3^2} = 45$	A1
	$\left\{\lambda^2 = 9^2 - 1 \Rightarrow\right\} \quad \lambda = \pm 4\sqrt{5}$	A1
		(3)
		(8 marks)

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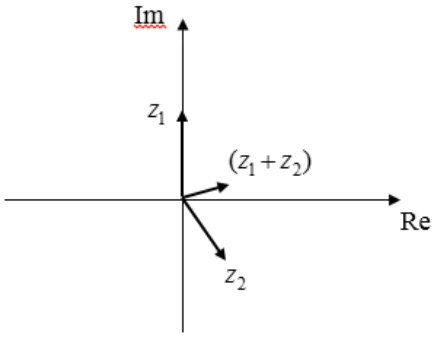
Question	Scheme	Marks
13(a)	$z_2 = \dfrac{6(1-\mathrm{i}\sqrt{3})}{(1+\mathrm{i}\sqrt{3})(1-\mathrm{i}\sqrt{3})} = \dfrac{6(1-\mathrm{i}\sqrt{3})}{4}$	M1
	$z_2 = \dfrac{6(1-\mathrm{i}\sqrt{3})}{4}\left(= \dfrac{3}{2} - \mathrm{i}\dfrac{3}{2}\sqrt{3}\right)$	A1
		(2)
13(b)	$\lvert z_2\rvert = \sqrt{\left(\dfrac{3}{2}\right)^2 + \left(\dfrac{3\sqrt{3}}{2}\right)^2} = \sqrt{\dfrac{9}{4} + \dfrac{27}{4}}$	M1
	The modulus of z_2 is 3	A1
	$\tan\theta = (\pm)\sqrt{3}$ and attempts to find θ	M1
	and the argument is $-\dfrac{\pi}{3}$	A1
		(4)
13(c)		M1 A1
		(2)
		(8 marks)

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Question	Scheme	Marks
14(a)	$z = \dfrac{p+2\mathrm{i}}{3+p\mathrm{i}} \cdot \dfrac{3-p\mathrm{i}}{3-p\mathrm{i}}$	M1
	$= \dfrac{3p - p^2\mathrm{i} + 6\mathrm{i} + 2p}{9+p^2}$	M1
	$= \dfrac{5p}{p^2+9}, \qquad + \dfrac{6-p^2}{p^2+9}\mathrm{i}$	A1A1
		(4)
14(b)	$\arg(z) = \arctan\left(\dfrac{\dfrac{6-p^2}{p^2+9}}{\dfrac{5p}{p^2+9}}\right)$	M1
	$\dfrac{6-p^2}{5p} = 1$	M1
	$p^2 + 5p - 6 = 0$	A1
	$(p+6)(p-1) = 0 \Rightarrow x =$	M1
	$p = 1, p = -6$	A1
		(5)
		(9 marks)

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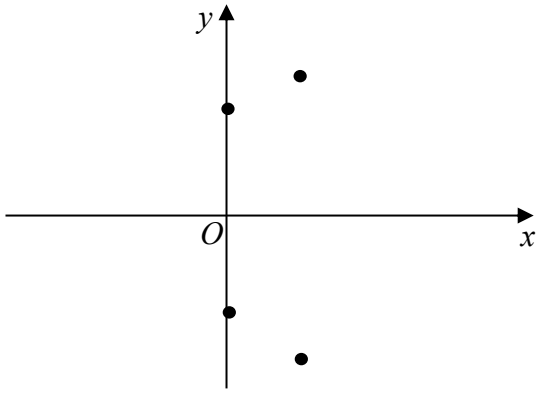
Question	Scheme	Marks
15(a)	$f(x) = (4x^2 + 9)(x^2 - 2x + 5) = 0$	
	$(4x^2 + 9) = 0 \Rightarrow x = \frac{3i}{2}, -\frac{3i}{2}$	M1 A1
	$(x^2 - 2x + 5) = 0 \Rightarrow x = \frac{2 \pm \sqrt{4 - 4(1)(5)}}{2(1)}$	M1
	$\Rightarrow x = \frac{2 \pm \sqrt{-16}}{2}$	
	$\Rightarrow x = 1 \pm 2i$	A1
		(4)
15(b)		B1ft B1ft
		(2)
		(6 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	FP1 2015	1		Complex numbers	1.1b, 3.1a
2	FP1 2012	1		Complex numbers	1.1b, 3.1a
3	FP1 2014R	1		General complex numbers	1.1b, 3.1a
4	FP1 2014	1		General complex numbers	1.1b, 3.1a
5	FP1 2013R	1		Complex numbers	1.1b, 3.1a
6	FP1 Jan 2012	1		Complex numbers	1.1b, 3.1a
7	FP1 2011	1		Complex numbers	1.1b
8	FP1 2011	2		Complex numbers	1.1b
9	FP1 Jan 2013	2		Complex numbers	1.1b, 3.1a
10	FP1 2014	3		General complex numbers	1.1b, 3.1a
11	FP1 2013	3		Complex numbers	1.1b, 3.1a
12	FP1 2017	4		Complex numbers	1.1b, 3.1a
13	FP1 2015	4		Complex numbers	1.1b, 3.1a
14	FP1 2014R	4		General complex numbers	1.1b, 3.1a
15	FP1 2013R	4		Complex numbers	1.1b, 3.1a