AS and A level Further Mathematics Practice Paper – Complex numbers (part 2) – Mark scheme

Question	Scheme	Marks
1(a)	$z = \dfrac{4(1-\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})}$	M1
	$z = 2(1-\mathrm{i})$ or 2 – 2i or exact equivalent.	A1
		(2)
1(b)	$z^2 = (2-2\mathrm{i})(2-2\mathrm{i}) = 4-8\mathrm{i}+4\mathrm{i}^2$	M1
	$= -8\mathrm{i}$	A1 cao
		(2)
1(c)	If z is a root so is z^* So $(x-2+2\mathrm{i})(x-2-2\mathrm{i})$ (or $x^2-2\mathrm{Re}(z).x+\|z\|^2$)	M1
	So $(x-2+2\mathrm{i})(x-2-2\mathrm{i})=0$ (or $x^2-2\mathrm{Re}(z).x+\|z\|^2=0$) and so $p = q =$	M1
	Equation is $x^2-4x+8(=0)$ or $p=-4$ and $q=8$	A1
		(3)
		(7 marks)

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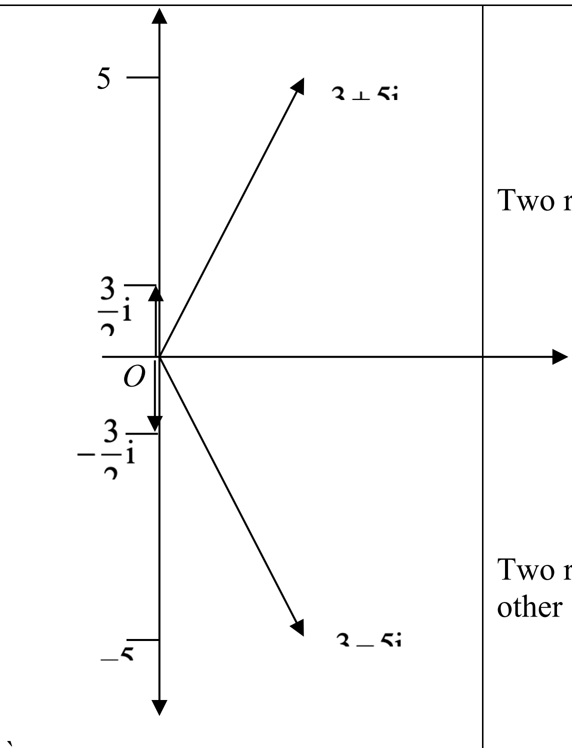
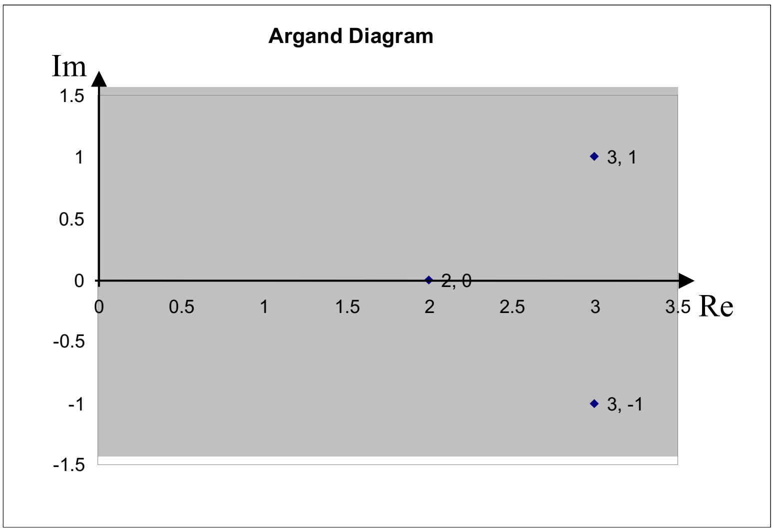
Question	Scheme	Marks
2(a)	$4x^2+9=0 \quad \Rightarrow \quad x=k\text{i}, \quad x=\pm\frac{3}{2}\text{i}$ or equivalent	M1A1
	Solving 3-term quadratic by formula or completion of the square $x=\frac{6\pm\sqrt{36-136}}{2}$ or $(x-3)^2-9+34=0$	M1
	$=3+5\text{i}$ and $3-5\text{i}$	A1 A1ft **(5)**
2(b)		
	 Two roots on imaginary axis	B1ft
	Two roots – one the conjugate of the other	B1ft
	Accept points or vectors	**(2)**
		(7 marks)

Question	Scheme	Marks
3(a)	$\left(z_2\right) = 3 - i$	B1
	$(z-(3+i))(z-(3-i)) = z^2 - 6z + 10$ Attempt to expand $(z-(3+i))(z-(3-i))$ or any valid method to establish the quadratic factor e.g. $z = 3 \pm i \Rightarrow z - 3 = \pm i \Rightarrow z^2 - 6z + 9 = -1$ $z = 3 \pm \sqrt{-1} = \dfrac{6 \pm \sqrt{-4}}{2} \Rightarrow b = -6, c = 10$ Sum of roots 6, product of roots 10 $\therefore z^2 - 6z + 10$	M1
	$(z^2 - 6z + 10)(z - 2) = 0$ Attempt at linear factor with their *cd* in $(z^2 + az + c)(z + d) = \pm 20$ Or $(z^2 - 6z + 10)(z + a) \Rightarrow 10a = -20$ Or attempts f(2)	M1
	$\left(z_3\right) = 2$	A1
	Showing that *f*(2) = 0 is equivalent to scoring both M's so it is possible to gain all 4 marks quite easily e.g. $z_2 = 3 - i$ **B1, shows *f*(2) = 0 M2,** $z_3 = 2$ **A1. Answers only can score 4/4**	**(4)**
3(b)		B1 B1
	First B1 for plotting (3, 1) and (3, -1) correctly with an indication of scale or labelled with coordinates (allow points/lines/crosses/vectors etc.) Allow *i*/-*i* for 1/-1 marked on imaginary axis. Second B1 for plotting (2, 0) correctly relative to the conjugate pair with an indication of scale or labelled with coordinates or just 2	**(2)**
		(6 marks)

Question	Scheme		Marks
4(a)	$x^3 + ax^2 + bx - 52 = 0, \quad a, b \in \mathbb{R}$		
	$-2\mathrm{i} - 3$		B1
			(1)
4(b)	$\left(x-(2\mathrm{i}-3)\right)\left(x-\text{"}(-2\mathrm{i}-3)\text{"}\right); = x^2+6x+13$ or		
	$x=-3\pm 2\mathrm{i} \Rightarrow (x+3)^2=-4; \; = x^2+6x+13 (=0)$		M1A1
	$\left(x-4\right)\left(x-(2\mathrm{i}-3)\right); = x^2-(1+2\mathrm{i})x+4(2\mathrm{i}-3)$		
	$\left(x-4\right)\left(x-\text{"}(-2\mathrm{i}-3)\text{"}\right); = x^2-(1-2\mathrm{i})x+4(-2\mathrm{i}-3)$		
	$\left(x-4\right)\left(x^2+6x+13\right) \; \left\{= x^3+ax^2+bx-52\right\}$		M1
	$a=2,\; b=-11$ or $x^3+2x^2-11x-52$		A1A1
			(5)
			(6 marks)
5	$z + 3\mathrm{i}z^* = -1 + 13\mathrm{i}$		
	$\left(x+\mathrm{i}y\right) + 3\mathrm{i}\left(x-\mathrm{i}y\right)$	$z^* = x - \mathrm{i}y$	B1
		Substituting $z = x + \mathrm{i}y$ and their z^* into $z + 3\mathrm{i}z^*$	M1
	$x + \mathrm{i}y + 3\mathrm{i}x + 3y = -1 + 13\mathrm{i}$	Correct equation in x and y with $\mathrm{i}^2 = -1$. Can be implied.	A1
	$\left(x+3y\right) + \mathrm{i}\left(y+3x\right) = -1 + 13\mathrm{i}$		
	Re part: $\quad x + 3y = -1$ Im part: $\quad y + 3x = 13$	An attempt to equate real **and** imaginary parts.	M1
		Correct equations.	A1
	$3x + 9y = -3$ $3x + y = 13$		
	$8y = -16 \Rightarrow y = -2$	Attempt to solve simultaneous equations to find one of x or y. **At least one of the equations must contain both *x* and *y* terms.**	M1
	$x + 3y = -1 \;\Rightarrow\; x - 6 = -1 \;\Rightarrow\; x = 5$	Both $x = 5$ and $y = -2$.	A1
	$\left\{z = 5 - 2\mathrm{i}\right\}$		**(7)**
			(7 marks)

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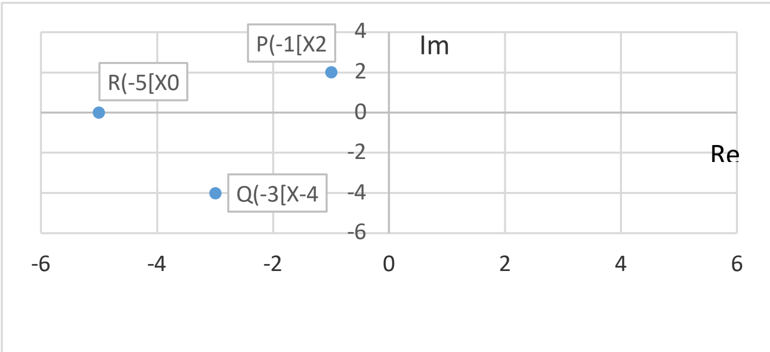
Question	Scheme	Marks
6(a)	$z^2 = (a+2\mathrm{i})(a+2\mathrm{i}) = (a^2-4)+4\mathrm{i}a$ So $z^2+2z = (a^2-4+2a)+\mathrm{i}(4a+4)$ or $x=(a^2+2a-4)$ and $y=4a+4$	M1M1 A1A1 **(4)**
6(b)	and so $4a+4=0 \rightarrow a=-1$	B1 **(1)**
6(c)	$\lvert z\rvert = \sqrt{5}$ or awrt 2.24 $\arctan,(-2) = 2.03$	B1 M1A1 **(3)**
6(d)		M1A1 B1ft **(3)**
6(e)	OP and QR are parallel , and QR is twice the length of OP **Or** Enlargement with Scale Factor 2 (centre O), followed by translation $\begin{pmatrix}-3\\-4\end{pmatrix}$	B1B1 **(2)**
		(13 marks)

AS and A level Further Mathematics Practice Paper – Complex numbers (part 2) – Mark scheme

Question	Scheme	Marks
7(a)	$z_1 = 2+3\text{i}, \quad z_2 = 3+2\text{i}$	
	$z_1+z_2 = 5+5\text{i} \Rightarrow \lvert z_1+z_2 \rvert = \sqrt{5^2+5^2}$	M1
	$\sqrt{50}\ (=5\sqrt{2})$	A1 cao
		(2)
7(b)	$\dfrac{z_1 z_3}{z_2} = \dfrac{(2+3\text{i})(a+b\text{i})}{3+2\text{i}}$	
	$= \dfrac{(2+3\text{i})(a+b\text{i})(3-2i)}{(3+2\text{i})(3-2\text{i})}$	M1
	$(3+2\text{i})(3-2\text{i}) = 13$	B1
	$\dfrac{z_1 z_3}{z_2} = \dfrac{(12a-5b)+(5a+12b)\text{i}}{13}$	dM1A1
		(4)
7(c)	$12a-5b=17$ $5a+12b=-7$	M1
	$\begin{matrix} 60a-25b=85 \\ 60a+144b=-84 \end{matrix} \Rightarrow b=-1$	dM1
	$a = 1$, $b = -1$	A1
		(3)
7(d)	$\arg(w) = -\tan^{-1}\left(\dfrac{7}{17}\right)$	M1
	=awrt -0.391 or awrt 5.89	A1
		(2)
		(11 marks)

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Question	Scheme	Marks
8(a)	$\arg z = -\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$	M1
	$= -0.7137243789.. = -0.71$ (2 dp)	A1
		(2)
8(b)	$z^2 = \left(2 - \mathrm{i}\sqrt{3}\right)\left(2 - \mathrm{i}\sqrt{3}\right)$	M1
	$= 4 - 2\mathrm{i}\sqrt{3} - 2\mathrm{i}\sqrt{3} + 3\mathrm{i}^2$	
	$= 2 - \mathrm{i}\sqrt{3} + \left(4 - 4\mathrm{i}\sqrt{3} - 3\right)$	
	$= 2 - \mathrm{i}\sqrt{3} + \left(1 - 4\mathrm{i}\sqrt{3}\right)$	M1A1
	$= 3 - 5\mathrm{i}\sqrt{3}$ (Note: $a = 3, b = -5$.)	
		(3)
8(c)	$\dfrac{z+7}{z-1} = \dfrac{2 - \mathrm{i}\sqrt{3} + 7}{2 - \mathrm{i}\sqrt{3} - 1}$	M1
	$= \dfrac{\left(9 - \mathrm{i}\sqrt{3}\right)}{\left(1 - \mathrm{i}\sqrt{3}\right)} \times \dfrac{\left(1 + \mathrm{i}\sqrt{3}\right)}{\left(1 + \mathrm{i}\sqrt{3}\right)}$	**d**M1
	$= \dfrac{9 + 9\mathrm{i}\sqrt{3} - \mathrm{i}\sqrt{3} + 3}{1 + 3}$	
	$= \dfrac{12 + 8\mathrm{i}\sqrt{3}}{4}$	M1
	$= 3 + 2\mathrm{i}\sqrt{3}$ (Note: $c = 3, d = 2$.)	A1
		(4)
8(d)	$w = \lambda - 3\mathrm{i}$, and $\arg(4 - 5\mathrm{i} + 3w) = -\dfrac{\pi}{2}$	
	$\left(4 - 5i + 3w = 4 + 3\lambda - 14\mathrm{i}\right)$	
	So real part of $(4 - 5\mathrm{i} + 3w) = 0$ or $4 + 3\lambda = 0$	M1
	So, $\lambda = -\frac{4}{3}$	A1
		(2)
		(11 marks)

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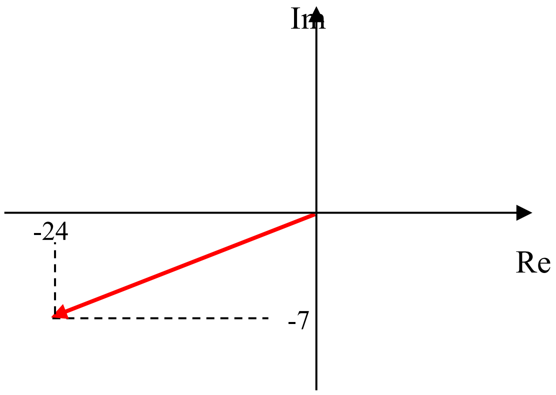
Question	Scheme		Marks
9(a)		Correct quadrant with $(-24, -7)$ indicated.	B1
			(1)
9(b)	$\arg z = -\pi + \tan^{-1}\left(\frac{7}{24}\right)$	$\tan^{-1}\left(\frac{7}{24}\right)$ or $\tan^{-1}\left(\frac{24}{7}\right)$	M1
	$= -2.857798544... = -2.86$ (2 dp)	awrt -2.86 or awrt 3.43	A1
			(2)
9(c)	$\lvert w\rvert = 4, \arg w = \frac{5\pi}{6} \Rightarrow r = 4, \theta = \frac{5\pi}{6}$		
	$w = r\cos\theta + \mathrm{i}r\sin\theta$		
	$w = 4\cos\left(\frac{5\pi}{6}\right) + 4\mathrm{i}\sin\left(\frac{5\pi}{6}\right)$	Attempt to apply $r\cos\theta + \mathrm{i}r\sin\theta$.	M1
		Correct expression for w.	A1
	$= 4\left(\frac{-\sqrt{3}}{2}\right) + 4\mathrm{i}\left(\frac{1}{2}\right)$		
	$= -2\sqrt{3} + 2\mathrm{i}$	either $-2\sqrt{3} + 2\mathrm{i}$ or awrt $-3.5 + 2\mathrm{i}$	A1
			(3)
9(d)	$a = -2\sqrt{3},\ b = 2$		
	$\lvert z\rvert = \sqrt{(-24)^2 + (-7)^2} = \underline{25}$	$\underline{\lvert z\rvert = 25}$ or $zw = (48\sqrt{3} + 14) + (14\sqrt{3} - 48)\mathrm{i}$ or awrt 97.1-23.8i	B1
	$\lvert zw\rvert = \lvert z\rvert \times \lvert w\rvert = (25)(4)$	Applies $\lvert z\rvert \times \lvert w\rvert$ or $\lvert zw\rvert$	M1
	$= \underline{100}$	$\underline{100}$	A1
			(3)
			(9 marks)

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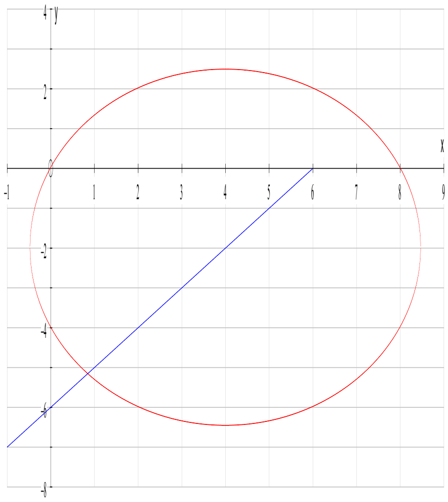
Question	Scheme	Marks
10(a)	$\lvert x+\mathrm{i}y-6\mathrm{i}\rvert = 2\lvert x+\mathrm{i}y-3\rvert$	M1
	$x^2+(y-6)^2 = 4\left[(x-3)^2+y^2\right]$	M1 A1
	$x^2+y^2-12y+36=4x^2-24x+36+4y^2$	
	$3x^2+3y^2-24x+12y=0$	
	$(x-4)^2+(y+2)^2=20$	M1
	Centre $(4,-2)$, Radius $\sqrt{20}=2\sqrt{5}=$ awrt 4.47	A1 A1
		(6)
10(b)		
	Centre in correct quad for their circle	M1
	Passes through O centre in 4th quad.	A1cao
	Half line with positive gradient	B1
	Correct position, clearly through (6, 0)	B1
		(4)
10(c)	Equation of line $y=x-6$	B1
	Attempting simultaneous solution of $(x-4)^2+(y+2)^2=20$ and $y=x-6$	M1
	$x=4\pm\sqrt{10}$	A1
	$\left(4-\sqrt{10}\right)+\mathrm{i}\left(-2-\sqrt{10}\right)$	A1cao
		(4)
		(14 marks)

Question	Scheme	Marks
11(a)	$w = 10 - 5\text{i}$	
	$\lvert w\rvert = \left\{\sqrt{10^2 + (-5)^2}\right\} = \sqrt{125}$ or $5\sqrt{5}$ or $11.1803...$	B1
	$\arg w = -\tan^{-1}\left(\frac{5}{10}\right)$	M1
	$= -0.463647609... = -0.46$ (2 dp)	A1 oe
		(2)
11(b)	$(2 + \text{i})(z + 3\text{i}) = w$	
	$z + 3\text{i} = \dfrac{10 - 5\text{i}}{(2 + \text{i})}$	B1
	$z + 3\text{i} = \dfrac{(10 - 5\text{i})}{(2 + \text{i})} \times \dfrac{(2 - \text{i})}{(2 - \text{i})}$	M1
	$z + 3\text{i} = \dfrac{20 - 10\text{i} - 10\text{i} - 5}{1 + 4}$	M1
	$z + 3\text{i} = \dfrac{15 - 20\text{i}}{5}$	
	$z + 3\text{i} = 3 - 4\text{i}$	
	$z = 3 - 7\text{i}$ (Note: $a = 3, b = -7$.)	A1
		(4)
11(c)	$\arg(\lambda + 9\text{i} + w) = \dfrac{\pi}{4}$	
	$\lambda + 9\text{i} + w = \lambda + 9\text{i} + 10 - 5\text{i} = (\lambda + 10) + 4\text{i}$	
	$\arg(\lambda + 9\text{i} + w) = \dfrac{\pi}{4} \Rightarrow \lambda + 10 = 4$	M1
	So, $\lambda = -6$	A1
		(2)
		(9 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	FP1 2016	4		Complex numbers	1.1b, 3.1a
2	FP1 Jan 2013	5		Complex numbers	1.1b, 3.1a
3	FP1 Jan 2012	5		Complex numbers	1.1b, 3.1a
4	FP1 2017	6		Complex numbers	1.1b, 3.1a
5	FP1 2011	6		Complex numbers	1.1b
6	FP1 2016	7		Complex numbers	1.1b, 2.1, 3.1a
7	FP1 2013	7		Complex numbers	1.1b, 3.1a
8	FP1 2012	7		Complex numbers	1.1b, 3.1a
9	FP1 2011	7		Complex numbers	1.1b, 3.1a
10	FP2 2012	8		Further complex numbers	1.1b, 2.1, 3.1a
11	FP1 2013R	9		Complex numbers	1.1b, 3.1a