

$$1a) \quad y = 2x^3 + 5x^2 - 7x + 10$$

$$\frac{dy}{dx} = 6x^2 + 10x - 7$$

$$b) \quad \text{when } x = 2 \quad \frac{dy}{dx} = 6(2)^2 + 10(2) - 7$$

$$= \underline{\underline{37}}$$

$$2a) \quad y = 3x + \frac{1}{x}$$

$$y = 3x + x^{-1}$$

$$\frac{dy}{dx} = 3 - x^{-2}$$

$$b) \quad 3 - x^{-2} = 0$$

$$3 = x^{-2}$$

$$3 = \frac{1}{x^2}$$

$$x^2 = \frac{1}{3}$$

$$x = \pm \sqrt{\frac{1}{3}}$$

$$x = \frac{\sqrt{3}}{3} \quad \text{or} \quad x = -\frac{\sqrt{3}}{3}$$

$$3) \quad f(x) = 3x^{\frac{3}{2}} + \frac{3}{x^2} - 6x$$

$$f(x) = 3x^{\frac{3}{2}} + 3x^{-2} - 6x$$

$$f'(x) = \frac{9}{2}x^{\frac{1}{2}} - 6x^{-3} - 6$$

4a)

$$y = 4\sqrt{x} + \frac{1}{2x} + 10$$

$$y = 4x^{\frac{1}{2}} + \frac{1}{2}x^{-1} + 10$$

$$\frac{dy}{dx} = 2x^{-\frac{1}{2}} - \frac{1}{2}x^{-2}$$

b/

$$\frac{d^2y}{dx^2} = -x^{-\frac{3}{2}} + \cancel{2}x^{-3}$$

5a/

$$y = \frac{2x^2 - 5x + 3}{x}$$

$$\cancel{dy} y = 2x - 5 + 3x^{-1}$$

$$\frac{dy}{dx} = 2 - 3x^{-2}$$

b/ when $x=3$

$$\frac{dy}{dx} = 2 - 3(3)^{-2}$$

$$= \frac{5}{3}$$

6a/

$$y = x^3 - 4x^2 - 3x + 9$$

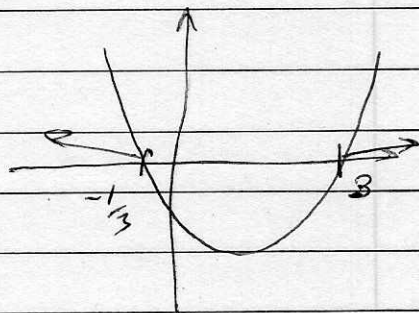
$$\frac{dy}{dx} = 3x^2 - 8x - 3$$

b/ Increasing when $\frac{dy}{dx} > 0$

$$3x^2 - 8x - 3 > 0$$

$$(3x+1)(x-3) > 0$$

$$x = -\frac{1}{3} \quad x = 3$$



$$\underline{\underline{x < -\frac{1}{3}}}$$

$$\underline{\underline{x > 3}}$$

7/

$$y = 2x^3 + 9x^2 - 24x + 13$$

$$\frac{dy}{dx} = 6x^2 + 18x - 24$$

or minimum

$$\text{Maximum}^{\vee} \text{ when } \frac{dy}{dx} = 0$$

$$6x^2 + 18x - 24 = 0$$

$$x^2 + 3x - 4 = 0$$

$$(x + 4)(x - 1) = 0$$

$$x = -4 \quad x = 1$$

Stationary points when $x = -4$ and $x = 1$

$$\frac{d^2y}{dx^2} = 12x + 18$$

$$\text{when } x = -4 \quad \frac{d^2y}{dx^2} = 12(-4) + 18 = -30 \quad \begin{array}{l} \text{-ve so:} \\ \text{MAX} \end{array}$$

$$x = 1 \quad \frac{d^2y}{dx^2} = 12(1) + 18 = 30 \quad \begin{array}{l} \text{+ve so:} \\ \text{MIN} \end{array}$$

Maximum when $x = -4$

$$y = 2(-4)^3 + 9(-4)^2 - 24(-4) + 13$$

$$= 125$$

$$\underline{\underline{(-4, 125)}}$$

8)

$$y = 2x^3 + 5x^2 - 7x + 10$$

$$\frac{dy}{dx} = 6x^2 + 10x - 7$$

when $x = 1$:

$$\frac{dy}{dx} = 6(1)^2 + 10(1) - 7$$

$$= 9$$

$$m = 9$$

when $x = 1$:

$$y = 2(1)^3 + 5(1)^2 - 7(1) + 10$$

$$y = 10$$

$$y = 9x + c \quad (1, 10)$$

$$10 = 9(1) + c$$

$$c = 1$$

$$\underline{\underline{y = 9x + 1}}$$

9/

$$f(x) = 2x^3 + x^2 - 18x + 2$$

$$f'(x) = 6x^2 + 2x - 18$$

A and B are where $f'(x) = 2$

$$6x^2 + 2x - 18 = 2$$

$$6x^2 + 2x - 20 = 0$$

$$3x^2 + x - 10 = 0$$

$$(3x - 5)(x + 2) = 0$$

$$\underline{\underline{x = \frac{5}{3}}} \quad \underline{\underline{x = -2}}$$

$$y = 2\left(\frac{5}{3}\right)^3 + \left(\frac{5}{3}\right)^2 - 18\left(\frac{5}{3}\right) + 2$$

$$= -\frac{431}{27}$$

$$y = 2(-2)^3 + (-2)^2 - 18(-2) + 2$$

$$= 26$$

$$\left(\frac{5}{3}, -\frac{431}{27}\right) \quad \text{and} \quad (-2, 26)$$

10/

$$y = \frac{(4x-1)(x+2)}{2x}$$

$$y = \frac{4x^2 + 8x - x - 2}{2x}$$

$$y = \frac{4x^2 + 7x - 2}{2x}$$

$$y = 2x + \frac{7}{2} - x^{-1}$$

$$\frac{dy}{dx} = 2 + x^{-2}$$

when $x = -2$

$$\frac{dy}{dx} = 2 + (-2)^{-2}$$

$$= \frac{9}{4}$$

$\therefore$ gradient of normal $= -\frac{4}{9}$

$$y = -\frac{4}{9}x + c$$

when $x = -2$

$$y = 2(-2) + \frac{7}{2} - (-2)^{-1}$$

$$= 0$$

$$0 = -\frac{4}{9}(-2) + c$$

$$0 = \frac{8}{9} + c$$

$$c = -\frac{8}{9}$$

$$y = -\frac{4}{9}x - \frac{8}{9}$$

$$9y = -4x - 8$$

$$\underline{4x + 9y + 8 = 0}$$

11/

$$C = \frac{4500}{v} + v + 10$$

$$C = 4500v^{-1} + v + 10$$

$$\frac{dC}{dv} = -4500v^{-2} + 1$$

Minimum where $\frac{dC}{dv} = 0$

$$-\frac{4500}{v^2} + 1 = 0$$

$$1 = \frac{4500}{v^2}$$

$$v^2 = 4500$$

$$v = 67.1 \text{ mph} \quad (30\sqrt{5})$$

b/

$$\frac{d^2C}{dv^2} = 9000v^{-3}$$

$$= 9000(30\sqrt{5})^{-3}$$

$$= 2716822593$$

positive $\therefore$ it is a minimum.

c/

$$C = \frac{4500}{30\sqrt{5}} + 30\sqrt{5} + 10$$

$$= £144.16$$

12/

$$\text{Surface area} = 2\pi r^2 + 2\pi rh$$

$$500 = 2\pi r^2 + 2\pi rh$$

$$\text{volume} = \pi r^2 h$$

$$2\pi rh = 500 - 2\pi r^2$$

$$h = \frac{500 - 2\pi r^2}{2\pi r}$$

$$\text{volume} = \pi r^2 \left(\frac{500 - 2\pi r^2}{2\pi r} \right)$$

$$V = \frac{\pi r^2 (500 - 2\pi r^2)}{2\pi r}$$

$$V = \frac{500r - 2\pi r^3}{2}$$

$$V = \underline{250r - \pi r^3}$$

b/

$$\frac{dV}{dr} = 250 - 3\pi r^2$$

$$\text{Max where } \frac{dV}{dr} = 0$$

$$250 - 3\pi r^2 = 0$$

$$250 = 3\pi r^2$$

$$\frac{250}{3\pi} = r^2$$

$$r = \sqrt{\frac{250}{3\pi}}$$

$$= 5.15 \text{ cm}$$

$$\text{Max } V = 250(5.15) - 3\pi(5.15)^2$$

$$= \underline{1038 \text{ cm}^3}$$

c/

$$\frac{d^2V}{dr^2} = -6\pi r$$

$$= -6\pi(5.15)$$

$$= -97.1$$

Negative $\therefore$ it is a maximum.

13)

$$y = 4x^3 + 15x^2 - 18x + 5$$

$$\frac{dy}{dx} = 12x^2 + 30x - 18$$

stationary points are where $\frac{dy}{dx} = 0$

$$12x^2 + 30x - 18 = 0$$

$$4x^2 + 10x - 6 = 0$$

$$2x^2 + 5x - 3 = 0$$

$$(2x - 1)(x + 3) = 0$$

$$x = \frac{1}{2} \quad x = -3$$

$$\frac{d^2y}{dx^2} = 24x + 30$$

when $x = \frac{1}{2}$ $\frac{d^2y}{dx^2} = 42$ positive $\therefore$ Minimum

when $x = -3$ $\frac{d^2y}{dx^2} = -42$ negative $\therefore$ Maximum

$$\begin{aligned} \text{when } x = \frac{1}{2} \quad y &= 4\left(\frac{1}{2}\right)^3 + 15\left(\frac{1}{2}\right)^2 - 18\left(\frac{1}{2}\right) + 5 \\ &= \frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{when } x = -3 \quad y &= 4(-3)^3 + 15(-3)^2 - 18(-3) + 5 \\ &= 86 \end{aligned}$$

Minimum at $\left(\frac{1}{2}, \frac{1}{4}\right)$

Maximum at $(-3, 86)$