

Topic Test Summer 2022

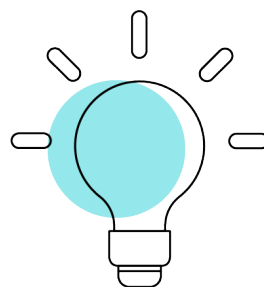
Pearson Edexcel GCE Mathematics (9MA0)

Paper 3 – Mechanics

Topic 3: Kinematics-Projectiles

Contents

<u>General guidance to Topic Tests</u>	<u>3</u>
<u>Revise Revision Guide content coverage</u>	<u>4</u>
<u>Questions</u>	<u>5</u>
<u>Mark Scheme</u>	<u>27</u>



Mark Scheme

Question T3_Q1

Question	Scheme	Marks	AOs
6.	Integrate $\mathbf{v}$ w.r.t. time	M1	1.1a
	$\mathbf{r} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j} \text{ (+ C)}$	A1	1.1b
	Substitute $t = 4$ and $t = 1$ into their $\mathbf{r}$	M1	1.1b
	$t = 4, \mathbf{r} = 4\mathbf{i} - 32\mathbf{j} \text{ (+ C)}; t = 1, \mathbf{r} = 2\mathbf{i} - 2\mathbf{j} \text{ (+ C)}$ or $(4, -32); (2, -2)$	A1	1.1b
	$\sqrt{2^2 + (-30)^2}$	M1	1.1b
	$\sqrt{904} = 2\sqrt{226}$	A1	1.1b
		(6)	
(6 marks)			
Notes: Allow column vectors throughout			
M1: At least one power increasing by 1. A1: Any correct (unsimplified) expression M1: Must have attempted to integrate $\mathbf{v}$. Substitute $t = 4$ and $t = 1$ into their $\mathbf{r}$ to produce 2 vectors (or 2 points if just working with coordinates). A1: $4\mathbf{i} - 32\mathbf{j} \text{ (+ C)}$ and $2\mathbf{i} - 2\mathbf{j} \text{ (+ C)}$ or $(4, -32)$ and $(2, -2)$. These can be seen or implied. M1: Attempt at distance of form $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for their points. Must have 2 non zero terms. A1: $\sqrt{904} = 2\sqrt{226}$ or any equivalent surd (exact answer needed)			

Question T3_Q2

Question	Scheme	Marks	AOs
10(a)	Using the model and vertical motion: $0^2 = (U \sin \alpha)^2 - 2g \times (3-2)$	M1	3.3
	$U^2 = \frac{2g}{\sin^2 \alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut \cos \alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-\frac{5}{4} = Ut \sin \alpha - \frac{1}{2}gt^2$	A1	1.1b
	sub for t : $-\frac{5}{4} = U \sin \alpha \left(\frac{20}{U \cos \alpha} \right) - \frac{1}{2}g \left(\frac{20}{U \cos \alpha} \right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\frac{5}{4} = 20 \tan \alpha - 100 \tan^2 \alpha$	A1(I)	1.1b
	$(4 \tan \alpha - 1)(100 \tan \alpha + 5) = 0$	M1(III)	1.1b
	$\tan \alpha = \frac{1}{4} \Rightarrow \alpha = 14^\circ$ or better	A1(II)	2.2a
		(9)	
	N.B. For the last 5 marks, they may set up a quadratic in t , by substituting for $U \sin \alpha$ first, then solve the quadratic to find the value of t , then use $20 = Ut \cos \alpha$ to find α . The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		
	Sub for $U \sin \alpha$ to give equation in t only	M1(II)	
	$-\frac{5}{4} = \sqrt{2g}t - \frac{1}{2}gt^2$	A1(I)	
	Solve for t	M1(III)	
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20 = Ut \cos \alpha$	M1(I)	
	$\alpha = 14^\circ$ or better	A1(II)	
(b)	ALTERNATIVE		

	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut \cos \alpha$	A1	1.1b
	A to top: $s = vt - \frac{1}{2}at^2$ <u>and</u> top to T: $s = ut + \frac{1}{2}at^2$		
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ <u>and</u> $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	M1	3.4
	$= \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}} \quad (= \frac{5}{\sqrt{2g}})$	A1	1.1b
	$20 = U \frac{5}{\sqrt{2g}} \cos \alpha$ (sub. for t)	M1	3.1b
	$20 = \sqrt{\frac{2g}{\sin^2 \alpha}} \frac{5}{\sqrt{2g}} \cos \alpha$ (sub. for U)	M1	3.1b
	$\tan \alpha = \frac{1}{4}$	A1	1.1b
	Solve for α	M1	1.1b
	$\square \alpha = 14^\circ$ or better	A1	2.2a
		(9)	
(c)	The target will have dimensions so in practice there would be a range of possible values of α Or There will be air resistance Or The ball will have dimensions Or Wind effects Or Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U = \sqrt{\frac{2g}{\sin^2 \alpha}}$	M1	3.1b
	Use $20 = Ut \cos \alpha$ (or use vertical motion equation)	A1 M1	1.1b
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 A1	1.1b
		(3)	
(d)	ALTERNATIVE		

	$A \text{ to top: } s = vt - \frac{1}{2}at^2 \quad \text{and} \quad \text{top to } T: s = ut + \frac{1}{2}at^2$	M1	3.1b
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}} \quad \text{and} \quad \frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	A1 M1	1.1b
	$= \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}} \quad (= \frac{5}{\sqrt{2g}}) = 1.1 \text{ or } 1.13 \text{ (s)}$	B1 A1	1.1b
		(3)	
(15 marks)			
Notes:			
(a) M1: Or any other complete method to obtain an equation in U , g and α only A1*: Correct GIVEN ANSWER			
(b) M1: Using horizontal motion A1: Correct equation M1: Using vertical motion . N.B. M0 if they use $s = \pm 2$ or ± 3 , but allow $s = \pm 1.25$ or ± 0.75 or ± 2.25 or ± 2.75 A1: Correct equation M1: Using $20 = Ut \cos \alpha$ to sub. for t M1: Substituting for U^2 using (a) A1: Correct quadratic equation (in $\tan \alpha$ or $\cot \alpha$) M1: Solve a 3 term quadratic, either by factorisation or formula (or by calculator (implied) if answer is correct) and find α A1: $\alpha = 14^\circ$ or better (No restriction on accuracy since g 's cancel) N.B. If answer is correct, previous M mark can be implied, but if answer is incorrect, an explicit attempt to solve must be seen to earn the previous M mark. (b) ALTERNATIVE M1: Using the model with the usual rules applying to the equation A1: Correct equation M1: Using the model to obtain the total time from A to T A1: Correct total time t M1: Substitute for t in $20 = Ut \cos \alpha$ M1: Substitute for U in $20 = Ut \cos \alpha$, using part (a) A1: Correct equation in $\tan \alpha$ only M1: Solve equation for α A1: $\alpha = 14^\circ$ or better (No restriction on accuracy since g 's cancel)			

N.B. If they quote the equation of the trajectory $y = x \tan \alpha - \frac{gx^2}{2U^2 \cos^2 \alpha}$ or AND put in values for x and y , could score first 5 marks, M1A1M1A1M1 (nothing for the equation only); wrong x value loses first A mark and wrong y value loses second A mark

(c)

B1: Give one limitation of the model e.g. the ball will have dimensions, or there will be air resistance or wind effects or spin

N.B. B0 if any incorrect extra(s) but ignore extra consequences.

(d)

M1: Using their α to find a value for U

A1: Treat as M1: Using their U to find a value for t

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

(d) ALTERNATIVE

M1: Using their α to find a value for U

A1: Treat as M1: Using their U to find a value for t

B1: Treat as A1 : $t = 1.1$ or 1.10 (since depends on $g = 9.8$)

Question T3_Q3

Question	Scheme	Marks	AO
	In this question mark parts (a) and (b) together.		
5(a)	Horizontal speed = $20 \cos 30^\circ$	B1	3.4
	Vertical velocity <u>at $t = 2$</u>	M1	3.4
	$= 20 \sin 30^\circ - 2g$	A1	1.1b
	$\theta = \tan^{-1} \left(\pm \frac{9.6}{10\sqrt{3}} \right)$	M1	1.1b
	Speed = $\sqrt{100 \times 3 + 9.6^2}$ or e.g. speed = $\frac{9.6}{\sin \theta}$	M1	1.1b
	19.8 or 20 (m s^{-1}) at 29.0° or 29° to the horizontal oe	A1	2.2a
		(6)	
(b)	Using sum of horizontal distances = 50 at $t = 2$	M1	3.3
	$(u \cos \theta) \times 2 + (20 \cos 30^\circ) \times 2 = 50$ $(u \cos \theta = 25 - 20 \cos 30^\circ)$	A1	1.1b
	Vertical distances equal	M1	3.4
	$\Rightarrow (20 \sin 30^\circ) \times 2 - \frac{g}{2} \times 4 = (u \sin \theta) \times 2 - \frac{g}{2} \times 4$ $(20 \sin 30^\circ = u \sin \theta)$	A1	1.1b
	Solving for both θ and u	M1	3.1b
	$\theta = 52^\circ$ or better (52.47756849....°) $u = 13$ or better (12.6085128...)	A1	2.2a
		(6)	
(c)	It does not take account of the fact that they are not particles (moving freely under gravity) It does not take account of the size(s) of the balls It does not take account of the spin of the balls It does not take account of the wind g is not exactly 9.8 m s^{-2} N.B. If they refer to the mass or weight of the balls give B0	B1	3.5b
		(1)	
		(13)	

Question T3_Q4

Question		Scheme	Marks	AOs
5(a)		Using horizontal motion	M1	3.3
		$U \cos 45^\circ t = 100$	A1	1.1b
		Using vertical motion	M1	3.4
		$U \sin 45^\circ t - \frac{1}{2}gt^2 = -25$	A1	1.1b
		Solve problem by eliminating t and solving for U	M1	3.1b
		$U = 28^*$	A1*	1.1b
			(6)	
5(b)		Using vertical motion	M1	3.4
		$0^2 = (28 \sin 45^\circ)^2 - 2gh$	A1	1.1b
		Greatest height = 45 m	A1	1.1b
			(3)	
5(c)		New value > 28	B1	3.5a
			(1)	
5(d)		e.g. wind effects, more accurate value of g , spin of ball, include size of the ball, not model as a particle, shape of ball	B1	3.5c
			(1)	
(11 marks)				
Notes:				
5a	M1	Complete method to give equation in U and t only, condone sin/cos confusion and sign errors		
	A1	Correct equation		
	M1	Complete method to give equation in U and t only, condone sin/cos confusion and sign errors		
	A1	Correct equation (g does not need to be substituted)		
	M1	Must have earned the previous two M marks. Eliminate t and solve for U . N.B. They may solve for t first ($100 - \frac{1}{2}gt^2 = -25$) and then use it to find U .		
	A1*	Exact given answer correctly obtained with no wrong working (e.g. $g = 9.81$ used) or approximation seen.		
5b	M1	Complete method to give equation in h only (allow if U not substituted), condone sin/cos confusion and sign errors		

Marks		Notes
5a	B1	Seen or implied, possibly on a diagram
	M1	Use of $v = u + at$ or any other complete method <u>using $t = 2$</u> Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation in v or v^2
	M1	Correct use of trig to find a relevant angle for the direction. Must have found a horizontal and a vertical velocity component
	M1	Use Pythagoras or trig to find the magnitude Must have found a horizontal and a vertical velocity component
	A1	Or equivalent. Need magnitude and direction stated or implied in a diagram. (0.506 or 0.51 rads)
5b	M1	First equation, in terms of u and θ (could be implied by subsequent working), using the horizontal motion with $t = 2$ used Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation – any equivalent form
	M1	Second equation, in terms of u and θ (could be implied by subsequent working), using the vertical motion – equating distances or just vertical components of velocities. Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation – any equivalent form
	M1	Complete strategy: all necessary equations formed and solve for u and θ N.B. This is an independent method mark but can only be earned if 50 m has been used in their solution.
	A1	Both values correct. (Here we accept 2SF or better, since the g 's cancel) Allow radians for θ : 0.92 or better (0.915906..) rads.
5c	B1	Any factor related to the model as stated in the question. Penalise incorrect extras but ignore consequences e.g. 'AB (or the ground) is not horizontal' should be penalised or 'they do not move in a vertical plane' should be penalised

	A1	Correct equation (g does not need to be substituted) (A0 if U is used instead of 28)
	A1	cao
5c	B1	Clear statement
5d	B1	Penalise incorrect extras i.e. B0 if there are incorrect extras. The ground being horizontal, the cliff being vertical, .. are not part of the model so B0 Include weight/mass of the ball B0

Question T3_Q5

Question	Scheme		Marks	AOs
	Note that $g = 10$; penalise once for whole question if $g = 9.8$			
4(a)	Use $s = ut + \frac{1}{2}at^2$ vertically or any complete method to give an equation in t only		M1	3.4
	$-70 = 65 \sin \alpha \times t - \frac{1}{2} \times g \times t^2$	A1	1.1b	
		M (A)1	1.1b	
	$t = 7$ (s)	A1	1.1b	
		(4)		
4(b)	Horizontal velocity component at $A = 65 \cos \alpha$ (60)		B1	3.4
	Complete method to find vertical velocity component at A		M1	3.4
	$65 \sin \alpha - g \times 7$ OR $\sqrt{(-25)^2 + 2g \times 70}$ (45)		A1ft	1.1b
	Sub for trig and square, add and square root : $\sqrt{60^2 + (-45)^2}$		M1	3.1b
	75 Accept 80 (m s ⁻¹)		A1	1.1b
			(5)	
4(c)	e.g. an approximate value of g has been used, the dimensions of the stone could affect its motion, spin of the stone, $g = 10$ instead of 9.8 has been used, g has been assumed to be constant, wind effect, shape of the stone		B1	3.5b
			(1)	
(10 marks)				
Notes:				
4a	M1	Complete method, correct no. of terms, condone sign errors and sin/cos confusion		
	A1	Correct equation in t only with at most one error		
	M(A)1	Correct equation in t only		
		N.B. For ‘up and down’ methods etc, the two A marks are for all the equations that they use, lose a mark for each error.		
	A1	Cao ($g = 9.8$, 7.1 or 7.11) ($g = 9.81$, 7.1 or 7.12)		
4b	B1	Seen, including on a diagram.		
	M1	Condone sign errors and sin/cos confusion		
	A1ft	Correct expression; accept negative of this, follow their t		
	M1	Sub for trig and use Pythagoras		
	A1	Cao ($g = 9.8$ or 9.81 , 75 or 74.8)		

4c	B1	B0 if incorrect extras
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