

A level Mathematics Practice Paper – Trigonometry (part 2) – Mark scheme

Question	Scheme	Marks
1(a)	$R = \sqrt{5}$	B1
	$\tan \alpha = \frac{1}{2} \Rightarrow \alpha = 26.57^\circ$	M1 A1
		(3)
1(b)	$\frac{2}{2 \cos \theta - \sin \theta - 1} = 15 \Rightarrow \frac{2}{\sqrt{5} \cos(\theta + 26.6^\circ) - 1} = 15$	
	$\Rightarrow \cos(\theta + 26.6^\circ) = \frac{17}{15\sqrt{5}} = (\text{awrt } 0.507)$	M1 A1
	$\theta + 26.57^\circ = 59.54^\circ$	
	$\Rightarrow \theta = \text{awrt } 33.0^\circ \text{ or } \text{awrt } 273.9^\circ$	A1
	$\theta + 26.6^\circ = 360^\circ - \text{their } '59.5^\circ'$	dM1
	$\Rightarrow \theta = \text{awrt } 273.9^\circ \text{ and } \text{awrt } 33.0^\circ$	A1
		(5)
1(c)	$\theta - \text{their } 26.57^\circ = \text{their } 59.54^\circ \Rightarrow \theta = \dots$	M1
	$\theta = \text{awrt } 86.1^\circ$	A1
		(2)
		(10 marks)
2(a)	$4 \cos 2\theta + 2 \sin 2\theta = R \cos(2\theta - \alpha)$	
	$R = \sqrt{4^2 + 2^2} = \sqrt{20} = (2\sqrt{5})$	B1
	$\alpha = \arctan\left(\frac{1}{2}\right) = 26.565^\circ \dots = \text{awrt } 26.57^\circ$	M1A1
		(3)
2(b)	$\sqrt{20} \cos(2\theta - 26.6^\circ) = 1 \Rightarrow \cos(2\theta - 26.57^\circ) = \frac{1}{\sqrt{20}}$	M1
	$\Rightarrow (2\theta - 26.57^\circ) = +77.1^\circ \Rightarrow \theta = \dots$	dM1
	$\theta = \text{awrt } 51.8^\circ$	A1
	$2\theta - 26.57^\circ = -77.1^\circ \Rightarrow \theta = -\text{awrt } 25.3^\circ$	ddM1A1
		(5)
2(c)	$k < -\sqrt{20}, k > \sqrt{20}$	B1ft either B1ft both
		(2)
		(10 marks)

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Question	Scheme		Marks
3(a)	$2 \cos x \cos 50 - 2 \sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40$		M1
	$\sin x(\cos 40 + 2 \sin 50) = \cos x(2 \cos 50 - \sin 40)$		
	$\div \cos x \Rightarrow \tan x(\cos 40 + 2 \sin 50) = 2 \cos 50 - \sin 40$		M1
	$\tan x = \frac{2 \cos 50 - \sin 40}{\cos 40 + 2 \sin 50}, \quad (\text{or numerical answer awrt } 0.28)$		A1
	States or uses $\cos 50 = \sin 40$ and $\cos 40 = \sin 50$ and so $\tan x^\circ = \frac{1}{3} \tan 40^\circ$ * cao		A1 *
			(4)
3(b)	Deduces $\tan 2\theta = \frac{1}{3} \tan 40$		M1
	$2\theta = 15.6$ so $\theta =$ awrt 7.8(1) One answer		A1
	Also $2\theta = 195.6, 375.6, 555.6$ $\Rightarrow \theta = ..$		M1
	$\theta =$ awrt 7.8, 97.8, 187.8, 277.8 All 4 answers		A1
			(4)
			(8 marks)
4	$2 \cos 2\theta = 1 - 2 \sin \theta$		
	$2(1 - 2 \sin^2 \theta) = 1 - 2 \sin \theta$	Substitutes either $1 - 2 \sin^2 \theta$ or $2 \cos^2 \theta - 1$ or $\cos^2 \theta - \sin^2 \theta$ for $\cos 2\theta$.	M1
	$2 - 4 \sin^2 \theta = 1 - 2 \sin \theta$		
	$4 \sin^2 \theta - 2 \sin \theta - 1 = 0$	Forms a “quadratic in sine” = 0	M1(*)
	$\sin \theta = \frac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	Applies the quadratic formula See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$		
	$\theta = \{54, 126, 198, 342\}$	Any one correct answer	A1
		180-their pv	dM1(*)
		All four solutions correct.	A1
			(6 marks)

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Question	Scheme	Marks
5(a)	$R = \sqrt{29}$	B1
	$\tan \alpha = \frac{2}{5} \Rightarrow \alpha = \text{awrt } 0.381$	M1A1
		(3)
5(b)	$5 \cot 2x - 3 \operatorname{cosec} 2x = 2 \Rightarrow 5 \frac{\cos 2x}{\sin 2x} - \frac{3}{\sin 2x} = 2$	M1
	$\Rightarrow 5 \cos 2x - 2 \sin 2x = 3$	A1
		(2)
5(c)	$5 \cos 2x - 2 \sin 2x = 3 \Rightarrow \cos(2x + 0.381) = \frac{3}{\sqrt{29}}$	M1
	$2x + 0.381 = \arccos\left(\frac{3}{\sqrt{29}}\right) \Rightarrow x = \dots$	dM1
	$x = \text{awrt } 0.30, 2.46$	A1A1
		(4)
		(9 marks)
6(a)	$R^2 = 6^2 + 8^2 \Rightarrow R = 10$	M1A1
	$\tan \alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$	M1A1
		(4)
6(b)(i)	$p(x) = \frac{4}{12 + 10 \cos(\theta - 0.927)}$	
	$p(x) = \frac{4}{12 - 10}$	M1
	Maximum = 2	A1
		(2)
6(b)(ii)	$\theta - 'their \alpha' = \pi$	M1
	$\theta = \text{awrt } 4.07$	A1
		(2)
		(8 marks)

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Question	Scheme	Marks
7(i)	$(\sin 22.5 + \cos 22.5)^2 = \sin^2 22.5 + \cos^2 22.5 + \dots$	M1
	$= \sin^2 22.5 + \cos^2 22.5 + 2 \sin 22.5 \cos 22.5$	
	States or uses $\sin^2 22.5 + \cos^2 22.5 = 1$	B1
	Uses $2 \sin x \cos x = \sin 2x \Rightarrow 2 \sin 22.5 \cos 22.5 = \sin 45$	M1
	$(\sin 22.5 + \cos 22.5)^2 = 1 + \sin 45$	A1
	$= 1 + \frac{\sqrt{2}}{2}$ or $1 + \frac{1}{\sqrt{2}}$ cso	A1
		(5)
7(ii)(a)	$\cos 2\theta + \sin \theta = 1 \Rightarrow 1 - 2 \sin^2 \theta + \sin \theta = 1$	M1
	$\sin \theta - 2 \sin^2 \theta = 0$	
	$2 \sin^2 \theta - \sin \theta = 0$ or $k = 2$	A1*
		(2)
7(ii)(b)	$\sin \theta (2 \sin \theta - 1) = 0$	M1
	$\sin \theta = 0, \quad \sin \theta = \frac{1}{2}$	A1
	Any two of 0,30,150,180	B1
	All four answers 0,30,150,180	A1
		(4)
		(11 marks)

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Question	Scheme	Marks
8(a)	$R = \sqrt{6^2 + 2.5^2} = 6.5$	B1
	$\tan \alpha = \frac{2.5}{6}, \Rightarrow \alpha = \text{awrt } 0.395$	M1A1
		(3)
8(b)	(0, 6),	B1
	awrt (1.97, 0) (5.11, 0)	M1A1
		(3)
8(c)	$H_{\max} = 18.5, H_{\min} = 5.5$	M1A1A1
		(3)
8(d)	Sub $H = 16$ and proceed to ' $6.5 \cos\left(\frac{2\pi t}{52} \pm '0.395'$)' = 4	M1
	$\left(\frac{2\pi t}{52} - '0.395'\right) = \text{awrt } 0.91$	A1
	$t = (\text{awrt } 0.908 \pm '0.395') \times \frac{52}{2\pi} = 11 \text{ (10.78)}$	dM1A1
	$\left(\frac{2\pi t}{52} \pm '0.395'\right) = \text{awrt } 2\pi - 0.908 \Rightarrow t = 48 \text{ (47.75)}$	ddM1A1
		(6)
		(15 marks)

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Question	Scheme	Marks
9(a)	$R = \sqrt{7^2 + 24^2} = 25$	B1
	$\tan \alpha = \frac{24}{7}, \Rightarrow \alpha = \text{awrt } 73.74^\circ$	M1A1
		(3)
9(b)	maximum value of $24\sin x + 7\cos x = 25$ so $V_{\min} = \frac{21}{25} = (0.84)$	M1A1
		(2)
9(c)	Distance $AB = \frac{7}{\sin \theta}$, with $\theta = \alpha$	M1 B1
	So distance = $7.29\text{m} = \frac{175}{24}\text{m}$	A1
		(3)
9(d)	$R \cos(\theta - \alpha) = \frac{21}{1.68} \Rightarrow \cos(\theta - \alpha) = 0.5$	M1 A1
	$\theta - \alpha = 60 \Rightarrow \theta = .., \theta - \alpha = -60 \Rightarrow \theta = ..$	dM1 dM1
	$\theta = \text{awrt } 133.7, 13.7$	A1 A1
		(6)
		(14 marks)
10(a)	$R = \sqrt{20}$	B1
	$\tan \alpha = \frac{4}{2} \Rightarrow \alpha = \text{awrt } 1.107$	M1A1
		(3)
10(b)	' $4 + 5R^2$ ' = 104	B1ft
	$3\theta - '1.107' = \frac{\pi}{2} \Rightarrow \theta = \text{awrt } 0.89$	M1A1
		(3)
10(c)	4	B1
	$3\theta - '1.107' = 2\pi \Rightarrow \theta = \text{awrt } 2.46$	M1A1
		(3)
		(9 marks)

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	Source paper	Question number	New spec references	Question description	New AOs