A level Mathematics Practice Paper – Differential equations – Mark scheme

Question	Scheme	Marks
1	$\int y\,\mathrm{d}y = \int \frac{3}{\cos^2 x}\,\mathrm{d}x$ Can be implied. Ignore integral signs	B1
	$= \int 3\sec^2 x\,\mathrm{d}x$	
	$\frac{1}{2}y^2 = 3\tan x \quad (+C)$	M1 A1
	$y = 2,\ x = \frac{\pi}{4}$	
	$\frac{1}{2}2^2 = 3\tan\frac{\pi}{4} + C$	M1
	Leading to	
	$C = -1$	
	$\frac{1}{2}y^2 = 3\tan x - 1$ or equivalent	A1
		(5 marks)

Question	Scheme		Marks
2(i)	$\int xe^{4x}\,dx = \frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	$\pm\alpha xe^{4x} - \int \beta e^{4x}\{dx\},\quad \alpha \neq 0,\ \beta > 0$	M1
		$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\,\{dx\}$	A1
	$= \frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}\ \{+c\}$	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$	A1
			(3)
2(ii)	$\int \frac{8}{(2x-1)^3}\,dx = \frac{8(2x-1)^{-2}}{(2)(-2)}\ \{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1
	$\{= -2(2x-1)^{-2}\ \{+c\}\}$	*{Ignore subsequent working}*.	
			(2)
2(iii)	$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y \qquad y = \frac{\pi}{6}$ at $x = 0$		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,dy = \int e^x\,dx$ **or** $\int \sin 2y \sin y\,dy = \int e^x\,dx$		B1 oe
	$\int 2\sin y\cos y \sin y\,dy = \int e^x\,dx$	Applying $\frac{1}{\operatorname{cosec}2y}$ or $\sin 2y \to 2\sin y\cos y$	M1
	$\frac{2}{3}\sin^3 y = e^x\ \{+c\}$	Integrates to give $\pm\mu\sin^3 y$	M1
		$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$	A1
		$e^x \to e^x$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = e^0 + c$ **or** $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$	A1
			(7)
			(12 marks)

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Question	Scheme	Marks
3(a)	$\int \frac{1}{x}\,\mathrm{d}x = \int -\frac{5}{2}\,\mathrm{d}t$	B1
	$\ln x = -\frac{5}{2}t + c$	M1 A1
	$\{t = 0,\ x = 60 \Rightarrow\}\ \ln 60 = c$ $\ln x = -\frac{5}{2}t + \ln 60 \Rightarrow \underline{x = 60\mathrm{e}^{-\frac{5}{2}t}}$ or $\underline{x = \frac{60}{\mathrm{e}^{\frac{5}{2}t}}}$	A1 **cso**
		(4)
3(b)	$20 = 60\mathrm{e}^{-\frac{5}{2}t}$ or $\ln 20 = -\frac{5}{2}t + \ln 60$	M1
	$t = -\frac{2}{5}\ln\left(\frac{20}{60}\right)$ $\{= 0.4394449\ldots \text{ (days)}\}$	dM1
	$\Rightarrow t = 632.8006\ldots = 633$ (to the nearest minute)	A1 **cso**
		(3)
		(7 marks)
4(a)	$\frac{\mathrm{d}h}{\mathrm{d}t} = k\sqrt{(h-9)},\ 9 < h \leq 200;\quad h = 130,\ \frac{\mathrm{d}h}{\mathrm{d}t} = -1.1$	
	$-1.1 = k\sqrt{(130-9)} \Rightarrow k = \ldots$	M1
	so, $k = -\frac{1}{10}$ or -0.1	A1
		(2)
4(b)	$\int \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int k\,\mathrm{d}t$	B1
	$\int (h-9)^{-\frac{1}{2}}\,\mathrm{d}h = \int k\,\mathrm{d}t$	
	$\frac{(h-9)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} = kt\,(+c)$	M1 A1
	$\{t = 0,\ h = 200 \Rightarrow\}\ 2\sqrt{(200-9)} = k(0) + c$	M1
	$\Rightarrow c = 2\sqrt{191} \Rightarrow 2(h-9)^{\frac{1}{2}} = -0.1t + 2\sqrt{191}$ $\{h = 50 \Rightarrow\}\ 2\sqrt{(50-9)} = -0.1t + 2\sqrt{191}$ $t = \ldots$	dM1
	$t = 20\sqrt{191} - 20\sqrt{41}$ or $t = 148.3430145\ldots = 148$ (minutes) (nearest minute)	A1 cso
		(6)
		(8 marks)

Question	Scheme	Marks
5(a)	$\dfrac{2}{P(P-2)} = \dfrac{A}{P} + \dfrac{B}{(P-2)}$	
	$2 \equiv A(P-2) + BP$	M1
	$A = -1,\ B = 1$	A1
	giving $\dfrac{1}{(P-2)} - \dfrac{1}{P}$	A1
		(3)
5(b)	$\dfrac{\mathrm{d}P}{\mathrm{d}t} = \dfrac{1}{2}P(P-2)\cos 2t$	
	$\displaystyle\int \frac{2}{P(P-2)}\,\mathrm{d}P = \int \cos 2t\,\mathrm{d}t$	B1 oe
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t\ \ (+c)$	M1
		A1
	$\{t = 0,\ P = 3 \Rightarrow\}\ \ \ln 1 - \ln 3 = 0 + c\ \ \ \left\{\Rightarrow\ \ c = -\ln 3 \text{ or } \ln(\tfrac{1}{3})\right\}$	M1
	$\ln(P-2) - \ln P = \dfrac{1}{2}\sin 2t - \ln 3$	
	$\ln\left(\dfrac{3(P-2)}{P}\right) = \dfrac{1}{2}\sin 2t$	
	$\dfrac{3(P-2)}{P} = \mathrm{e}^{\frac{1}{2}\sin 2t}$	M1
	$3(P-2) = P\mathrm{e}^{\frac{1}{2}\sin 2t} \Rightarrow 3P - 6 = P\mathrm{e}^{\frac{1}{2}\sin 2t}$ gives $\ 3P - P\mathrm{e}^{\frac{1}{2}\sin 2t} = 6 \Rightarrow P(3 - \mathrm{e}^{\frac{1}{2}\sin 2t}) = 6$	dM1
	$P = \dfrac{6}{(3 - \mathrm{e}^{\frac{1}{2}\sin 2t})}$ *	A1 * cso
		(7)
5(c)	$\{\text{population} = 4000 \Rightarrow\}\ P = 4$	M1
	$\dfrac{1}{2}\sin 2t = \ln\left(\dfrac{3(4-2)}{4}\right)\ \left\{= \ln\left(\dfrac{3}{2}\right)\right\}$	M1
	$t = 0.4728700467...$	A1
		(3)
		(13 marks)

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Question	Scheme		Marks
6(a)	$\dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{(kt-1)(5000-N)}{t}, \quad t>0,\ 0<N<5000$		
	$\displaystyle\int \frac{1}{5000-N}\,\mathrm{d}N = \int \frac{(kt-1)}{t}\,\mathrm{d}t \quad \left\{\text{or } = \int\left(k-\frac{1}{t}\right)\mathrm{d}t\right\}$		B1
	$-\ln(5000-N) = kt - \ln t; +c$		M1 A1 A1
	then eg either...	***or...***	
	$-kt + c = \ln(5000-N) - \ln t$	$kt + c = \ln t - \ln(5000-N)$	
	$-kt + c = \ln\left(\dfrac{5000-N}{t}\right)$	$kt + c = \ln\left(\dfrac{t}{5000-N}\right)$	
	$\mathrm{e}^{-kt+c} = \dfrac{5000-N}{t}$	$\mathrm{e}^{kt+c} = \dfrac{t}{5000-N}$	
	leading to $N = 5000 - At\mathrm{e}^{-kt}$ with **no incorrect working/statements**.		A1 * **cso**
			(5)
6(b)	$\{t=1,\ N=1200 \Rightarrow\}\quad 1200 = 5000 - A\mathrm{e}^{-k}$ $\{t=2,\ N=1800 \Rightarrow\}\quad 1800 = 5000 - 2A\mathrm{e}^{-2k}$		B1
	So $A\mathrm{e}^{-k} = 3800$ and $2A\mathrm{e}^{-2k} = 3200$ or $A\mathrm{e}^{-2k} = 1600$		
	Eg. $\dfrac{\mathrm{e}^{-k}}{2\mathrm{e}^{-2k}} = \dfrac{3800}{3200}$ or $\dfrac{2\mathrm{e}^{-2k}}{\mathrm{e}^{-k}} = \dfrac{3200}{3800}$		M1
	So $\dfrac{1}{2}\mathrm{e}^{k} = \dfrac{3800}{3200}$ or $2\mathrm{e}^{-k} = \dfrac{3200}{3800}$		
	$k = \ln\left(\dfrac{7600}{3200}\right)$ or equivalent $\left\{\text{eg } k = \ln\left(\dfrac{19}{8}\right)\right\}$		A1
	$\left\{A = 3800(\mathrm{e}^{k}) = 3800\left(\dfrac{19}{8}\right) \Rightarrow\right\}\ A = 9025$		A1
			(4)
6(c)	$\left\{t=5,\ N = 5000 - 9025(5)\mathrm{e}^{-5\ln\left(\frac{19}{8}\right)}\right\}$		
	$N = 4402.828401... = 4400$ (fish) (nearest 100)		B1
			(1)
			(10 marks)

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Question	Scheme	Marks
7(a)	$\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}x = \dfrac{(4y+3)^{\frac{1}{2}}}{(4)(\frac{1}{2})} \quad (+C)$	M1 A1
		(2)
	$\left(= \frac{1}{2}(4y+3)^{\frac{1}{2}} + C\right)$	
7(b)	$\displaystyle\int \frac{1}{\sqrt{(4y+3)}}\,\mathrm{d}y = \int \frac{1}{x^2}\,\mathrm{d}x$	B1
	$\int (4y+3)^{-\frac{1}{2}}\,\mathrm{d}y = \int x^{-2}\,\mathrm{d}x$	
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} \quad (+C)$	M1
	Using $(-2, 1.5)$ $\quad \dfrac{1}{2}(4\times 1.5+3)^{\frac{1}{2}} = -\dfrac{1}{-2} + C$	M1
	leading to $\quad C = 1$	A1
	$\dfrac{1}{2}(4y+3)^{\frac{1}{2}} = -\dfrac{1}{x} + 1$	
	$(4y+3)^{\frac{1}{2}} = 2 - \dfrac{2}{x}$	M1
	$y = \dfrac{1}{4}\left(2 - \dfrac{2}{x}\right)^2 - \dfrac{3}{4}$ $\quad$ or equivalent	A1
		(6)
		(8 marks)

Question	Scheme		Marks
8(a)	$\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \lambda(120-\theta), \quad \theta \leqslant 100$		
	$\displaystyle\int \frac{1}{120-\theta}\,\mathrm{d}\theta = \int \lambda\,\mathrm{d}t$ or $\displaystyle\int \frac{1}{\lambda(120-\theta)}\,\mathrm{d}\theta = \int \mathrm{d}t$		B1
	$-\ln(120-\theta);\ = \lambda t + c$ or $-\dfrac{1}{\lambda}\ln(120-\theta);\ = t + c$		M1 A1 M1 A1
	$\{t = 0,\ \theta = 20 \Rightarrow\}\ -\ln(120-20) = \lambda(0) + c$		M1
	$c = -\ln 100 \Rightarrow -\ln(120-\theta) = \lambda t - \ln 100$		
	then either...	***or...***	
	$-\lambda t = \ln(120-\theta) - \ln 100$	$\lambda t = \ln 100 - \ln(120-\theta)$	
	$-\lambda t = \ln\left(\dfrac{120-\theta}{100}\right)$	$\lambda t = \ln\left(\dfrac{100}{120-\theta}\right)$	
	$\mathrm{e}^{-\lambda t} = \dfrac{120-\theta}{100}$	$\mathrm{e}^{\lambda t} = \dfrac{100}{120-\theta}$	dddM1
	$100\mathrm{e}^{-\lambda t} = 120-\theta$	$(120-\theta)\mathrm{e}^{\lambda t} = 100$ $\Rightarrow 120-\theta = 100\mathrm{e}^{-\lambda t}$	A1 *
	leading to $\theta = 120 - 100\mathrm{e}^{-\lambda t}$		
			(8)
8(b)	$\{\lambda = 0.01,\ \theta = 100 \Rightarrow\}\quad 100 = 120 - 100\mathrm{e}^{-0.01t}$		M1
	$\Rightarrow$ $100\mathrm{e}^{-0.01t} = 120-100 \Rightarrow -0.01t = \ln\left(\dfrac{120-100}{100}\right)$ $t = \dfrac{1}{-0.01}\ln\left(\dfrac{120-100}{100}\right)$	Uses correct order of operations by moving from $100 = 120 - 100\mathrm{e}^{-0.01t}$ to give $t = ...$ and $t = A\ln B$, where $B > 0$	dM1
	$\left\{t = \dfrac{1}{-0.01}\ln\left(\dfrac{1}{5}\right) = 100\ln 5\right\}$		
	$t = 160.94379... =$ 161 (s) (nearest second)	awrt 161	A1
			(3)
			(11 marks)

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Question	Scheme		Marks
9(a)	$\left\{\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{(3-\theta)}{125}\right\} \Rightarrow \displaystyle\int \frac{1}{3-\theta}\,\mathrm{d}\theta = \int \frac{1}{125}\,\mathrm{d}t$ or $\displaystyle\int \frac{125}{3-\theta}\,\mathrm{d}\theta = \int \mathrm{d}t$		B1
	$-\ln(\theta - 3) = \dfrac{1}{125}t \ \{+ c\}$ or $-\ln(3 - \theta) = \dfrac{1}{125}t \ \{+ c\}$	See notes.	M1 A1
	$\ln(\theta - 3) = -\dfrac{1}{125}t + c$	***Correct*** completion to $\theta = A\mathrm{e}^{-0.008t} + 3$.	
	$\theta - 3 = \mathrm{e}^{-\frac{1}{125}t + c}$ or $\mathrm{e}^{-\frac{1}{125}t}\mathrm{e}^{c}$		A1
	$\theta = A\mathrm{e}^{-0.008t} + 3$ *		
			(4)
9(b)	$\{t = 0\ , \theta = 16 \Rightarrow\}$ $16 = A\mathrm{e}^{-0.008(0)} + 3\,;\ \Rightarrow \underline{A = 13}$	***See notes.***	M1 A1
	$10 = 13\mathrm{e}^{-0.008t} + 3$	Substitutes $\theta = 10$ into an equation of the form $\theta = A\mathrm{e}^{-0.008t} + 3$, or equivalent.	M1
	$\mathrm{e}^{-0.008t} = \dfrac{7}{13} \quad \Rightarrow \quad -0.008t = \ln\left(\dfrac{7}{13}\right)$	Correct algebra to $-0.008t = \ln k$, where k is a positive value.	M1
	$\left\{t = \dfrac{\ln\left(\frac{7}{13}\right)}{(-0.008)}\right\} = 77.3799... = 77$ (nearest minute)	awrt 77	A1
			(5)
			(9 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	C4 2012	4	8.7	First order differential equations	1.1b, 2.1
2	C4 June 2014	6	8.2, 8.5, 8.7, 5.6	Integration, First order differential equations	1.1b, 2.1, 3.1a
3	C4 2016	4	8.8, 6.3, 6.5	First order differential equations	1.1b, 2.1, 3.1a, 3.4
4	C4 2017	7	8.7	First order differential equations	1.1b, 2.1, 3.1a
5	C4 2015	7	2.10, 8.7, 8.8,	First order differential equations	1.1b, 2.1, 3.1a, 3.2a, 3.4
6	C4 June 2014R	7	8.7, 8.8, 6.3	Differential equations	1.1b, 3.1a, 3.2a, 3.4
7	C4 2011	8	8.7	First order differential equations	1.1b, 2.1
8	C4 2013	6	8.7, 8.8	Solution of a differential equation	1.1b, 2.1, 3.1a, 3.4
9	C4 Jan 2013	8	8.7, 8.8, 6.3	First order differential equations	1.1b, 2.1, 3.1a, 3.2a, 3.4