Factor Theorem - Section Test (Answers)

1) The roots of $(x-2)(x+1)x = 0$ are $x = 2, -1, 0$. In ascending order this is $-1, 0, 2$.

2) The factor theorem, $(x - a)$ is a factor of $f(x)$ if $f(a) = 0$. Therefore, it can be deduced that $f(3) = 0$. None of the other statements follow from the information given.

3) With $f(x) = x2 + ax + 2$ since $x + 1$ is a factor $f(-1) = 0$.
So $-1^2 - a + 2 = 0$ and $a = 3$.

4) The false statement is that $(x + 2)$ is a factor of $x^3 + 2x^2 - 4x + 8$.

By the factor theorem, $(x - a)$ is a factor of $f(x)$ if $f(a) = 0$.

With $f(x) = x^3 + x^2 - 4x - 4$,
$f(2) = 2^3 + 2^2 - 4 \times 2 - 4 = 8 + 4 - 8 - 4 = 0$, so $(x - 2)$ is a factor.

With $f(x) = x^3 + 2x^2 - 8x - 21$,
$f(3) = 3^3 + 2 \times 3^2 - 8 \times 3 - 21 = 27 + 18 - 24 - 21 = 0$, so $(x - 3)$ is a factor

With $f(x) = x^4 - x^3 + 2x^2 - 4x - 8$
$f(-1) = (-1)^4 - (-1)^3 + 2(-1)^2 - 4 \times (-1) - 8 = 1 + 1 + 2 + 4 - 8 = 0$, so $(x + 1)$ is a factor.

With $f(x) = x^3 + 2x^2 - 4x + 8$
$f(-2) = (-2)^3 + 2(-2)^2 - 4 \times (-2) + 8 = -8 + 8 + 8 + 8 = 16$, so $(x + 2)$ is not a factor.

5) The correct answer is $x^4 - x^3 + 2x^2 - 4x - 8$

By the factor theorem, $(x - 2)$ is a factor of $f(x)$ if $f(2) = 0$
With $f(x) = x^3 + x^2 - 4x - 2$
$f(2) = 2^3 + 2^2 - 4 \times 2 - 2 = 8 + 4 - 8 - 2 = 2$
With $x^3 + 2x^2 - 8x - 2$
$f(2) = 2^3 + 2 \times 2^2 - 8 \times 2 - 2 = 8 + 8 - 16 - 2 = -2$
$x^3 + 2x^2 - 4x + 8$
$f(2) = 2^3 + 2 \times 2^2 - 4 \times 2 + 8 = 8 + 8 - 8 + 8 = 16$
$x^4 - x^3 + 2x^2 - 4x - 8$
$f(2) = 2^4 - 2^3 + 2 \times 2^2 - 4 \times 2 - 8 = 16 - 8 + 8 - 8 - 8 = 0$

6) The correct answer is (x – 2) (x – 3) (x + 1)

$$f(x) = x^3 - 4x^2 + x + 6$$

$$f(1) = 1^3 - 4 \times 1^2 + 1 + 6 = 1 - 4 + 1 + 6 = 4$$

$$f(2) = 2^3 - 4 \times 2^2 + 2 + 6 = 8 - 16 + 2 + 6 = 0$$ so (x – 2) is a factor.

By inspection, $x^3 - 4x^2 + x + 6 = (x-2)(x^2 - 2x - 3)$

$$= (x-2)(x+1)(x-3)$$

7)

$$f(x) = x^3 + x^2 + 2x + 8$$

$$f(-1) = (-1)^3 + (-1)^2 + 2 \times -1 + 8 = -1 + 1 - 2 + 8 = 6$$

so (x + 1) is not a factor

$$f(1) = 1^3 + 1^2 + 2 \times 1 + 8 = 1 + 1 + 2 + 8 = 12$$

so (x – 1) is not a factor.

$$f(-2) = (-2)^3 + (-2)^2 + 2 \times -2 + 8 = -8 + 4 - 4 + 8 = 0$$

so (x + 2) is a factor

$$f(2) = 2^3 + 2^2 + 2 \times 2 + 8 = 8 + 4 + 4 + 8 = 24$$

so (x - 2) is not a factor

8)

$$f(x) = x^3 - 5x^2 + ax + 2$$

$$f(2) = 2^3 - 5 \times 2^2 + a \times 2 + 2$$

$$= 8 - 20 + 2a + 2$$

$$= 2a - 10$$

x – 2 is a factor, so f(2) = 0.

$$2a - 10 = 0$$

$$a = 5$$

9)

$$x^3 + x^2 - 5x + 3 = (x-1)(x^2 + px + q)$$

$$= x^3 - x^2 + px^2 - px + qx - q$$

$$= x^3 + (p-1)x^2 + (q-p)x - q$$

Equating constant terms

$\Rightarrow c = -3$

Equating coefficients of x^2

$\Rightarrow p - 1 = 1 \Rightarrow p = 2$

Check: coefficient of x

$= q - p = -3 - 2 = -5$

$$x^3 + x^2 - 5x + 3 = (x-1)(x^2 + 2x - 3)$$

10)

$f(x) = x^3 - x^2 - 34x - 56$

$f(1) = 1 - 1 - 34 - 56 \neq 0$

$f(-1) = -1 - 1 + 34 - 56 \neq 0$

$f(2) = 8 - 4 - 68 - 56 \neq 0$

$f(-2) = -8 - 4 + 68 - 56 = 0$

so $(x + 2)$ is a factor

$$x^3 - x^2 - 34x - 56 = (x+2)(x^2 - 3x - 28)$$
$$= (x+2)(x+4)(x-7)$$

Since $a \leq b$, $a=-7$ and $b=2$