Differentiation Chain, Product and Quotient Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$\frac{1}{(x^2+3x+5)} \times \ldots , = \frac{2x+3}{(x^2+3x+5)}$	M1,A1 (2)
(b)	Applying $\frac{vu' - uv'}{v^2}$	M1,
	$\frac{x^2 \times -\sin x - \cos x \times 2x}{(x^2)^2} = \frac{-x^2 \sin x - 2x\cos x}{x^4} = \frac{-x\sin x - 2\cos x}{x^3}$ oe	A2,1,0 (3)
		5 Marks

Q2.

Question No	Scheme	Marks
	(a)	
	$\frac{d}{dx}(\ln(3x)) \to \frac{B}{x}$ for any constant B	M1
	Applying vu'+uv' , $\ln(3x) \times 2x + x$	M1, A1 A1 (4)
	(b)	
	Applying $\frac{vu' - uv'}{v^2}$	
	$\frac{x^3 \times 4\cos(4x) - \sin(4x) \times 3x^2}{x^6}$	M1 $\underline{\text{A1+A1}}$ A1
	$= \frac{4x\cos(4x) - 3\sin(4x)}{x^4}$	A1 (5)
		(9 MARKS)

Q3.

Question Number	Scheme	Marks
(a)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{(5x-1)}\right)=\dfrac{\mathrm{d}}{\mathrm{d}x}\left((5x-1)^{\frac{1}{2}}\right)$	
	$=5\times\dfrac{1}{2}(5x-1)^{-\frac{1}{2}}$	M1 A1
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{(5x-1)}+\dfrac{5}{2}x^2(5x-1)^{-\frac{1}{2}}$	M1 A1ft
	At $x=2$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=4\sqrt{9}+\dfrac{10}{\sqrt{9}}=12+\dfrac{10}{3}$	M1
	$=\dfrac{46}{3}$ Accept awrt 15.3	A1 (6)
(b)	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\sin 2x}{x^2}\right)=\dfrac{2x^2\cos 2x-2x\sin 2x}{x^4}$	M1 $\dfrac{\text{A1+A1}}{\text{A1}}$ (4) [10]
	Alternative to (b)	
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sin 2x\times x^{-2}\right)=2\cos 2x\times x^{-2}+\sin 2x\times(-2)x^{-3}$	M1 A1 + A1
	$=2x^{-2}\cos 2x-2x^{-3}\sin 2x$ $\left(=\dfrac{2\cos 2x}{x^2}-\dfrac{2\sin 2x}{x^3}\right)$	A1 (4)

Q4.

Question	Scheme	Marks	AOs
	Attempts the product and chain rule on $y = x(2x+1)^4$	M1	2.1
	$\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$	A1	1.1b
	Takes out a common factor $\frac{dy}{dx} = (2x+1)^3\{(2x+1)+8x\}$	M1	1.1b
	$\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$	A1	1.1b
			(4 marks)

Notes:

M1: Applies the product rule to reach $\frac{dy}{dx} = (2x+1)^4 + Bx(2x+1)^3$

A1: $\frac{dy}{dx} = (2x+1)^4 + 8x(2x+1)^3$

M1: Takes out a common factor of $(2x+1)^3$

A1: The form of this answer is given. Look for $\frac{dy}{dx} = (2x+1)^3(10x+1) \Rightarrow n = 3, A = 10, B = 1$

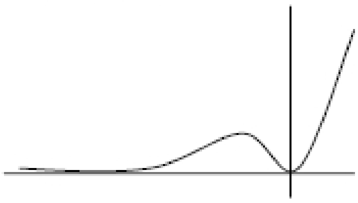
Q5.

Question Number	Scheme	Marks
	(a) $-32 = (2w-3)^5 \Rightarrow w = \frac{1}{2}$ oe	M1A1
		(2)
	(b) $\frac{dy}{dx} = 5 \times (2x-3)^4 \times 2$ or $10(2x-3)^4$	M1A1
	When $x = \frac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160' = \frac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y = 160x - 112$ cso	A1
		(5)
		(7 marks)

Q6.

Question Number	Scheme		Marks
(a)	$y = \dfrac{3+\sin 2x}{2+\cos 2x}$		
	Apply quotient rule: $\left\{\begin{matrix} u = 3+\sin 2x & v = 2+\cos 2x \\ \dfrac{du}{dx} = 2\cos 2x & \dfrac{dv}{dx} = -2\sin 2x \end{matrix}\right\}$		
	$\dfrac{dy}{dx} = \dfrac{2\cos 2x(2+\cos 2x) - -2\sin 2x(3+\sin 2x)}{(2+\cos 2x)^2}$	Applying $\dfrac{vu' - uv'}{v^2}$ Any one term correct on the numerator Fully correct (unsimplified).	M1 A1 A1
	$= \dfrac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2+\cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2+\cos 2x)^2}$		
	$= \dfrac{4\cos 2x + 6\sin 2x + 2}{(2+\cos 2x)^2}$ (as required)	For correct proof with an understanding that $\cos^2 2x + \sin^2 2x = 1$. No errors seen in working.	A1*
			(4)
(b)	When $x = \frac{\pi}{2}$, $y = \dfrac{3+\sin\pi}{2+\cos\pi} = \dfrac{3}{1} = 3$	$y = 3$	B1
	At $(\frac{\pi}{2}, 3)$, $m(T) = \dfrac{6\sin\pi + 4\cos\pi + 2}{(2+\cos\pi)^2} = \dfrac{-4+2}{1^2} = -2$	$m(T) = -2$	B1
	Either T: $y - 3 = -2(x - \frac{\pi}{2})$ or $y = -2x + c$ and $3 = -2(\frac{\pi}{2}) + c \Rightarrow c = 3+\pi$;	$y - y_1 = m(x - \frac{\pi}{2})$ with 'their TANGENT gradient' and their y_1; or uses $y = mx + c$ with 'their TANGENT gradient';	M1
	T: $y = -2x + (\pi + 3)$	$y = -2x + \pi + 3$	A1
			(4) [8]

Q7.

Question Number	Scheme	Marks
(*a*)	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^2\mathrm{e}^x + 2x\mathrm{e}^x$	M1,A1,A1 (3)
(*b*)	If $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $\quad \mathrm{e}^x(x^2 + 2x) = 0 \qquad$ setting $(a) = 0$ $[\mathrm{e}^x \neq 0] \qquad x(x+2) = 0$ $(x = 0) \qquad x = -2$ $x = 0, y = 0$ **and** $x = -2, y = 4\mathrm{e}^{-2}$ $(= 0.54\ldots)$	M1 A1 A1 √ (3)
(*c*)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x^2\mathrm{e}^x + 2x\mathrm{e}^x + 2x\mathrm{e}^x + 2\mathrm{e}^x \qquad \left[= (x^2 + 4x + 2)\mathrm{e}^x\right]$	M1, A1 (2)
(*d*)	$x = 0,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0\ (=2) \qquad x = -2,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0\ \ [= -2\mathrm{e}^{-2}\ (= -0.270\ldots)]$ M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b) $\therefore$minimum $\qquad \therefore$maximum	M1 A1 (cso) (2)
Alt.(*d*)	For M1: Evaluate, or state sign of, $\frac{\mathrm{d}y}{\mathrm{d}x}$ at two appropriate values – on either side of at least one of their answers from (b) or Evaluate y at two appropriate values – on either side of at least one of their answers from (b) or Sketch curve 	**(10 marks)**

Q8.

Question Number	Scheme	Marks
(a)(i)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{3x}(\sin x+2\cos x)\right)=3\mathrm{e}^{3x}(\sin x+2\cos x)+\mathrm{e}^{3x}(\cos x-2\sin x)$	M1 A1 A1 (3)
	$\left(=\mathrm{e}^{3x}(\sin x+7\cos x)\right)$	
(ii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3\ln(5x+2)\right)=3x^2\ln(5x+2)+\frac{5x^3}{5x+2}$	M1 A1 A1 (3)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+1)^2(6x+6)-2(x+1)(3x^2+6x-7)}{(x+1)^4}$	M1 $\frac{\text{A1}}{\text{A1}}$
	$=\frac{(x+1)(6x^2+12x+6-6x^2-12x+14)}{(x+1)^4}$	M1
	$=\frac{20}{(x+1)^3}$ *	A1 cso (5)
(c)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{60}{(x+1)^4}=-\frac{15}{4}$	M1
	$(x+1)^4=16$	M1
	$x=1,-3$ both	A1 (3)
		(14 marks)