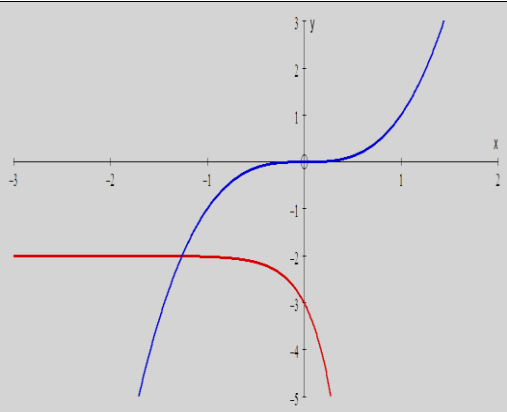


A level Mathematics Practice Paper – Numerical methods – Mark scheme

Question	Scheme		Marks
1(a)	$\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$		M1 A1
	Puts $\frac{dy}{dx} = 0$ to give $x^3 = -2 - e^{4x}$		A1 *
			(3)
1(b)		$y = x^3$	B1
		Shape of $y = -2 - e^{4x}$	B1
		$y = -2 - e^{4x}$ cuts y axis at (0, -3)	B1
		$y = -2 - e^{4x}$ has asymptote at $y = -2$	B1
			(4)
1(c)	Only one crossing point		B1
			(1)
1(d)	-1.26376, -1.26126 Accept answers which round to these answers to 5dp		M1 A1
			(2)
1(e)	$\alpha = -1.26$ and so turning point is at (-1.26, -2.55)		M1 A1cao
			(2)
			(12 marks)

A level Mathematics Practice Paper – Numerical methods – Mark scheme

Question	Scheme	Marks
2(a)	$x^3 + 3x^2 + 4x - 12 = 0 \Rightarrow x^3 + 3x^2 = 12 - 4x$	
	$\Rightarrow x^2(x+3) = 12 - 4x$	M1
	$\Rightarrow x^2 = \frac{12-4x}{(x+3)} \Rightarrow x = \sqrt{\frac{4(3-x)}{(x+3)}}$	dM1 A1*
		(3)
2(b)	$x_1 = 1.41, \text{ awrt } x_2 = 1.20 \quad x_3 = 1.31$	M1 A1 A1
		(3)
2(c)	Choosing (1.2715, 1.2725)	
	or tighter containing root 1.271998323	M1
	$f(1.2725) = (+)0.00827... \quad f(1.2715) = -0.00821....$	M1
	Change of sign $\Rightarrow \alpha = 1.272$	A1
		(3)
		(9 marks)
3(a)	$0 = e^{x-1} + x - 6 \Rightarrow x = \ln(6-x) + 1$	M1 A1*
		(2)
3(b)	Sub $x_0 = 2$ into $x_{n+1} = \ln(6-x_n) + 1 \Rightarrow x_1 = 2.3863$	M1 A1
	AWRT 4 dp. $x_2 = 2.2847 \quad x_3 = 2.3125$	A1
		(3)
3(c)	Chooses interval [2.3065, 2.3075]	M1
	$g(2.3065) = -0.0002(7), g(2.3075) = 0.004(4)$	dM1
	Sign change, hence root (correct to 3dp)	A1
		(3)
		(8 marks)

A level Mathematics Practice Paper – Numerical methods – Mark scheme

Question	Scheme	Marks
4(a)	$f'(x) = 50x^2e^{2x} + 50xe^{2x}$ oe.	M1A1
	Puts $f'(x) = 0$ to give $x = -1$ and $x = 0$ or one coordinate	dM1A1
	Obtains 0, -16 and -1, $25e^{-2}$ -16 CSO	A1
		(5)
4(b)	Puts $25x^2e^{2x} - 16 = 0 \Rightarrow x^2 = \frac{16}{25}e^{-2x} \Rightarrow x = \pm \frac{4}{5}e^{-x}$	B1*
		(1)
4(c)	Subs $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \text{awrt } 0.485$	M1A1
	$\Rightarrow x_2 = \text{awrt } 0.492, x_3 = \text{awrt } 0.489$	A1
		(3)
4(d)	$\alpha = 0.49$ $f(0.485) = -0.487, f(0.495) = (+)0.485$, sign change and deduction	B1 B1
		(2)
		(11 marks)
5(a)	(i) 21	B1
	(ii) $4e^{2x} - 25 = 0 \Rightarrow e^{2x} = \frac{25}{4} \Rightarrow 2x = \ln\left(\frac{25}{4}\right) \Rightarrow x = \frac{1}{2}\ln\left(\frac{25}{4}\right), \Rightarrow x = \ln\left(\frac{5}{2}\right)$	M1 A1 A1
	(iii) 25	B1
		(5)
5(b)	$4e^{2x} - 25 = 2x + 43 \Rightarrow e^{2x} = \frac{1}{2}x + 17$	M1
	$\Rightarrow 2x = \ln\left(\frac{1}{2}x + 17\right) \Rightarrow x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$	A1*
		(2)
5(c)	$x_1 = \frac{1}{2}\ln\left(\frac{1}{2} \times 1.4 + 17\right) = \text{awrt } 1.44$	M1
	awrt $x_1 = 1.4368, x_2 = 1.4373$	A1
		(2)
5(d)	Defines a suitable interval 1.4365 and 1.4375	M1
	...and substitutes into a suitable function, e.g. $4e^{2x} - 2x - 68$, obtains correct values with both a reason and conclusion	A1
		(2)
		(11 marks)

A level Mathematics Practice Paper – Numerical methods – Mark scheme

Question	Scheme	Marks
6(a)	At P $x = -2 \Rightarrow y = 3$	B1
	$\frac{dy}{dx} = \frac{4}{2x+5} - \frac{3}{2}$	M1 A1
	$\frac{dy}{dx}\bigg _{x=-2} = \frac{5}{2} \Rightarrow$ Equation of normal is $y - '3' = -\frac{2}{5}(x - (-2))$	M1
	$\Rightarrow 2x + 5y = 11$	A1
		(5)
6(b)	Combines $5y + 2x = 11$ and $y = 2\ln(2x+5) - \frac{3x}{2}$ to form equation in x	
	$5\left(2\ln(2x+5) - \frac{3x}{2}\right) + 2x = 11$	M1
	$\Rightarrow x = \frac{20}{11}\ln(2x+5) - 2$	dM1 A1*
		(3)
6(c)	Substitutes $x_1 = 2 \Rightarrow x_2 = \frac{20}{11}\ln 9 - 2$	M1
	Awrt $x_2 = 1.9950$ and $x_3 = 1.9929$.	A1
		(2)
		(10 marks)

A level Mathematics Practice Paper – Numerical methods – Mark scheme

Question	Scheme		Marks
7(a)	Crosses x-axis $\Rightarrow f(x) = 0 \Rightarrow (8 - x)\ln x = 0$		
	Either $(8 - x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of $\{x\} = 1$ OR $x = \{8\}$	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A(1, \{0\})$ and $B(8, \{0\})$	B1
			(2)
7(b)	Apply product rule: $\left\{ \begin{array}{l} u = (8 - x) \quad v = \ln x \\ \frac{du}{dx} = -1 \quad \frac{dv}{dx} = \frac{1}{x} \end{array} \right\}$	$vu' + uv'$	M1
	$f'(x) = -\ln x + \frac{8-x}{x}$	Any one term correct	A1
		Both terms correct	A1
			(3)
7(c)	$f'(3.5) = 0.032951317...$ $f'(3.6) = -0.058711623...$ Sign change (and as $f'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate both $f'(3.5)$ and $f'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1
			(2)
7(d)	At Q, $f'(x) = 0 \Rightarrow -\ln x + \frac{8-x}{x} = 0$	Setting $f'(x) = 0$	M1
	$\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to $x =$	M1
	$\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \frac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1
			(3)

A level Mathematics Practice Paper – Numerical methods – Mark scheme

Question	Scheme		Marks
7(e)	Iterative formula: $x_{n+1} = \frac{8}{\ln x_n + 1}$		
	$x_1 = \frac{8}{\ln(3.55) + 1}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$	Both $x_1 = \text{awrt } 3.529$ and $x_2 = \text{awrt } 3.538$	A1
	$x_2 = 3.538246011...$		
	$x_3 = 3.534144722...$		
	$x_1 = 3.529, x_2 = 3.538, x_3 = 3.534, \text{ to } 3 \text{ dp.}$	x_1, x_2, x_3 all stated correctly to 3 dp	A1
			(3)
			(13 marks)
8(a)	$2^{x+1} - 3 = 17 - x \Rightarrow 2^{x+1} = 20 - x$		M1
	$(x+1) \ln 2 = \ln(20-x) \Rightarrow x = ...$		dM1
	$x = \frac{\ln(20-x)}{\ln 2} - 1$		A1*
			(3)
8(b)	Sub $x_0 = 3$ into $x_{n+1} = \frac{\ln(20-x_n)}{\ln 2} - 1 \Rightarrow x_1 = 3.087 \text{ (awrt)}$		M1 A1
	$x_2 = 3.080, x_3 = 3.081 \text{ (awrt)}$		A1
			(3)
8(c)	$A = (3.1, 13.9) \text{ cao}$		M1 A1
			(2)
			(8 marks)

A level Mathematics Practice Paper – Numerical methods – Mark scheme

Question	Scheme	Marks
9(a)	$y_{2,1} = -0.224$, $y_{2,2} = (+)0.546$	M1
	Change of sign $\Rightarrow Q$ lies between	A1
		(2)
9(b)	At R $\frac{dy}{dx} = -2x \sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3$	M1A1
	$-2x \sin\left(\frac{1}{2}x^2\right) + 3x^2 - 3 = 0 \Rightarrow x = \sqrt{1 + \frac{2}{3}x \sin\left(\frac{1}{2}x^2\right)}$ cso	M1A1*
		(4)
9(c)	$x_1 = \sqrt{1 + \frac{2}{3} \times 1.3 \sin\left(\frac{1}{2} \times 1.3^2\right)}$	M1
	$x_1 = \text{awrt } 1.284$ $x_2 = \text{awrt } 1.276$	A1
		(2)
		(8 marks)
10(a)	$f(0.8) = 0.082, f(0.9) = -0.089$	M1
	Change of sign $\Rightarrow$ root (0.8,0.9)	A1
		(2)
10(b)	$f'(x) = 2x - 3 - \sin\left(\frac{1}{2}x\right)$	M1 A1
	Sets $f'(x) = 0 \Rightarrow x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$	M1 A1*
		(4)
10(c)	Sub $x_0 = 2$ into $x_{n+1} = \frac{3 + \sin\left(\frac{1}{2}x_n\right)}{2}$	M1
	$x_1 = \text{awrt } 1.921, x_2 = \text{awrt } 1.91(0)$ and $x_3 = \text{awrt } 1.908$	A1 A1
		(3)
10(d)	[1.90775, 1.90785]	M1
	$f''(1.90775) = -0.00016...$ AND $f''(1.90785) = 0.0000076...$	M1
	Change of sign $\Rightarrow x = 1.9078$	A1
		(3)
		(12 marks)

A level Mathematics Practice Paper – Numerical methods – Mark scheme

	Source paper	Question number	New spec references	Question description	New AOs
1	C3 June 2014R	2	7.2, 2.7, 6.1, 9.1, 9.2	Differentiation of exponential, graph of exponential, using iterative methods	1.1b, 2.2a, 2.4
2	C3 2012	2	9.1, 9.2	Numerical methods	1.1b, 2.1, 2.4
3	C3 Jan 2013	2	9.1, 9.2, 6.3	Numerical methods	1.1b, 2.1, 2.4
4	C3 2013	4	7.2, 7.4, 9.1, 9.2	Differentiation and numerical analysis	1.1b, 2.1, 2.2a, 2.4
5	C3 2016	4	2.9, 6.3, 9.1, 9.2	Modulus of exponential, iterative equation	1.1b, 2.1, 2.4
6	C3 2017	5	7.2, 7.3, 2.4, 9.2	Normal to a function, leading to iterative methods	1.1b, 2.1
7	C3 Jan 2011	5	6.3, 7.3, 7.4, 9.1, 9.2	Exponentials and logarithms, Differentiation, Numerical methods	1.1b, 2.1, 2.4
8	C3 2015	6	6.5, 9.2	Intersection of graphs leading to iterative equation	1.1b, 2.2a
9	C3 June 2014	6	7.2, 7.4, 9.1, 9.2	Numerical methods	1.1b, 2.1, 2.4
10	C3 Jan 2012	6	7.2, 9.1, 9.2	Numerical methods	1.1b, 2.1, 2.4