

A level Pure Maths: Practice Paper F mark scheme

1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that: $A(2x + 5) + B(5x - 1) \equiv 6x + 42$	M1	2.2a	5th Decompose algebraic fractions into partial fractions – two linear factors.
	Equates the various terms. Equating the coefficients of x : $2A + 5B = 6$ Equating constant terms: $5A - B = 42$	M1*	2.2a	
	Multiplies both of the equations in an effort to equate one of the two variables.	M1*	1.1b	
	Finds $A = 8$	A1	1.1b	
	Find $B = -2$	A1	1.1b	
	(5 marks)			
Notes				
Alternative method				
Uses the substitution method, having first obtained this equation: $A(2x + 5) + B(5x - 1) \equiv 6x + 42$				
Substitutes $x = -\frac{5}{2}$ to obtain $-\frac{27}{2}B = 27$ (M1)				
Substitutes $x = \frac{1}{5}$ to obtain $\frac{27}{5}A = 43.2$ (M1)				

A level Pure Maths: Practice Paper F mark scheme

2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. ‘Assumption: there do exist integers a and b such that $25a + 15b = 1$ ’	B1	3.1	7th Complete proofs using proof by contradiction.
	Understands that $25a + 15b = 1 \Rightarrow 5a + 3b = \frac{1}{5}$ ‘As both 25 and 15 are multiples of 5, divide both sides by 5 to leave $5a + 3b = \frac{1}{5}$,’	M1	2.2a	
	Understands that if a and b are integers, then $5a$ is an integer, $3b$ is an integer and $5a + 3b$ is also an integer.	M1	1.1b	
	Recognises that this contradicts the statement that $5a + 3b = \frac{1}{5}$, as $\frac{1}{5}$ is not an integer. Therefore there do not exist integers a and b such that $25a + 15b = 1$ ’	B1	2.4	
(4 marks)				
Notes				

A level Pure Maths: Practice Paper F mark scheme

3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Finds $\frac{dx}{dt} = -2\sin 2t$ and $\frac{dy}{dt} = \cos t$	M1	1.1b	6th Differentiate simple functions defined parametrically including application to tangents and normals.
	Writes $-2\sin 2t = -4\sin t \cos t$	M1	2.2a	
	Calculates $\frac{dy}{dx} = \frac{\cos t}{-4\sin t \cos t} = -\frac{1}{4}\operatorname{cosec} t$	A1	1.1b	
		(3)		
(b)	Evaluates $\frac{dy}{dx}$ at $t = -\frac{5\pi}{6}$ $\frac{dy}{dx} = \frac{-1}{4\sin\left(-\frac{5\pi}{6}\right)} = \frac{1}{2}$	A1 ft	1.1b	6th Differentiate simple functions defined parametrically including application to tangents and normals.
	Understands that the gradient of the tangent is $\frac{1}{2}$, and then the gradient of the normal is -2 .	M1 ft	1.1b	
	Finds the values of x and y at $t = -\frac{5\pi}{6}$ $x = \cos\left(2 \times -\frac{5\pi}{6}\right) = \frac{1}{2}$ and $y = \sin\left(-\frac{5\pi}{6}\right) = -\frac{1}{2}$	M1 ft	1.1b	
	Attempts to substitute values into $y - y_1 = m(x - x_1)$ For example, $y + \frac{1}{2} = -2\left(x - \frac{1}{2}\right)$ is seen.	M1 ft	2.2a	
	Shows logical progression to simplify algebra, arriving at: $y = -2x + \frac{1}{2}$ or $4x + 2y - 1 = 0$	A1	2.4	
		(5)		
(8 marks)				
Notes				
(b) Award ft marks for a correct answer using an incorrect answer from part a.				

A level Pure Maths: Practice Paper F mark scheme

4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	States that $\cot 3x = \frac{\cos 3x}{\sin 3x}$	M1	2.2a	6th Integrate using trigonometric identities.
	Makes an attempt to find $\int \left(\frac{\cos 3x}{\sin 3x} \right) dx$ Writing $\int \frac{f'(x)}{f(x)} dx = \ln [f(x)]$ or writing $\ln (\sin x)$ constitutes an attempt.	M1	2.2a	
	States a fully correct answer $\frac{1}{3} \ln \sin 3x + C$	A1	1.1b	
(3 marks)				
Notes				

A level Pure Maths: Practice Paper F mark scheme

5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Demonstrates an attempt to find the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$	M1	2.2a	5th Find the magnitude of a vector in 3 dimensions.
	Finds $\overrightarrow{AB} = (0, 4, -2)$, $\overrightarrow{AC} = (5, 4, 8)$ and $\overrightarrow{BC} = (5, 0, 10)$	A1	1.1b	
	Demonstrates an attempt to find $ \overrightarrow{AB} $, $ \overrightarrow{AC} $ and $ \overrightarrow{BC} $	M1	2.2a	
	Finds $ \overrightarrow{AB} = \sqrt{(0)^2 + (4)^2 + (-2)^2} = \sqrt{20}$ Finds $ \overrightarrow{AC} = \sqrt{(5)^2 + (4)^2 + (8)^2} = \sqrt{105}$ Finds $ \overrightarrow{BC} = \sqrt{(5)^2 + (0)^2 + (10)^2} = \sqrt{125}$	A1	1.1b	
	States or implies in a right-angled triangle $c^2 = a^2 + b^2$	M1	2.2a	
	States that $ \overrightarrow{AB} ^2 + \overrightarrow{AC} ^2 = \overrightarrow{BC} ^2$	B1	2.1	
	(6 marks)			
Notes				

A level Pure Maths: Practice Paper F mark scheme

6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	States or implies that $pq(x) = (5 - 2x)^2$	M1	2.2a	5th Find composite functions.
	States or implies that $qp(x) = 5 - 2x^2$	M1	2.2a	
	Makes an attempt to solve $(5 - 2x)^2 = 5 - 2x^2$. For example, $25 - 20x + 4x^2 = 5 - 2x^2$ or $6x^2 - 20x + 20 = 0$ is seen.	M1	1.1b	
	States that $3x^2 - 10x + 10 = 0$. Must show all steps and a logical progression.	A1	1.1b	
		(4)		
(b)	$b^2 - 4ac = 100 - 4(3)(10) = -20 < 0$	M1*	2.2a	5th Find the domain and range of composite functions.
	States that as $b^2 - 4ac < 0$ there are no real solutions to the equation.	B1*	3.2b	
		(2)		
(6 marks)				
Notes (b) Alternative Method M1: Uses the method of completing the square to show that $3\left(x - \frac{5}{3}\right)^2 + \frac{65}{9} = 0$ or $3\left(x - \frac{5}{3}\right)^2 = -\frac{65}{9}$ B1: Concludes that this equation will have no real solutions.				

A level Pure Maths: Practice Paper F mark scheme

7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. ‘Assumption: there is a finite amount of prime numbers.’	B1	3.1	7th Complete proofs using proof by contradiction.
	Considers what having a finite amount of prime numbers means by making an attempt to list them: Let all the prime numbers exist be $p_1, p_2, p_3, \dots p_n$	M1	2.2a	
	Consider a new number that is one greater than the product of all the existing prime numbers: Let $N = (p_1 \times p_2 \times p_3 \times \dots \times p_n) + 1$	M1	1.1b	
	Understands the implication of this new number is that division by any of the existing prime numbers will leave a remainder of 1. So none of the existing prime numbers is a factor of N .	M1	1.1b	
	Concludes that either N is prime or N has a prime factor that is not currently listed.	B1	2.4	
	Recognises that either way this leads to a contradiction, and therefore there is an infinite number of prime numbers.	B1	2.4	
(6 marks)				
Notes If N is prime, it is a new prime number separate to the finite list of prime numbers, $p_1, p_2, p_3, \dots p_n$. If N is divisible by a previously unknown prime number, that prime number is also separate to the finite list of prime numbers.				

A level Pure Maths: Practice Paper F mark scheme

8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Attempts to write a differential equation. For example, $\frac{dF}{dt} \propto F$ or $\frac{dF}{dt} \propto -F$ is seen.	M1	3.1a	7th Construct simple differential equations.
	States $\frac{dF}{dt} = -kF$	A1	3.1a	
(2 marks)				
Notes				

A level Pure Maths: Practice Paper F mark scheme

9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Recognises that it is a geometric series with a first term $a = 100$ and common ratio $r = 1.05$	M1	3.1a	6th Use geometric sequences and series in context.
	Attempts to use the sum of a geometric series. For example, $S_9 = \frac{100(1-1.05^9)}{1-1.05}$ or $S_9 = \frac{100(1.05^9-1)}{1.05-1}$ is seen.	M1*	2.2a	
	Finds $S_9 = \text{£}1102.66$	A1	1.1b	
		(3)		
(b)	States $\frac{100(1.05^n-1)}{1.05-1} > 6000$ or $\frac{100(1-1.05^n)}{1-1.05} > 6000$	M1	3.1a	5th Use arithmetic sequences and series in context.
	Begins to simplify. $1.05^n > 4$ or $-1.05^n < -4$	M1	1.1b	
	Applies law of logarithms correctly $n\log 1.05 > \log 4$ or $-n\log 1.05 < -\log 4$	M1	2.2a	
	States $n > \frac{\log 4}{\log 1.05}$	A1	1.1b	
		(4)		
(c)	Uses the sum of an arithmetic series to state $\frac{29}{2}[100 + (28)d] = 6000$	M1	3.1a	5th Use arithmetic sequences and series in context.
	Solves for d . $d = \text{£}11.21$	A1	1.1b	
		(2)		
(9 marks)				
Notes				
M1				
Award mark if attempt to calculate the amount of money after 1, 2, 3, ..., 8 and 9 months is seen.				

A level Pure Maths: Practice Paper F mark scheme

10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Selects $\cos 2x \equiv 2 \cos^2 x - 1$ as the appropriate trigonometric identity.	M1	2.2a	6th Integrate using trigonometric identities.
	Manipulates the identity to the question: $\cos 12x \equiv 2 \cos^2 6x - 1$	M1	1.1b	
	States that $\int (\cos^2 6x) \mathrm{d}x = \frac{1}{2} \int (1 + \cos 12x) \mathrm{d}x$	M1	1.1b	
	Makes an attempt to integrate the expression, x and $\sin x$ are seen.	M1	1.1b	
	Correctly states $\frac{1}{2} \left(x + \frac{1}{12} \sin 12x \right) + C$	A1	1.1b	
(5 marks)				
Notes				
Student does not need to state ‘+C’ to be awarded the third method mark. Must be stated in the final answer.				

A level Pure Maths: Practice Paper F mark scheme

11	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Writes tanx and secx in terms of sinx and cosx. For example, $\frac{\tan x - \sec x}{1 - \sin x} = \frac{\left(\frac{\sin x}{\cos x} - \frac{1}{\cos x}\right)}{\left(\frac{1 - \sin x}{1}\right)}$	M1	2.1	5th Understand the functions sec, cosec and cot.
	Manipulates the expression to find $\left(\frac{\sin x - 1}{\cos x}\right) \times \left(\frac{1}{1 - \sin x}\right)$	M1	1.1b	
	Simplifies to find - $\frac{1}{\cos x} = - \sec x$	A1	1.1b	
		(3)		
(b)	States that - $\sec x = \sqrt{2}$ or $\sec x = - \sqrt{2}$	B1	2.2a	6th Use the functions sec, cosec and cot to solve simple trigonometric problems.
	Writes that $\cos x = - \frac{1}{\sqrt{2}}$ or $x = \cos^{-1}\left(- \frac{1}{\sqrt{2}}\right)$	M1	1.1b	
	Finds $x = \frac{3\pi}{4}, \frac{5\pi}{4}$	A1	1.1b	
		(3)		
(6 marks)				
Notes				

A level Pure Maths: Practice Paper F mark scheme

12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Rearranges $x = 8(t + 10)$ to obtain $t = \frac{x - 80}{8}$	M1	1.1b	8th Use parametric equations in modelling in a variety of contexts.
	Substitutes $t = \frac{x - 80}{8}$ into $y = 100 - t^2$ For example, $y = 100 - \left(\frac{x - 80}{8}\right)^2$ is seen.	M1	1.1b	
	Finds $y = -\frac{1}{64}x^2 + \frac{5}{2}x$	A1	1.1b	
		(3)		
(b)	Deduces that the width of the arch can be found by substituting $t = \pm 10$ into $x = 8(t + 10)$	M1	3.4	8th Use parametric equations in modelling in a variety of contexts.
	Finds $x = 0$ and $x = 160$ and deduces the width of the arch is 160 m.	A1	3.2a	
		(2)		
(c)	Deduces that the greatest height occurs when $\frac{dy}{dt} = 0 \Rightarrow -2t = 0 \Rightarrow t = 0$	M1	3.4	8th Use parametric equations in modelling in a variety of contexts.
	Deduces that the height is 100 m.	A1	3.2a	
		(2)		
(7 marks)				
Notes				

A level Pure Maths: Practice Paper F mark scheme

13	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to set up a long division. For example: $x + 6 \overline{) x^3 + 8x^2 - 9x + 12}$ is seen.	M1	2.2a	5th Divide polynomials by linear expressions with a remainder.
	Award 1 accuracy mark for each of the following: x^2 seen, $2x$ seen, -21 seen. For the final accuracy mark either $D = 138$ or $\frac{138}{x + 6}$ or the remainder is 138 must be seen. $\begin{array}{r} x^2 + 2x - 21 \\ x + 6 \overline{) x^3 + 8x^2 - 9x + 12} \\ \underline{x^3 + 6x^2} \\ 2x^2 - 9x \\ \underline{2x^2 + 12x} \\ -21x + 12 \\ \underline{-21x - 126} \\ 138 \end{array}$	A4	1.1b	
(5 marks)				
Notes This question can be solved by first writing $(Ax^2 + Bx + C)(x + 6) + D \equiv x^3 + 8x^2 - 9x + 12$ and then solving for A, B, C and D. Award 1 mark for the setting up the problem as described. Then award 1 mark for each correct coefficient found. For example: Equating the coefficients of x^3: $A = 1$ Equating the coefficients of x^2: $6 + B = 8$, so $B = 2$ Equating the coefficients of x: $12 + C = -9$, so $C = -21$ Equating the constant terms: $-126 + D = 12$, so $D = 138$.				

A level Pure Maths: Practice Paper F mark scheme

14	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Recognises the need to use the chain rule to find $\frac{dV}{dt}$ For example $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dS} \times \frac{dS}{dt}$ is seen.	M1	3.1a	8th Construct differential equations in a range of contexts.
	Finds $\frac{dV}{dr} = 4\pi r^2$ and $\frac{dS}{dr} = 8\pi r$	M1	2.2a	
	Makes an attempt to substitute known values. For example, $\frac{dV}{dt} = \frac{4\pi r^2}{1} \times \frac{1}{8\pi r} \times \frac{-12}{1}$	M1	1.1b	
	Simplifies and states $\frac{dV}{dt} = -6r$	A1	1.1b	
(4 marks)				
Notes				

A level Pure Maths: Practice Paper F mark scheme

	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
15	Recognises the need to write $\sin^3 x \equiv \sin x(\sin^2 x)$	M1	2.2a	6th Integrate using trigonometric identities.
	Selects the correct trigonometric identity to write $\sin x(\sin^2 x) \equiv \sin x(1 - \cos^2 x)$. Could also write $\sin x - \sin x \cos^2 x$	M1	2.2a	
	Makes an attempt to find $\int (\sin x - \sin x \cos^2 x) dx$	M1	1.1b	
	Correctly states answer $-\cos x + \frac{1}{3} \cos^3 x + C$	A1	1.1b	
(4 marks)				
Notes				

A level Pure Maths: Practice Paper F mark scheme

16	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Finds $h(19.3) = (+)0.974...$ and $h(19.4) = - 0.393...$	M1	3.1a	7th
	Change of sign and continuous function in the interval $[19.3, 19.4] \Rightarrow \text{root}$	A1	2.4	Use numerical methods to solve problems in context.
		(2)		
(b)	Makes an attempt to differentiate $h(t)$	M1	2.2a	7th
	Correctly finds $h'(t) = \frac{40}{t+1} + 8\cos\left(\frac{t}{5}\right) - \frac{1}{2}t$	A1	1.1b	Use numerical methods to solve problems in context.
	Finds $h(19.35) = 0.2903...$ and $h'(19.35) = - 13.6792...$	M1	1.1b	
	Attempts to find x_1 $x_1 = x_0 - \frac{h(x_0)}{h'(x_0)} \Rightarrow x_1 = 19.35 - \frac{0.2903...}{-13.6792...}$	M1	1.1b	
	Finds $x_1 = 19.371$	A1	1.1b	
		(5)		
(c)	Demonstrates an understanding that $x = 19.3705$ and $x = 19.3715$ are the two values to be calculated.	M1	2.2a	7th
	Finds $h(19.3705) = (+)0.0100...$ and $h(19.3715) = - 0.00366...$	M1	1.1b	Use numerical methods to solve problems in context.
	Change of sign and continuous function in the interval $[19.3705, 19.3715] \Rightarrow \text{root}$	A1	2.4	
		(3)		
(10 marks)				
Notes				
(a) Minimum required is that answer states there is a sign change in the interval and that this implies a root in the given interval.				

A level Pure Maths: Practice Paper F mark scheme

17	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Demonstrates an attempt to find the vectors $\vec{KL}$, $\vec{LM}$ and $\vec{KM}$	M1	2.2a	6th Solve geometric problems using vectors in 3 dimensions.
	Finds $\vec{KL} = (3, 0, -6)$, $\vec{LM} = (2, 5, 4)$ and $\vec{KM} = (5, 5, -2)$	A1	1.1b	
	Demonstrates an attempt to find $ \vec{KL} $, $ \vec{LM} $ and $ \vec{KM} $	M1	2.2a	
	Finds $ \vec{KL} = \sqrt{(3)^2 + (0)^2 + (-6)^2} = \sqrt{45}$ Finds $ \vec{LM} = \sqrt{(2)^2 + (5)^2 + (4)^2} = \sqrt{45}$ Finds $ \vec{KM} = \sqrt{(5)^2 + (5)^2 + (-2)^2} = \sqrt{54}$	A1	1.1b	
	Demonstrates an understanding of the need to use the Law of Cosines. Either $c^2 = a^2 + b^2 - 2ab \times \cos C$ (or variation) is seen, or attempt to substitute into formula is made $(\sqrt{54})^2 = (\sqrt{45})^2 + (\sqrt{45})^2 - 2(\sqrt{45})(\sqrt{45})\cos \theta$	M1 ft	2.2a	
	Makes an attempt to simplify the above equation. For example, $-36 = -90\cos \theta$ is seen.	M1 ft	1.1b	
	Shows a logical progression to state $\theta = 66.4^\circ$	B1	2.4	
		(7)		
(b)	States or implies that ΔKLM is isosceles.	M1	2.2a	6th Solve geometric problems using vectors in 3 dimensions.
	Makes an attempt to find the missing angles $\angle LKM = \angle LMK = \frac{180 - 66.421...}{2}$	M1	1.1b	
	States $\angle LKM = \angle LMK = 56.789...^\circ$. Accept awrt 56.8°	A1	1.1b	
		(3)		
(10 marks)				
Notes				
(b) Award ft marks for a correct answer to part a using their incorrect answer from earlier in part a.				