

Topic Test

Summer 2022

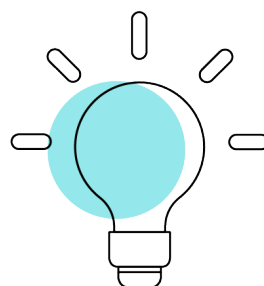
Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 10: Vectors

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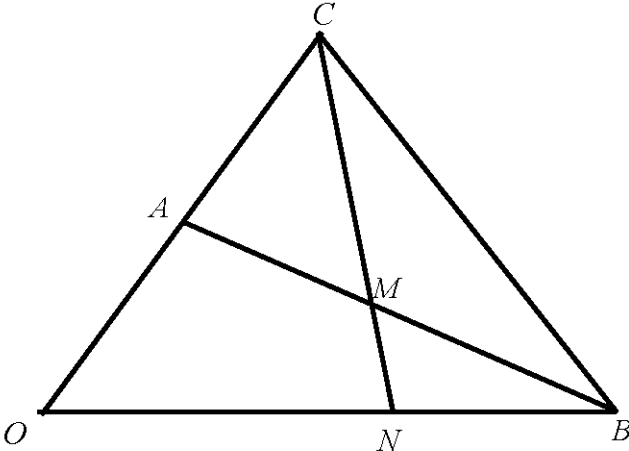


Mark Scheme

Question T10_Q1

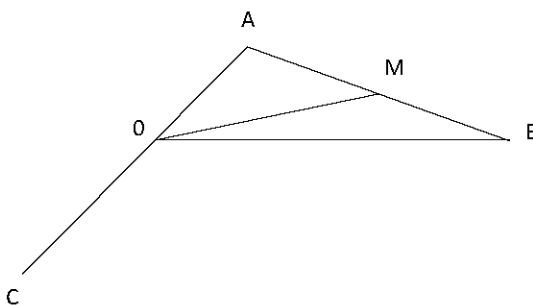
Question	Scheme	Marks	AOs
2	$\vec{OA} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, $\vec{OB} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\vec{OC} = a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $a < 0$ $\vec{AB} = \vec{BD}$, $ \vec{AB} = 4$		
(a)	E.g. $\vec{OD} = \vec{OB} + \vec{BD} = \vec{OB} + \vec{AB}$ or $\vec{OD} = \vec{OB} + \vec{BD} = \vec{OB} + \vec{AB} = \vec{OB} + \vec{OB} - \vec{OA} = 2\vec{OB} - \vec{OA}$ or $\vec{OD} = \vec{OB} + \vec{BD} = \vec{OB} + \vec{AB} = \vec{OA} + \vec{AB} + \vec{AB} = \vec{OA} + 2\vec{AB}$		
	$= \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \left\{ = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -5 \\ 7 \end{pmatrix} \right\}$ or $= \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + 2 \left(\begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} \right) \left\{ = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ -5 \\ 7 \end{pmatrix} \right\}$	M1	3.1a
	$= \begin{pmatrix} 6 \\ -7 \\ 10 \end{pmatrix} \text{ or } 6\mathbf{i} - 7\mathbf{j} + 10\mathbf{k}$	A1	1.1b
		(2)	
(b)	$(a-2)^2 + (5-3)^2 + (-2-4)^2$	M1	1.1b
	$\left\{ \vec{AC} = 4 \Rightarrow \right\} (a-2)^2 + (5-3)^2 + (-2-4)^2 = (4)^2$ $\Rightarrow (a-2)^2 = 8 \Rightarrow a = \dots$ or $\Rightarrow a^2 - 4a - 4 = 0 \Rightarrow a = \dots$	dM1	2.1
	(as $a < 0 \Rightarrow$) $a = 2 - 2\sqrt{2}$ (or $a = 2 - \sqrt{8}$)	A1	1.1b
		(3)	
(5 marks)			
Notes for Question 2			
(a)			
M1:	Complete applied strategy to find a vector expression for $\vec{OD}$		
A1:	See scheme		
Note:	Give M0 for subtracting the wrong way wrong to give e.g. $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + (2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) - (4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) = (4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + (-2\mathbf{i} + 5\mathbf{j} - 7\mathbf{k}) = (2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$		
Note:	Writing e.g. $\vec{OD} = \vec{OB} + \vec{AB}$ or $\vec{OD} = 2\vec{OB} - \vec{OA}$ with no other work is M0		
Note:	Finding coordinates , i.e. $(6, -7, 10)$ without reference to the correct position vectors is A0		
Note:	Allow M1A1 for writing down $6\mathbf{i} - 7\mathbf{j} + 10\mathbf{k}$ with no working		
Note:	M1 can be implied for at least two correct components in their position vector of D		
(b)			
M1:	Finds the difference between $\vec{OA}$ and $\vec{OC}$, then squares and adds each of the 3 components Note: Ignore labelling		
dM1:	Complete method of correctly applying Pythagoras' Theorem on $ \vec{AC} = 4$ and using a correct method of solving their resulting quadratic equation to find at least one of $a = \dots$		
Note:	Condone at least one of either awrt 4.8 or awrt -0.83 for the dM mark		
A1:	Obtains only one exact value, $a = 2 - 2\sqrt{2}$		
Note:	Writing $a = 2 \pm 2\sqrt{2}$, without evidence of rejecting $a = 2 + 2\sqrt{2}$ is A0		
Note:	Allow exact alternatives such as $2 - \sqrt{8}$ or $\frac{4 - \sqrt{32}}{2}$ for A1, and isw can be applied		
Note:	Writing $a = -0.828\dots$, without reference to a correct exact value is A0		

Question T10_Q2

Question	Scheme	Marks	AOs
10	 $\overrightarrow{OA} = \mathbf{a}, \overrightarrow{OB} = \mathbf{b}$		
(a)	$\left\{ \overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2} \overrightarrow{AB} \Rightarrow \right\} \overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$	M1	3.1a
	$\left\{ \overrightarrow{CM} = \overrightarrow{CB} + \overrightarrow{BM} = \overrightarrow{CB} + \frac{1}{2} \overrightarrow{BA} \Rightarrow \right\} \overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$		
	$\Rightarrow \overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ (needs to be simplified and seen in (a) only)	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OC} + \lambda \overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON} = 2\mathbf{a} + \lambda \left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \right) \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda \right) \mathbf{a} + \frac{1}{2}\lambda \mathbf{b} *$	A1*	2.1
		(2)	
(c) Way 1	$\left(2 - \frac{3}{2}\lambda \right) = 0 \Rightarrow \lambda = \dots$	M1	2.2a
	$\lambda = \frac{4}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON:NB = 2:1 *$	A1*	2.1
		(2)	
(c) Way 2	$\overrightarrow{ON} = \mu \mathbf{b} \Rightarrow \left(2 - \frac{3}{2}\lambda \right) \mathbf{a} + \frac{1}{2}\lambda \mathbf{b} = \mu \mathbf{b}$		
	$\mathbf{a}: \left(2 - \frac{3}{2}\lambda \right) = 0 \Rightarrow \lambda = \dots \left\{ \mathbf{b}: \frac{1}{2}\lambda = \mu \text{ \& } \lambda = \frac{4}{3} \Rightarrow \mu = \frac{2}{3} \right\}$	M1	2.2a
	$\lambda = \frac{4}{3} \text{ or } \mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON:NB = 2:1 *$	A1*	2.1
		(2)	

(6 marks)

Question	Scheme	Marks	AOs
10 (c) Way 3	$\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$		
	$\mathbf{a}: \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = \dots \quad \left\{ \mathbf{b}: 1 = \frac{1}{2}\lambda + K \quad \& \quad \lambda = \frac{4}{3} \Rightarrow K = \frac{1}{3} \right\}$	M1	2.2a
	$\lambda = \frac{4}{3} \text{ or } K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \text{ or } \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON:NB = 2:1 *$	A1	2.1
		(2)	
10 (c) Way 4	$\overrightarrow{ON} = \mu\mathbf{b} \text{ \& } \overrightarrow{CN} = k\overrightarrow{CM} \Rightarrow \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$		
	$-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3}, \quad \mathbf{b}: \mu = \frac{1}{2}k \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = \dots$	M1	2.2a
	$\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b} \left\{ \Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b} \right\} \Rightarrow ON:NB = 2:1 *$	A1	2.1
		(2)	
Notes for Question 10			
(a)			
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors $\mathbf{a}$ and $\mathbf{b}$		
A1:	A simplified correct answer for $\overrightarrow{CM}$		
Note:	Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or for $\left\{ \overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow \right\} \overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ only o.e.		
(b)			
M1:	Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$		
A1*:	Correct proof		
Note:	Special Case Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \left\{ = \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b} \right\}$		
Note:	Alternative 1: Give M1 A1 for the following alternative solution: $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$ $\mu = \lambda - 1 \Rightarrow \overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$		
(c)	Way 1, Way 2 and Way 3		
M1:	Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of λ		
A1*:	Correct proof		
(c)	Way 4		
M1:	Complete attempt to find the value of μ		
A1*:	Correct proof		

Notes for Question 10 Continued	
Note:	Part (b) and part (c) can be marked together.
(a) Special Case	<u>Special Case where the point C is believed to be below the origin O</u>
	
	Give Special Case M1 A0 in part (a) for $\left\{ \overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} \Rightarrow \right\} \overrightarrow{CM} = 3\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$
	$\left\{ \text{which leads to } \overrightarrow{CM} = \frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} \right\}$

Question T10_Q3

Question	Scheme	Marks	AOs
3 (a)	$\overrightarrow{AB} = (3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}) - (2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$	M1	1.1b
	$= \mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$	A1	1.1b
		(2)	
(b)	States $\overrightarrow{OC} = 2 \times \overrightarrow{AB}$	M1	1.1b
	Explains that as OC is parallel to AB , so $OABC$ is a trapezium.	A1	2.4
		(2)	
(4 marks)			
Notes:			

(a)

M1: Attempts to subtract either way around. If no method is seen it is implied by two of $\pm\mathbf{i} \pm 8\mathbf{j} \pm 2\mathbf{k}$.

A1: $\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$ or $\begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$ but not $(1, -8, 2)$

(b)

M1: Compares their $\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}$ with $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$ by stating **any one of**

- $\overrightarrow{OC} = 2 \times \overrightarrow{AB}$
- $\begin{pmatrix} 2 \\ -16 \\ 4 \end{pmatrix} = 2 \times \begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$
- $\overrightarrow{OC} = \lambda \times \overrightarrow{AB}$ or vice versa

This may be awarded if AB was subtracted "the wrong way around" or if there was one numerical slip

A1: A full explanation as to why $OABC$ is a trapezium.

Requires fully correct calculations, so part (a) must be $\overrightarrow{AB} = (\mathbf{i} - 8\mathbf{j} + 2\mathbf{k})$

It requires a reason and minimal conclusion.

Example 1:

$\overrightarrow{OC} = 2 \times \overrightarrow{AB}$, therefore OC is parallel to AB so $OABC$ is a trapezium

Example 2:

A trapezium has one pair of parallel sides. As $\overrightarrow{OC} = 2 \times \overrightarrow{AB}$, they are parallel, so ✓.

Example 3

As $\begin{pmatrix} 2 \\ -16 \\ 4 \end{pmatrix} = 2 \times \begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$, OC and AB are parallel, so proven

Example 4

Accept as $\overrightarrow{OC} = \lambda \times \overrightarrow{AB}$, they are parallel so true

Note: There are two definitions for a trapezium. One stating that it is a shape with one pair of parallel sides, the other with **only one** pair of parallel sides. Any calculations to do with sides OA and CB in this question may be ignored, even if incorrect.

Question T10_Q4

Question Number	Scheme	Marks	AO's
2	Attempts any one of $(\pm \overrightarrow{PQ} =) \pm (\mathbf{q} - \mathbf{p}), (\pm \overrightarrow{PR} =) \pm (\mathbf{r} - \mathbf{p}), (\pm \overrightarrow{QR} =) \pm (\mathbf{r} - \mathbf{q})$ Or e.g. $(\pm \overrightarrow{PQ} =) \pm (\overrightarrow{OQ} - \overrightarrow{OP}), (\pm \overrightarrow{PR} =) \pm (\overrightarrow{OR} - \overrightarrow{OP}), (\pm \overrightarrow{QR} =) \pm (\overrightarrow{OR} - \overrightarrow{OQ})$	M1	1.1b
	Attempts e.g. $\mathbf{r} - \mathbf{q} = 2(\mathbf{q} - \mathbf{p})$ $\mathbf{r} - \mathbf{p} = 3(\mathbf{q} - \mathbf{p})$ $\frac{2}{3}(\mathbf{q} - \mathbf{p}) = \frac{1}{3}(\mathbf{r} - \mathbf{q})$ $\mathbf{q} = \mathbf{p} + \frac{1}{3}(\mathbf{r} - \mathbf{p})$ $\mathbf{q} = \mathbf{r} + \frac{2}{3}(\mathbf{p} - \mathbf{r})$	dM1	3.1a
	E.g. $\Rightarrow \mathbf{r} - \mathbf{q} = 2\mathbf{q} - 2\mathbf{p} \Rightarrow 2\mathbf{p} + \mathbf{r} = 3\mathbf{q} \Rightarrow \mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})^*$	A1*	2.1
		(3)	
(3 marks)			

Notes:

M1: Attempts any of the relevant vectors by subtracting either way around. This may be implied by sight of any one of $\pm(\mathbf{q} - \mathbf{p}), \pm(\mathbf{r} - \mathbf{p}), \pm(\mathbf{r} - \mathbf{q})$ ignoring how they are labelled

dM1: Uses the given information and writes it correctly in vector form that if rearranged would give the printed answer

A1*: Fully correct work leading to the given answer. Allow $OQ = \dots$ as long as OQ has been defined as $\mathbf{q}$ earlier.

In the working allow use of P instead of $\mathbf{p}$ and Q instead of $\mathbf{q}$ as long as the intention is clear.

Question T10_Q5

Question	Scheme	Marks	AOs
6(a)	$\vec{AC} = \vec{AB} + \vec{BC} = -3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k} + \mathbf{i} + \mathbf{j} + 4\mathbf{k} = \dots$	M1	1.1b
	$= -2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $(AC^2) = "2^2 + 3^2 + 1^2", (AB^2) = 3^2 + 4^2 + 5^2, (BC^2) = 1^2 + 1^2 + 4^2$	M1	1.1b
	$2^2 + 3^2 + 1^2 = 3^2 + 4^2 + 5^2 + 1^2 + 1^2 + 4^2 - 2\sqrt{3^2 + 4^2 + 5^2}\sqrt{1^2 + 1^2 + 4^2} \cos ABC$	M1	3.1a
	$14 = 50 + 18 - 2\sqrt{50}\sqrt{18} \cos ABC$ $\Rightarrow \cos ABC = \frac{50 + 18 - 14}{2\sqrt{50}\sqrt{18}} = \frac{9}{10}^*$	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2 = 3^2 + 4^2 + 5^2, BC^2 = 1^2 + 1^2 + 4^2$	M1	1.1b
	$\vec{BA} \cdot \vec{BC} = (3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) \cdot (\mathbf{i} + \mathbf{j} + 4\mathbf{k}) = 27 = \sqrt{3^2 + 4^2 + 5^2}\sqrt{1^2 + 1^2 + 4^2} \cos ABC$	M1	3.1a
	$27 = \sqrt{50}\sqrt{18} \cos ABC \Rightarrow \cos ABC = \frac{27}{\sqrt{50}\sqrt{18}} = \frac{9}{10}^*$	A1*	2.1
(5 marks)			
Notes			

(a)

M1: Attempts $\vec{AC} = \vec{AB} + \vec{BC}$

There must be attempt to add not subtract.

If no method shown it may be implied by **two** correct components

A1: Correct vector. Allow $-2\mathbf{i} - 3\mathbf{j} - \mathbf{k}$ and $\begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix}$ but not $\begin{pmatrix} -2\mathbf{i} \\ -3\mathbf{j} \\ -1\mathbf{k} \end{pmatrix}$

(b)

M1: Attempts to "square and add" for at least 2 of the 3 sides. Follow through on their $\vec{AC}$

Look for an attempt at either $a^2 + b^2 + c^2$ or $\sqrt{a^2 + b^2 + c^2}$

M1: A correct attempt to apply a correct cosine rule to the given problem; Condone **slips** on the lengths of the sides but the sides must be in the correct position to find angle ABC

A1*: Correct completion with sufficient intermediate work to establish the printed result.

Condone different labelling, e.g. $ABC \leftrightarrow \theta$ as long as it is clear what is meant

It is OK to move from a correct cosine rule $14 = 50 + 18 - 2\sqrt{50}\sqrt{18} \cos ABC$

$$\text{via } \cos ABC = \frac{54}{2\sqrt{50}\sqrt{18}} \text{ o.e. such as } \cos ABC = \frac{(5\sqrt{2})^2 + (3\sqrt{2})^2 - (\sqrt{14})^2}{2 \times 5\sqrt{2} \times 3\sqrt{2}} \text{ to } \cos ABC = \frac{9}{10}$$

Alternative:

M1: Correct application of Pythagoras for sides AB and BC or their squares

M1: Recognises the requirement for and applies the scalar product

A1*: Correct completion with sufficient intermediate work to establish the printed result