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|--------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Write your name here                                                                             |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| Surname                                                                                          | Other names                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
| <b>Pearson</b><br><b>Edexcel GCE</b>                                                             | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">           Centre Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; display: inline-block;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; display: inline-block;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; display: inline-block;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; display: inline-block;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; display: inline-block;"></div> </div> <div style="text-align: center;">           Candidate Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; display: inline-block;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; display: inline-block;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; display: inline-block;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; display: inline-block;"></div> </div> </div> |
| <b>A level Mathematics</b><br><br><b>Practice Paper</b><br><b>Pure Mathematics - Integration</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
| <b>You must have:</b><br>Mathematical Formulae and Statistical Tables (Pink)                     | Total Marks<br><div style="border: 1px solid black; width: 60px; height: 40px; margin: 0 auto;"></div>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |

### Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 12 questions in this question paper. The total mark for this paper is 119.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a \* sign.

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. 
$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}.$$

(a) Find the values of the constants  $A$ ,  $B$  and  $C$ . (4)

(b) (i) Hence find  $\int f(x) \, dx$ .

(ii) Find  $\int_1^2 f(x) \, dx$ , leaving your answer in the form  $a + \ln b$ , where  $a$  and  $b$  are constants. (6)

(Total 10 marks)

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2. Use integration to find the exact value of  $\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$ .

(Total 6 marks)

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3. (a) Use integration to find

$$\int \frac{1}{x^3} \ln x \, dx.$$

(5)

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, dx.$$

(2)

(Total 7 marks)

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4. (a) Use integration by parts to find  $\int x \sin 3x \, dx$ .

(3)

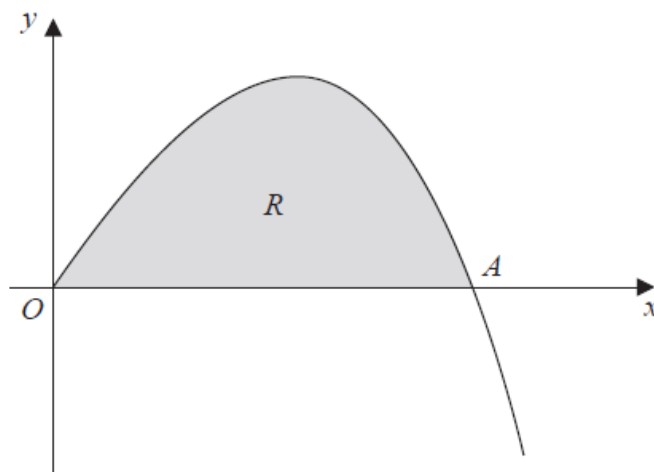
(b) Using your answer to part (a), find  $\int x^2 \cos 3x \, dx$ .

(3)

(Total 6 marks)

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5.



**Figure 1**

Figure 1 shows a sketch of part of the curve with equation  $y = 4x - xe^{\frac{1}{2}x}$ ,  $x \geq 0$ .

The curve meets the  $x$ -axis at the origin  $O$  and cuts the  $x$ -axis at the point  $A$ .

- (a) Find, in terms of  $\ln 2$ , the  $x$  coordinate of the point  $A$ . (2)

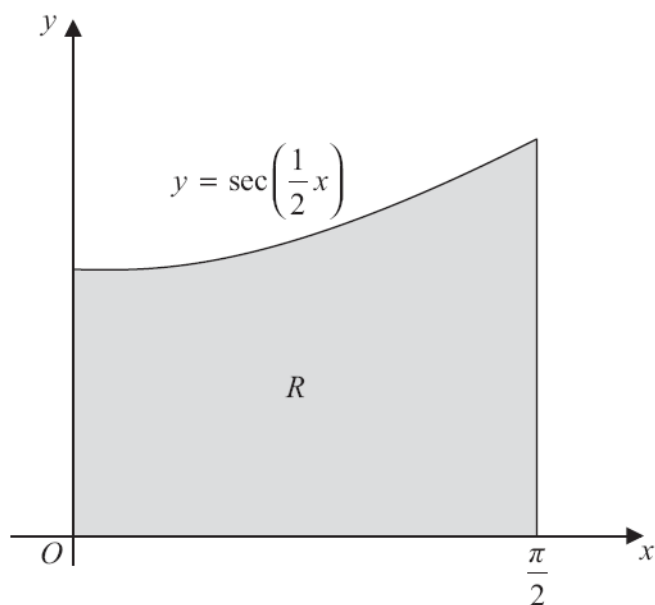
- (b) Find  $\int xe^{\frac{1}{2}x} dx$ . (3)

The finite region  $R$ , shown shaded in Figure 1, is bounded by the  $x$ -axis and the curve with equation  $y = 4x - xe^{\frac{1}{2}x}$ ,  $x \geq 0$ .

- (c) Find, by integration, the exact value for the area of  $R$ .  
Give your answer in terms of  $\ln 2$ . (3)

**(Total 8 marks)**

6.



**Figure 2**

Figure 2 shows the finite region  $R$  bounded by the  $x$ -axis, the  $y$ -axis, the line  $x = \frac{\pi}{2}$  and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \frac{\pi}{2}$$

The table shows corresponding values of  $x$  and  $y$  for  $y = \sec\left(\frac{1}{2}x\right)$ .

|     |   |                 |                 |                 |
|-----|---|-----------------|-----------------|-----------------|
| $x$ | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ |
| $y$ | 1 | 1.035276        |                 | 1.414214        |

- (a) Complete the table above giving the missing value of  $y$  to 6 decimal places. (1)
- (b) Using the trapezium rule, with all of the values of  $y$  from the completed table, find an approximation for the area of  $R$ , giving your answer to 4 decimal places. (3)

Region  $R$  is rotated through  $2\pi$  radians about the  $x$ -axis.

- (c) Use calculus to find the exact volume of the solid formed. (4)

(Total 8 marks)

7. (a) Express  $\frac{5}{(x-1)(3x+2)}$  in partial fractions. (3)

(b) Hence find  $\int \frac{5}{(x-1)(3x+2)} dx$ , where  $x > 1$ . (3)

- (c) Find the particular solution of the differential equation

$$(x-1)(3x+2) \frac{dy}{dx} = 5y, \quad x > 1,$$

for which  $y = 8$  at  $x = 2$ . Give your answer in the form  $y = f(x)$ . (6)

**(Total 12 marks)**

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8. (i) Given that  $y > 0$ , find

$$\int \frac{3y-4}{y(3y+2)} dy. \quad (6)$$

- (ii) (a) Use the substitution  $x = 4\sin^2\theta$  to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)} dx = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta d\theta,$$

where  $\lambda$  is a constant to be determined. (5)

- (b) Hence use integration to find

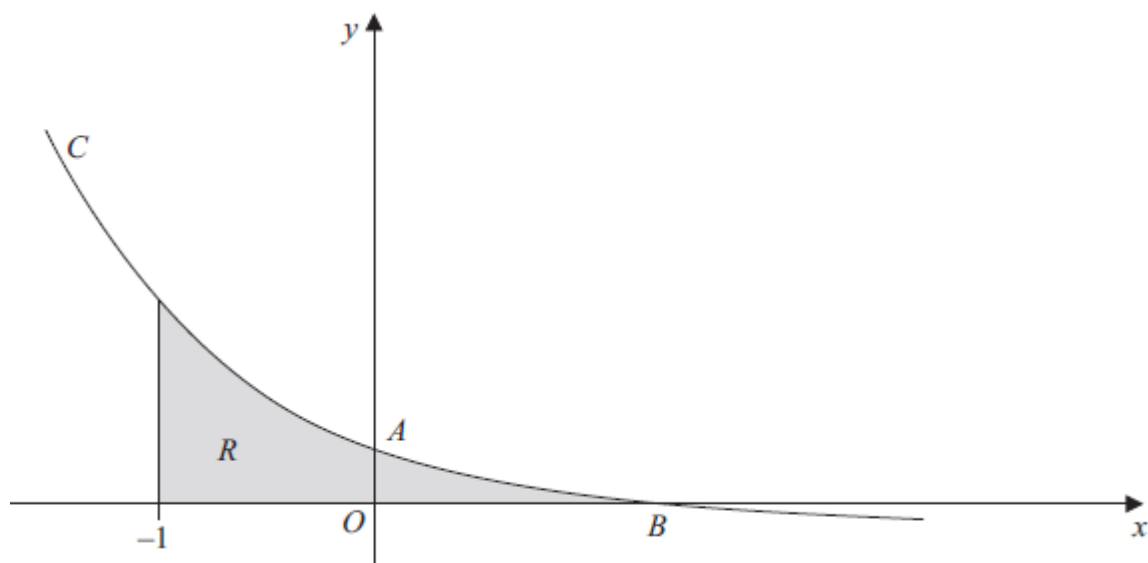
$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)} dx,$$

giving your answer in the form  $a\pi + b$ , where  $a$  and  $b$  are exact constants. (4)

**(Total 15 marks)**

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9.



**Figure 3**

Figure 3 shows a sketch of part of the curve  $C$  with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1.$$

The curve crosses the  $y$ -axis at the point  $A$  and crosses the  $x$ -axis at the point  $B$ .

(a) Show that  $A$  has coordinates  $(0, 3)$ . (2)

(b) Find the  $x$ -coordinate of the point  $B$ . (2)

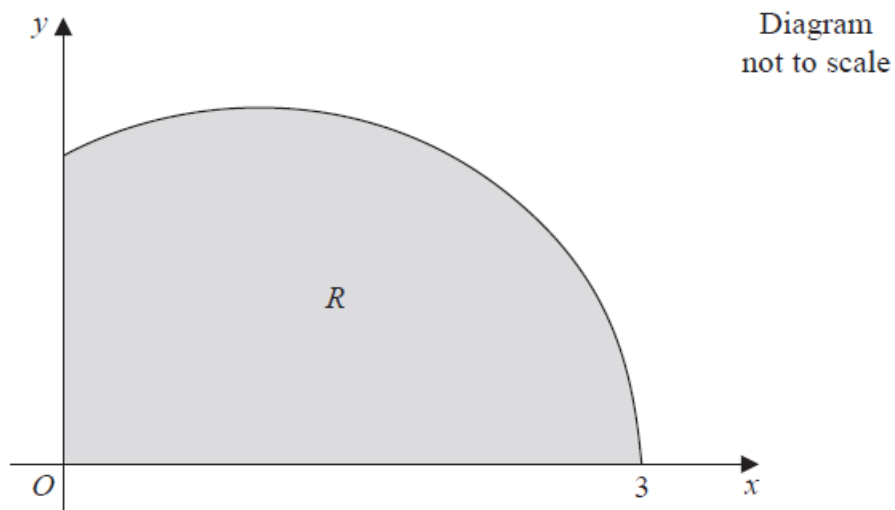
(c) Find an equation of the normal to  $C$  at the point  $A$ . (5)

The region  $R$ , as shown shaded in Figure 3, is bounded by the curve  $C$ , the line  $x = -1$  and the  $x$ -axis.

(d) Use integration to find the exact area of  $R$ . (6)

**(Total 15 marks)**

10.



**Figure 4**

Figure 4 shows a sketch of the curve with equation  $y = \sqrt{(3-x)(x+1)}$ ,  $0 \leq x \leq 3$ .

The finite region  $R$ , shown shaded in Figure 4, is bounded by the curve, the  $x$ -axis, and the  $y$ -axis.

(a) Use the substitution  $x = 1 + 2 \sin \theta$  to show that

$$\int_0^3 \sqrt{(3-x)(x+1)} \, dx = k \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta,$$

where  $k$  is a constant to be determined.

**(5)**

(b) Hence find, by integration, the exact area of  $R$ .

**(3)**

**(Total 8 marks)**

11.

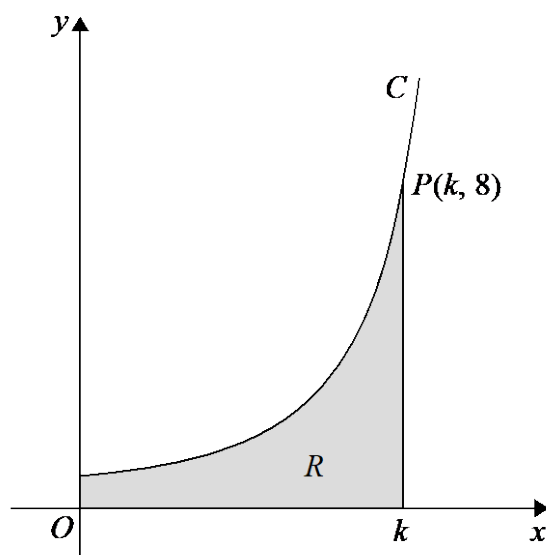


Diagram not  
drawn to scale

**Figure 5**

Figure 5 shows a sketch of part of the curve  $C$  with parametric equations

$$x = 3\theta \sin \theta, \quad y = \sec^3 \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point  $P(k, 8)$  lies on  $C$ , where  $k$  is a constant.

(a) Find the exact value of  $k$ .

(2)

The finite region  $R$ , shown shaded in Figure 5, is bounded by the curve  $C$ , the  $y$ -axis, the  $x$ -axis and the line with equation  $x = k$ .

(b) Show that the area of  $R$  can be expressed in the form

$$\lambda \int_{\alpha}^{\beta} (\theta \sec^2 \theta + \tan \theta \sec^2 \theta) d\theta$$

where  $\lambda$ ,  $\alpha$  and  $\beta$  are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of  $R$ .

(6)

**(Total 12 marks)**

12. a) Express  $\frac{1}{P(5-P)}$  in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15}P(5-P), \quad t \geq 0,$$

where  $P$ , in thousands, is the population of meerkats and  $t$  is the time measured in years since the study began.

Given that when  $t = 0$ ,  $P = 1$ ,

- (b) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + ce^{\frac{1}{3}t}}$$

where  $a$ ,  $b$  and  $c$  are integers.

(8)

- (c) Hence show that the population cannot exceed 5000.

(1)

(Total 12 marks)

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**TOTAL FOR PAPER: 119 MARKS**