Applied 2 - Projectiles Exam Questions (Answers)

Q1.

Question Number	Scheme	Marks	
a	After 4 seconds from O, horizontal speed $= u\cos\theta$	B1	
	Vertical component of speed at A $= u + at$	M1	Complete method using *suvat* to find *v*.
	$= u\sin\theta - 4g$	A1	
	At A, components are $15\cos 20$ (horizontal) and $15\sin 20$ (vertical)	B1	
	$u\cos\theta = 15\cos 20$ $u\sin\theta = 15\sin 20 + 4g$	DM1	Form simultaneous equations in u and θ and attempt to solve for u or θ. Depends on the previous M1
	$\theta = 72.4 \ \ (72)$	A1	Remember - A0 for the first overspecified answer
	$u = 46.5 \quad (47)$	A1	
		[7]	
Alt a	After 4 seconds from O, horizontal speed $= u\cos\theta$	B1	
	At $t = 4$, $s = vt - \frac{1}{2}gt^2$	M1	Complete method to find the vertical height at A
	$= 98.9.......$	A1	
	At A, components are $15\cos 20$ (horizontal) and $15\sin 20$ (vertical)	B1	
	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - 2gh$	DM1	Conservation of energy. The equation needs to include all three terms but condone sign error(s).
	$u = 46.5 \quad (47)$	A1	Remember - A0 for the first overspecified answer
	$\theta = 72.4 \ \ (72)$	A1	Beware inappropriate use of *suvat*
b	$-15\sin 20 = 15\sin 20 - gt$ or $0 = 15\sin 20t - \frac{1}{2}gt^2$	M1	Complete method using *suvat* or otherwise to find the time to travel from A to B
	$t = 1.05$ (s) or 1.0 (s)	A1	
		[2]	
c	Total time $= 4 + (1.05) + 4$	B1ft	Follow their *t* or $\frac{2u\sin\theta}{g}$ for their u, θ
	Range $= 46.5\times\cos 72.4\times(8+1.05)$ (or $15\cos 20\times 9.05$)	M1	Correct method to find *OC* for their *t*,*u* and θ
	$= 128$ (m) or 127 (m) (130)	A1	
		[3]	
		(12)	

Q2.

Question Number	Scheme	Marks	
(a)	$0^2 = u_V^{\ 2} - 2 \times 9.8 \times 10$	M1 A1	Complete method using *suvat* to form an equation in u_V. Correct equation e.g. $0 = u^2 - 20g$
	$u_V = 14$ *	A1	*Answer given* requires equation and working, including 196, seen.
		(3)	
OR	conservation of energy:	M1	Initial KE = gain in GPE + final KE
	$\frac{1}{2}m(u_h^{\ 2} + u_v^{\ 2}) = mg \times 10 + \frac{1}{2}mu_h^{\ 2}$, $\frac{1}{2}u_v^{\ 2} = 98$	A1	Correct equation
	$u_V = 14$ *	A1	*Answer given*
		(3)	
(b)	$(\uparrow)$, $-52.5 = 14t - \frac{1}{2}gt^2$	M1 A1 A1	Use the vertical distance travelled to find the total time taken. At most one error Correct equation
	$49t^2 - 140t - 525 = 0$ $(t-5)(49t+105) = 0 \quad t = 5$	DM1 A1	Solve for t. Dependent on the preceding M mark only
	$(\rightarrow)$, $50 = 5u_H$	M1	Use their time of flight to form an equation in u_H
	$u_H = 10$	A1	only
	$u = \sqrt{10^2 + 14^2}$	M1	Use of Pythagoras with two non-zero components, or solution of a pair of simultaneous equations in u and α.
	$= \sqrt{296}$; 17.2 m s^{-1}	A1	17.2 or 17 (method involves use of g = 9.8 so an exact surd answer is not acceptable)
		(9)	
			See next page for an alternative route to u, and (c).
OR			First 3 marks for the quadratic as above.
	$50 = u\cos\alpha t$ or $50 = u_H t$	M1	Used in their quadratic
	$49\left(\frac{50}{u_H}\right)^2 - 140\left(\frac{50}{u_H}\right) - 525 = 0$ $525(u_H)^2 + 140(u_H) - 122500 = 0$	A1	Correct quadratic in u_H
	Solve for u_H	DM1	Dependent on the M mark for setting up the initial quadratic equation in t.
	$u_H = 10$	A1	only
	etc.		Complete as above.
(c)	$\tan OBA = \frac{52.5}{50} = 1.05$	B1	Correct direction o.e. (accept reciprocal)
	$v_V = 1.05 \times 10 = 10.5$	M1	Use trig. with their u_H and correct interpretation of direction to find the vertical component of speed. Working with distances is M0. (condone $10 \div 1.05$)
	$(\uparrow)$, $-10.5 = 14 - gt$	DM1 A1	Use suvat to form an equation in t. Dependent on the preceding M. Correct equation for their u_H. For incorrect direction give A0 here.
	$t = 2.5$	A1	only
		(5) **17**	

Q3.

Question Number	Scheme	Marks
(a)	$\mathbf{i} \rightarrow$ distance $= 6t$	B1
	$\mathbf{j} \uparrow$ distance $= 12t - \frac{1}{2}gt^2$	M1 A1
	At B, $2\left(12t - \frac{1}{2}gt^2\right) = 6t$	M1 A1
	$(24-6)t = gt^2$	DM1
	$18 = gt$, $t = \frac{18}{g}(=1.84\text{s})$	A1
		(7)
(b)	$\mathbf{i} \rightarrow$ speed $= 6$	B1
	$\mathbf{j} \uparrow$ velocity $= 12 - gt = -6$	M1 A1
	$\therefore$ speed at A	
	$= \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2}(=8.49)(\text{ms}^{-1})$	M1 A1
		(5)
(c)	$\uparrow$ speed $= 12 - gt = +6$	M1 A1 ft
	$t = \frac{6}{g}(=0.61\text{s})$	A1
		(3)
		15

Q4.

(a)	Using $s = ut + \frac{1}{2}at^2$ Method must be clear $\mathbf{r} = (3t)\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$ Answer given	M1 A1 A1 (3)
(b)	$\mathbf{j}$ component $= 0$: $10 + 5t - 4.9t^2$ quadratic formula: $t = \frac{5 \pm \sqrt{25+196}}{9.8} = \frac{5 \pm \sqrt{221}}{9.8}$ $T = 2.03$(s), 2.0 (s) positive solution only.	M1 DM1 A1 (3)
(c)	Differentiating the position vector (or working from first principles) $\mathbf{v} = 3\mathbf{i} + (5 - 9.8t)\mathbf{j}$ (ms^{-1})	M1 A1 (2)
(d)	At B the $\mathbf{j}$ component of the velocity is the negative of the $\mathbf{i}$ component: $5 - 9.8t = -3$, $8 = 9.8t$, $t = 0.82$	M1 A1 (2)
(e)	$\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$, speed $= \sqrt{3^2 + 3^2} = \sqrt{18} = 4.24$ (m s^{-1})	M1A1 (2) **[12]**

Q5.

Question	Scheme	Marks	AOs
(a)	Using the model and vertical motion: $0^2 = (U\sin\alpha)^2 - 2g \leftrightarrow (3-2)$	M1	3.3
	$U^2 = \dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \frac{1}{2}at^2$	M1	3.4
	$-\frac{5}{4} = Ut\sin\alpha - \frac{1}{2}gt^2$	A1	1.1b
	sub for t: $-\frac{5}{4} = U\sin\alpha\left(\frac{20}{U\cos\alpha}\right) - \frac{1}{2}g\left(\frac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\frac{5}{4} = 20\tan\alpha - 100\tan^2\alpha$	A1(I)	1.1b
	$(4\tan\alpha - 1)(100\tan\alpha + 5) = 0$	M1(III)	1.1b
	$\tan\alpha = \frac{1}{4} \Rightarrow \alpha = 14^0$ or better	A1(II)	2.2a
		(9)	
	N.B. For the last 5 marks, they may set up a quadratic in t, by substituting for $U\sin\alpha$ first, then solve the quadratic to find the value of t, then use $20 = Ut\cos\alpha$ to find α. The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		

	Sub for $U\sin\alpha$ to give equation in t only	M1(II)	
	$-\frac{5}{4}=\sqrt{2g}t-\frac{1}{2}gt^2$	A1(I)	
	Solve for t	M1(III)	
	$t=\frac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20=Ut\cos\alpha$	M1(I)	
	$\alpha=14^0$ or better	A1(II)	
(b)	**ALTERNATIVE**		
	Using the model and horizontal motion: $s=ut$	M1	3.4
	$20=Ut\cos\alpha$	A1	1.1b
	A to top: $s=vt-\frac{1}{2}at^2$ <u>and</u> top to T: $s=ut+\frac{1}{2}at^2$		
	$1=\frac{1}{2}gt_1^{\ 2}$ => $t_1=\sqrt{\frac{2}{g}}$ <u>and</u> $\frac{9}{4}=\frac{1}{2}gt_2^{\ 2}$ => $t_2=\frac{3}{\sqrt{2g}}$ Total time $t=t_1+t_2$	M1	3.4
	$=\sqrt{\frac{2}{g}}+\frac{3}{\sqrt{2g}}$ $(=\frac{5}{\sqrt{2g}})$	A1	1.1b
	$20=U\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for t)	M1	3.1b
	$20=\sqrt{\frac{2g}{\sin^2\alpha}}\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for U)	M1	3.1b
	$\tan\alpha=\frac{1}{4}$	A1	1.1b
	Solve for α	M1	1.1b
	$\Rightarrow\alpha=14^0$ or better	A1	2.2a
		(9)	

(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U = \sqrt{\dfrac{2g}{\sin^2 \alpha}}$	M1	3.1b
	Use $20 = Ut\cos\alpha$ (or use vertical motion equation)	A1 M1	1.1b
	$t = \dfrac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 A1	1.1b
		(3)	
(d)	**ALTERNATIVE**		
	A to top: $s = vt - \dfrac{1}{2}at^2$ and top to T: $s = ut + \dfrac{1}{2}at^2$	M1	3.1b
	$1 = \dfrac{1}{2}gt_1^{\,2} \Rightarrow t_1 = \sqrt{\dfrac{2}{g}}$ <u>and</u> $\dfrac{9}{4} = \dfrac{1}{2}gt_2^{\,2} \Rightarrow t_2 = \dfrac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	A1 M1	1.1b
	$= = \sqrt{\dfrac{2}{g}} + \dfrac{3}{\sqrt{2g}} \quad \left(= \dfrac{5}{\sqrt{2g}}\right)$ = 1.1 or 1.13 (s)	B1 A1	1.1b
		(3)	
			(15 marks)