

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
Level 3 GCE**

Centre Number

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Candidate Number

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Wednesday 7 October 2020

Afternoon (Time: 2 hours)

Paper Reference **9MA0/01**

Mathematics

Advanced

Paper 1: Pure Mathematics 1

You must have:

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations.
Calculators must not have the facility for symbolic algebra manipulation,
differentiation and integration, or have retrievable mathematical formulae
stored in them.**

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

- (b) Explain how you could use $x = \frac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

$$a) \quad 1 + \left(\frac{1}{2}\right)(8x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(8x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}(8x)^3$$

$$\underline{1 + 4x - 8x^2 + 32x^3}$$

$$b) \quad \sqrt{1 + 8\left(\frac{1}{32}\right)}$$

$$= \sqrt{1 + \frac{1}{4}}$$

$$= \sqrt{\frac{5}{4}}$$

$$= \frac{\sqrt{5}}{\sqrt{4}}$$

$$= \frac{\sqrt{5}}{2}$$

Substituting $\frac{1}{32}$ would find $\frac{\sqrt{5}}{2}$, we could then double the answer to find an estimate for $\sqrt{5}$



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Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 5 marks)



2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

$$\ln 4^{3p-1} = \ln 5^{210}$$

$$(3p-1) \ln 4 = 210 \ln 5$$

$$3p-1 = \frac{210 \ln 5}{\ln 4}$$

$$3p = \frac{210 \ln 5}{\ln 4} + 1$$

$$3p = 244.80245$$

$$p = \underline{\underline{81.6}} \text{ 1dp}$$

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Question 2 continued

Lined area for writing the answer to Question 2.

(Total for Question 2 is 3 marks)



3. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(a) Find $\vec{AB}$

(2)

(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

(2)

$$a/ \vec{AB} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ -6 \end{pmatrix} = \begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$$

b/ $OABC$ is a trapezium if it has one set of parallel sides

AB and OC
or OA and CB



$$\vec{OC} = \begin{pmatrix} 2 \\ -16 \\ 4 \end{pmatrix}$$

$$\vec{OC} = 2 \vec{AB}$$

$$\vec{OA} = \begin{pmatrix} 2 \\ 5 \\ -6 \end{pmatrix}$$

$$\vec{CB} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ -16 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 13 \\ -8 \end{pmatrix}$$

$\vec{OA}$ and $\vec{CB}$ are not parallel.

$\vec{OC}$ and $\vec{AB}$ are parallel.

$\therefore OABC$ is a trapezium.



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Question 3 continued

Handwriting practice area with horizontal lines.

(Total for Question 3 is 4 marks)



4. The function f is defined by

$$f(x) = \frac{3x-7}{x-2} \quad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$ Input for inverse is output for original function (2)

(b) Show that $ff(x) = \frac{ax+b}{x-3}$ where a and b are integers to be found. (3)

$$a/ \quad 7 = \frac{3x-7}{x-2}$$

$$7(x-2) = 3x-7$$

$$7x-14 = 3x-7$$

$$4x-14 = -7$$

$$4x = 7$$

$$x = \frac{7}{4}$$

$$b/ \quad f(x) = \frac{3x-7}{x-2}$$

$$ff(x) = \frac{3\left(\frac{3x-7}{x-2}\right) - 7}{\frac{3x-7}{x-2} - 2}$$

$$= \frac{\frac{9x-21}{x-2} - 7}{\frac{3x-7}{x-2} - 2}$$

$$\frac{9x-21}{x-2} - 7$$

$$= \frac{9x-21}{x-2} - \frac{7(x-2)}{x-2}$$

$$\frac{9x-21}{x-2} - \frac{7(x-2)}{x-2}$$

$$= \frac{9x-21-7x+14}{x-2}$$

$$\frac{3x-7-7x+14}{x-2}$$

$$= \frac{9x-21-7x+14}{3x-7-7x+14}$$



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Question 4 continued

$$= \frac{2x - 7}{x - 3}$$

(Total for Question 4 is 5 marks)



P 6 6 7 8 5 A 0 9 5 2

5. A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

$$u_n = a + (n-1)d$$

(a) find the fastest speed of the car in 3rd gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

$$a) \quad u_n = a + (n-1)d$$

$$a = 28$$

$$115 = 28 + 5d$$

$$87 = 5d$$

$$d = 17.4$$

$$u_3 = 28 + 2(17.4) \\ = \underline{\underline{62.8 \text{ km/h}}}$$

$$b) \quad u_n = ar^{n-1}$$

$$115 = 28r^5$$

$$\frac{115}{28} = r^5$$

$$r = \sqrt[5]{\frac{115}{28}}$$

$$= 1.3265$$

$$u_5 = 28(1.3265)^4$$

$$= \underline{\underline{86.7 \text{ km/h}}}$$



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Question 5 continued

Lined area for writing the answer to Question 5.

(Total for Question 5 is 6 marks)



6. (a) Express $\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, $\theta^\circ\text{C}$, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \quad 0 \leq t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

- (b) deduce the maximum temperature of the room during this day,

(1)

- (c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

$$\begin{aligned} \text{a/ } \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ R \sin(x+\alpha) &= R \sin x \cos \alpha + R \cos x \sin \alpha \\ &\quad \underline{\sin x + 2 \cos x} \end{aligned}$$

$$R \cos \alpha = 1$$

$$R \sin \alpha = 2$$

$$\frac{R \sin \alpha}{R \cos \alpha} = \frac{2}{1}$$

$$\tan \alpha = 2$$

$$\alpha = \underline{\underline{1.107}}$$

$$\begin{aligned} R &= \sqrt{1^2 + 2^2} \\ &= \sqrt{5} \end{aligned}$$

$$\underline{\underline{\sqrt{5} \sin(x + 1.107)}}$$

$$\text{b/ } \theta = 5 + \sqrt{5} \sin\left(\frac{\pi t}{12} - 3 + 1.107\right)$$

$$\text{Max temp when } \sin\left(\frac{\pi t}{12} - 3 + 1.107\right) = 1$$



Question 6 continued

$$\theta = 5 + \sqrt{5}^{\circ}\text{C} = 7.24^{\circ}\text{C}$$

$$\text{c/} \quad \sin\left(\frac{\pi t}{12} - 3 + 1.107\right) = 1$$

$$\sin \theta = 1 \quad \text{when} \quad \theta = \frac{\pi}{2}$$

$$\frac{\pi t}{12} - 3 + 1.107 = \frac{\pi}{2}$$

$$\frac{\pi t}{12} - 1.893 = \frac{\pi}{2}$$

$$\pi t - 22.716 = 6\pi$$

$$\pi t = 6\pi + 22.716$$

$$t = \frac{6\pi + 22.716}{\pi}$$

$$= 13.23 \text{ hrs}$$

$$= 13 \text{ hrs } 14 \text{ mins}$$

$$= \underline{\underline{13:14}}$$



Question 6 continued

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Question 6 continued

Lined area for writing answers.

(Total for Question 6 is 7 marks)



7.

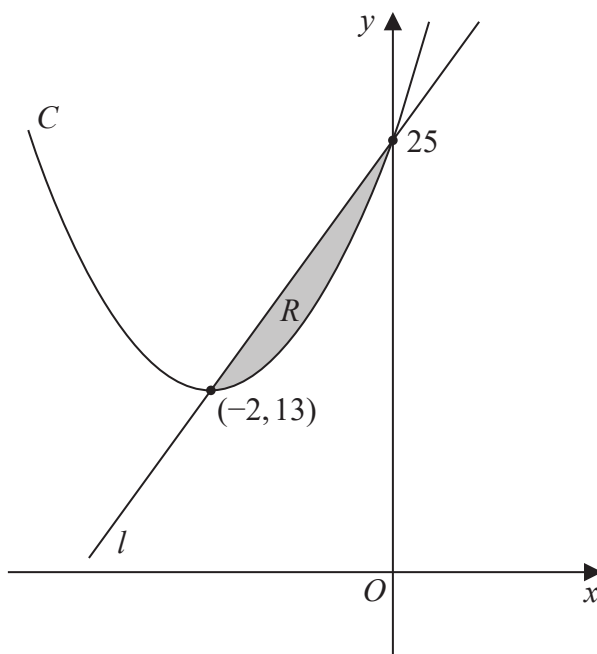


Figure 1

Figure 1 shows a sketch of a curve C with equation $y = f(x)$ and a straight line l .

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $f(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = f(x)$

use inequalities to define R .

(5)

$$f(x) = a(x+b)^2 + c$$

$$\text{min point} = (-2, 13)$$

$$\therefore f(x) = a(x+2)^2 + 13$$

$$f(x) \text{ passes through } (0, 25)$$

$$25 = a(0+2)^2 + 13$$

$$25 = 4a + 13$$

$$12 = 4a$$

$$a = 3$$

$$f(x) = 3(x+2)^2 + 13$$



Question 7 continued

l passes through $(-2, 13)$ and $(0, 25)$
 $x_1 \quad y_1 \quad x_2 \quad y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{25 - 13}{0 - (-2)}$$

$$= \frac{12}{2} = 6$$

$$m = 6 \quad c = 25$$

$$y = \underline{\underline{6x + 25}}$$

$$y = 3(x + 2)^2 + 13$$

$$3(x + 2)^2 + 13 \leq y \leq 6x + 25$$



Question 7 continued

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Question 7 continued

Lined area for writing the answer to Question 7.

(Total for Question 7 is 5 marks)



8. A new smartphone was released by a company.

The company monitored the total number of phones sold, n , at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t .

(You do not need to evaluate any unknown constants in your equation.)

(2)

$$\underline{\underline{n = Ae^{kt}}}$$



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Question 8 continued

Lined area for writing the answer to Question 8.

(Total for Question 8 is 2 marks)



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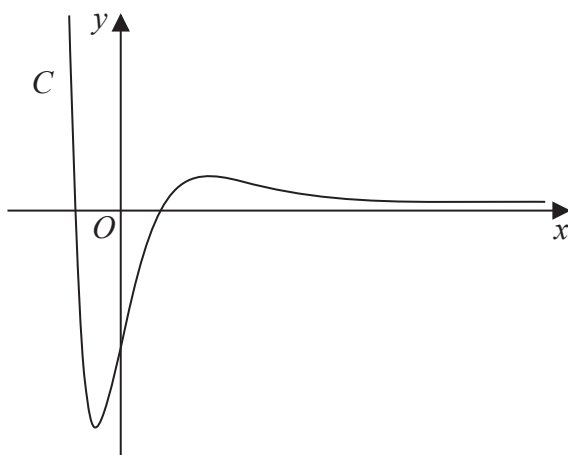


Figure 2

Figure 2 shows a sketch of the curve C with equation $y = f(x)$ where

$$f(x) = 4(x^2 - 2)e^{-2x} \quad x \in \mathbb{R}$$

- (a) Show that $f'(x) = 8(2 + x - x^2)e^{-2x}$ (3)
- (b) Hence find, in simplest form, the exact coordinates of the stationary points of C . (3)

The function g and the function h are defined by

$$g(x) = 2f(x) \quad x \in \mathbb{R}$$

$$h(x) = 2f(x) - 3 \quad \underline{x \geq 0}$$

- (c) Find (i) the range of g
(ii) the range of h (3)

$$a/ \quad f(x) = 4(x^2 - 2)e^{-2x}$$

$$u = 4x^2 - 8$$

$$v = e^{-2x}$$

$$\frac{du}{dx} = 8x$$

$$\frac{dv}{dx} = -2e^{-2x}$$

$$f'(x) = 8xe^{-2x} + (4x^2 - 8)(-2e^{-2x})$$

$$= 8xe^{-2x} - 8x^2e^{-2x} + 16e^{-2x}$$

$$= 8e^{-2x}(x - x^2 + 2)$$



Question 9 continued

$$= \underline{\underline{8(2+x-x^2)e^{-2x}}}$$

b/ stationary points where $f'(x)=0$

$$8(2+x-x^2)e^{-2x} = 0$$

$$8(1+x)(2-x)e^{-2x} = 0$$

$$x = -1 \quad x = 2$$

$$\begin{aligned} \text{when } x = -1 \quad y &= 4(1-1)^2 - 2) e^{-2(-1)} \\ &= -4e^2 \end{aligned}$$

$$\begin{aligned} \text{when } x = 2 \quad y &= 4(2)^2 - 2) e^{-2(2)} \\ &= 8e^{-4} \end{aligned}$$

$$\underline{\underline{(-1, -4e^2)}} \quad \text{and} \quad \underline{\underline{(2, 8e^{-4})}}$$

$$c/ i) f(x) \geq -4e^2$$

$$g(x) \geq -8e^2$$

$$ii) f(x) = 4(x^2 - 2)e^{-2x} \quad \text{crosses } y \text{ when } x = 0$$

$$y = 4(0^2 - 2)e^0$$

$$= -8$$

$$\begin{aligned} 2(-8) - 3 &\leq h(x) \leq 2(8e^{-4}) - 3 \\ -19 &\leq h(x) \leq 16e^{-4} - 3 \end{aligned}$$



Question 9 continued

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Question 9 continued

Lined area for writing the answer to Question 9.

(Total for Question 9 is 9 marks)



10. (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3 \, dx}{(x-1)(3+2\sqrt{x-1})} = \int_p^q \frac{6 \, du}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3 \, dx}{(x-1)(3+2\sqrt{x-1})} = \ln a$$

where a is a rational constant to be found.

(6)

$$\int_5^{10} \frac{3}{(x-1)(3+2\sqrt{x-1})} \, dx \quad x = u^2 + 1$$

$$\int_2^3 \frac{3}{(x-1)(3+2\sqrt{x-1})} \frac{dx}{du} \, du \quad \begin{array}{l} 10 = u^2 + 1 \\ u = 3 \\ 5 = u^2 + 1 \\ u = 2 \end{array}$$

$$\int_2^3 \frac{3}{u^2(3+2u)} 2u \, du \quad \begin{array}{l} \frac{dx}{du} = 2u \\ x-1 = u^2 \end{array}$$

$$\int_2^3 \frac{6u}{3u^2 + 2u^3} \, du$$

$$\int_2^3 \frac{6}{3u + 2u^2} \, du$$

$$\int_2^3 \frac{6}{u(3+2u)} \, du$$

$$\therefore \frac{6}{u(3+2u)} = \frac{A}{u} + \frac{B}{3+2u}$$

$$6 = A(3+2u) + Bu$$



Question 10 continued

when $u=0$ $6 = 3A$

$A = 2$

when $u = -\frac{3}{2}$ $6 = -\frac{3}{2}B$

$B = -4$

$$\int_2^3 \frac{2}{u} - \frac{4}{3+2u} du$$

$$\left[2 \ln|u| - \frac{4}{2} \ln|3+2u| \right]_2^3$$

$$(2 \ln 3 - 2 \ln 9) - (2 \ln 2 - 2 \ln 7)$$

$$2 \ln 3 - 2 \ln 3^2 - 2 \ln 2 + 2 \ln 7$$

$$2 \ln 3 - 4 \ln 3 - 2 \ln 2 + 2 \ln 7$$

$$-2 \ln 3 - 2 \ln 2 + 2 \ln 7$$

$$2 \ln 7 - 2 \ln 3 - 2 \ln 2$$

$$\ln 49 - \ln 9 - \ln 4$$

$$\underline{\underline{\ln\left(\frac{49}{36}\right)}}$$



Question 10 continued

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Question 10 continued

Lined area for writing the answer to Question 10.

(Total for Question 10 is 10 marks)



11.

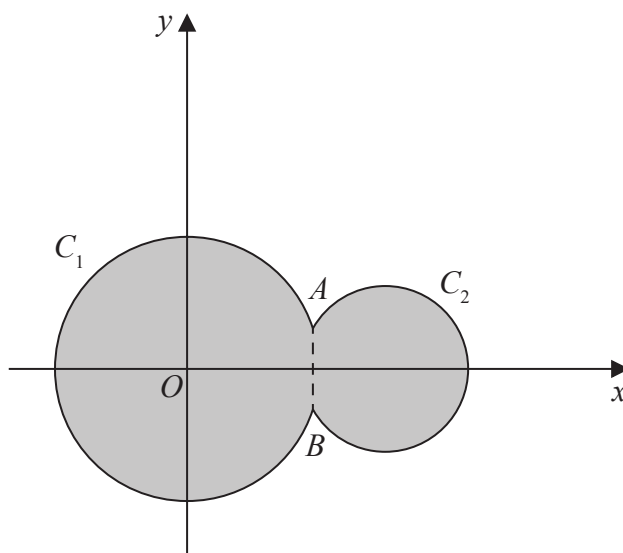


Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

$$\begin{aligned} a/ \quad x^2 + y^2 &= 100 & (x - 15)^2 + y^2 &= 40 \\ y^2 &= 100 - x^2 \end{aligned}$$

circles meet when:

$$(x - 15)^2 + 100 - x^2 = 40$$

$$(x - 15)(x - 15) + 100 - x^2 = 40$$

$$x^2 - 15x - 15x + 225 + 100 - x^2 = 40$$

$$325 - 30x = 40$$

$$285 = 30x$$

$$x = 9.5$$

when $x = 9.5$

$$(9.5)^2 + y^2 = 100$$

$$y^2 = \frac{39}{4}$$

$$y = \pm \sqrt{\frac{39}{4}}$$

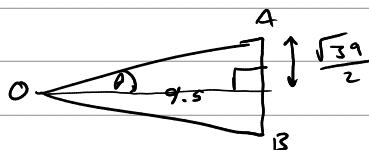


Question 11 continued

$$A : \left(9.5, \frac{\sqrt{39}}{2} \right)$$

$$B : \left(9.5, -\frac{\sqrt{39}}{2} \right)$$

$$O : (0, 0)$$



$$\tan \theta = \frac{\frac{\sqrt{39}}{2}}{9.5}$$

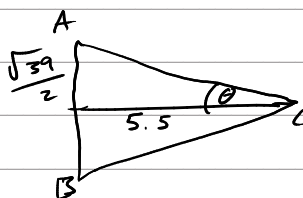
$$\theta = \tan^{-1} \left(\frac{\frac{\sqrt{39}}{2}}{9.5} \right)$$

$$= 0.31756 \dots$$

$$\angle AOB = 2\theta = \underline{\underline{0.635}}$$

b/ centre of C_2 $(15, 0)$

$$15 - 9.5 = 5.5$$



$$\tan \theta = \frac{\frac{\sqrt{39}}{2}}{5.5}$$

$$\theta = \tan^{-1} \left(\frac{\frac{\sqrt{39}}{2}}{5.5} \right)$$

$$= 0.516$$

$$\angle ACB = 2\theta = 1.0327 \text{ radians}$$

$$\text{Arc length} = r\theta$$

$$\text{For } C_1 = 10(2\pi - 0.635) = 56.5$$

$$\text{For } C_2 = \sqrt{40}(2\pi - 1.0327) = 33.2$$

$$\text{Total perimeter} = 56.5 + 33.2$$

$$= \underline{\underline{89.7}}$$



Question 11 continued

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Question 11 continued

Handwriting practice area with horizontal lines.

(Total for Question 11 is 8 marks)



12.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec} \theta - \sin \theta \equiv \cos \theta \cot \theta \quad \theta \neq (180n)^\circ \quad n \in \mathbb{Z} \quad (3)$$

(b) Hence, or otherwise, solve for $0 < x < 180^\circ$

$$\operatorname{cosec} x - \sin x = \cos x \cot(3x - 50^\circ) \quad (5)$$

$$a/ \quad \frac{1}{\sin \theta} - \sin \theta$$

$$\frac{1}{\sin \theta} - \frac{\sin^2 \theta}{\sin \theta}$$

$$\frac{(1 - \sin^2 \theta)}{\sin \theta}$$

$$\frac{\cos^2 \theta}{\sin \theta}$$

$$\frac{\cos \theta}{\sin \theta} \cos \theta$$

$$\cot \theta \cos \theta = \cos \theta \cot \theta$$

b/

$$\cos x \cot x = \cos x \cot(3x - 50)$$

$$0 = \cos x \cot(3x - 50) - \cos x \cot x$$

$$0 = \cos x (\cot(3x - 50) - \cot x)$$

$$\cos x = 0$$

$$\underline{\underline{x = 90}}$$

$$\text{or } \cot(3x - 50) - \cot x = 0$$

$$\cot(3x - 50) = \cot x$$

$$3x - 50 = x$$

$$2x = 50$$

$$\underline{\underline{x = 25}}$$



Question 12 continued

cot repeats every 180°

OR $3x - 50 = x + 180$

$2x = 230$

$x = \underline{115^\circ}$

$x = \underline{25^\circ, 90^\circ, 115^\circ}$



Question 12 continued

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Question 12 continued

Lined area for writing the answer to Question 12.

(Total for Question 12 is 8 marks)



13. A sequence of numbers $a_1, a_2, a_3, \dots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \quad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0 \quad (3)$$

(b) For this sequence explain why $k \neq 1$ (1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r \quad (3)$$

$$a/ \quad a_{n+1} = \frac{k(a_n + 2)}{a_n}$$

$$a_2 = \frac{k(2 + 2)}{2}$$

$$= 2k$$

$$a_3 = \frac{k(2k + 2)}{2k}$$

$$= \frac{2k + 2}{2}$$

$$= k + 1$$

$$a_4 = \frac{k(k + 1 + 2)}{k + 1}$$

$$a_4 = a_1 \quad \frac{k(k + 3)}{k + 1} = 2$$



Question 13 continued

$$k(k+3) = 2(k+1)$$

$$k^2 + 3k = 2k + 2$$

$$k^2 + k - 2 = 0$$

b/ If $k=1$ every term will be 2. (It will not be periodic of order 3)

c/ $k^2 + k - 2 = 0$

$$(k-1)(k+2) = 0$$

$$k=1 \quad k=-2$$

$$\times \quad \underline{\underline{\quad}}$$

$$a_1 = 2$$

$$a_2 = 2k = -4$$

$$a_3 = k+1 = -1$$

$$a_1 + a_2 + a_3 = 2 - 4 - 1 = -3$$

$$\sum_{r=1}^{78} a_r = 26(-3) = -78 \quad \left[\frac{78}{3} = 26 \right]$$

$$\sum_{r=1}^{80} a_r = -78 + 2 - 4 = \underline{\underline{-80}}$$



Question 13 continued

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DO NOT WRITE IN THIS AREA

Question 13 continued

Lined area for writing the answer to Question 13.

(Total for Question 13 is 7 marks)



14. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume V cm³

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{dr}{dt} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking r and t .

(5)

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2)

a/ $\text{Volume of sphere} = \frac{4}{3} \pi r^3$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = -c$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$

$$= \frac{1}{4\pi r^2} \times -c$$

$$= \frac{-c}{4\pi r^2}$$

$$= \frac{-c}{4\pi} \times \frac{1}{r^2}$$

$$\text{Let } \frac{c}{4\pi} = k$$

$$= -\frac{k}{r^2}$$



Question 14 continued

$$b/ \quad \frac{dr}{dt} = -\frac{k}{r^2} \quad \begin{array}{l} t=0 \quad r=40 \\ t=5 \quad r=20 \end{array}$$

$$\int r^2 dr = \int -k dt$$

$$\frac{1}{3} r^3 = -kt + C \quad \text{when } t=0 \quad r=40$$

$$\frac{1}{3} (40)^3 = -k(0) + C$$

$$\frac{64000}{3} = C$$

$$\frac{1}{3} r^3 = -kt + \frac{64000}{3} \quad \text{when } t=5 \quad r=20$$

$$\frac{1}{3} (20)^3 = -k(5) + \frac{64000}{3}$$

$$\frac{8000}{3} = -5k + \frac{64000}{3}$$

$$5k = \frac{56000}{3}$$

$$k = \frac{11200}{3}$$

$$\frac{1}{3} r^3 = -\frac{11200}{3} t + \frac{64000}{3}$$

$$r^3 = -11200t + 64000$$

$$c/ \quad \text{when } r=0$$

$$0 = -11200t + 64000$$

$$11200t = 64000$$

$$t = \frac{40}{7}$$

$$0 \leq t \leq \frac{40}{7}$$



Question 14 continued

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Question 14 continued

Lined area for writing the answer to Question 14.

(Total for Question 14 is 10 marks)



15. The curve C has equation

$$x^2 \tan y = 9 \quad 0 < y < \frac{\pi}{2}$$

(a) Show that

$$\tan y = \frac{9}{x^2}$$

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

$$a/ \quad x^2 \tan y = 9$$

$$u = x^2 \quad v = \tan y$$

$$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = \sec^2 y \frac{dy}{dx}$$

$$2x \tan y + x^2 \sec^2 y \frac{dy}{dx} = 0$$

$$x^2 \sec^2 y \frac{dy}{dx} = -2x \tan y$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\frac{dy}{dx} = -\frac{2x \tan y}{x^2 \sec^2 y}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\tan x = \frac{9}{x^2}$$

$$\tan^2 x = \frac{81}{x^4}$$

$$= \frac{-2x \left(\frac{9}{x^2} \right)}{x^2 (1 + \tan^2 x)}$$

$$= \frac{\left(\frac{-18x}{x^2} \right)}{x^2 \left(1 + \frac{81}{x^4} \right)}$$

$$= \frac{\left(\frac{-18x}{x^2} \right)}{\left(x^2 + \frac{81}{x^2} \right)} \quad \times x^2 \quad \times x^2$$

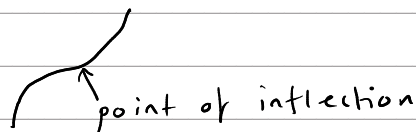
$$= \frac{-18x}{x^4 + 81}$$



Question 15 continued

b/ A point of inflection is where the concavity changes.

Where the gradient goes from increasing to decreasing or decreasing to increasing



$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

$$u = -18x \quad v = x^4 + 81$$

$$\frac{du}{dx} = -18 \quad \frac{dv}{dx} = 4x^3$$

$$\frac{d^2y}{dx^2} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$= \frac{-18(x^4 + 81) - (-18x)(4x^3)}{(x^4 + 81)^2}$$

$$= \frac{-18x^4 - 1458 + 72x^4}{(x^4 + 81)^2}$$

$$= \frac{54x^4 - 1458}{(x^4 + 81)^2}$$

$$\text{when } x = \sqrt[4]{27} \quad \frac{d^2y}{dx^2} = 0$$

$$\text{when } x < \sqrt[4]{27} \quad \frac{d^2y}{dx^2} < 0$$

$$\text{when } x > \sqrt[4]{27} \quad \frac{d^2y}{dx^2} > 0$$

$$\left[\begin{array}{l} \frac{54(27) - 1458}{(27 + 81)^2} = 0 \\ \frac{54(26) - 1458}{(26 + 81)^2} < 0 \\ \frac{54(28) - 1458}{(28 + 81)^2} > 0 \end{array} \right]$$

$\therefore$ when $x = \sqrt[4]{27}$ there is a point of inflection



Question 15 continued

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Question 15 continued

Lined area for writing the answer to Question 15.

(Total for Question 15 is 7 marks)



16. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

$$4p^2 - q^2 = 25$$

$$(2p + q)(2p - q) = 25$$

$$25 = 5 \times 5 \quad \text{or} \quad 1 \times 25$$

$$\text{Either } 2p + q = 5 \quad \text{and} \quad 2p - q = 5$$

$$\begin{array}{r} 2p + q = 5 \\ + \quad + \quad + \\ 2p - q = 5 \end{array}$$

$$4p = 10$$

$$p = \frac{10}{4} = \frac{5}{2} \quad (\text{not a +ve integer})$$

$$\text{Or } 2p + q = 25 \quad \text{and} \quad 2p - q = 1$$

$$\begin{array}{r} 2p + q = 25 \\ + \quad + \quad + \\ 2p - q = 1 \end{array}$$

$$4p = 26$$

$$p = \frac{26}{4} = \frac{13}{2} \quad (\text{not a +ve integer})$$

$\therefore p$ cannot be a positive integer



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Question 16 continued

Lined area for writing the answer to Question 16.



Question 16 continued

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(Total for Question 16 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS

