

Write your name here	
Surname	Other names
Pearson Edexcel GCE	Centre Number Candidate Number
A level Mathematics Practice Paper Pure Mathematics - Differentiation (part 2)	
You must have: Mathematical Formulae and Statistical Tables (Pink)	Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 11 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. The curve C has equation $y = f(x)$ where

$$f(x) = \frac{4x+1}{x-2}, \quad x > 2$$

- (a) Show that

$$f'(x) = \frac{-9}{(x-2)^2}$$

(3)

Given that P is a point on C such that $f'(x) = -1$,

- (b) find the coordinates of P .

(3)

(Total 6 marks)

2. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

- (a) the value of w ,

(2)

- (b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

(Total 7 marks)

3. The curve C has equation $x = 8y \tan 2y$.

The point P has coordinates $\left(\pi, \frac{\pi}{8}\right)$.

- (a) Verify that P lies on C .

(1)

- (b) Find the equation of the tangent to C at P in the form $ay = x + b$, where the constants a and b are to be found in terms of π .

(7)

(Total 8 marks)

4. (i) Given that $x = \sec^2 2y$, $0 < y < \frac{\pi}{4}$

show that
$$\frac{dy}{dx} = \frac{1}{4x\sqrt{(x-1)}}$$

(4)

- (ii) Given that $y = (x^2 + x^3) \ln 2x$

find the exact value of $\frac{dy}{dx}$ at $x = \frac{e}{2}$, giving your answer in its simplest form.

(5)

- (iii) Given that $f(x) = \frac{3 \cos x}{(x+1)^{\frac{1}{3}}}$, $x \neq -1$

show that
$$f'(x) = \frac{g(x)}{(x+1)^{\frac{4}{3}}}, \quad x \neq -1$$

where $g(x)$ is an expression to be found.

(3)

(Total 12 marks)

5. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2.$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

- (a) find the exact value of p .

(1)

The tangent to the curve at P cuts the y -axis at the point A .

- (b) Use calculus to find the coordinates of A .

(6)

(Total 7 marks)

6. (i) Find, using calculus, the x coordinate of the turning point of the curve with equation

$$y = e^{3x} \cos 4x, \quad \frac{\pi}{4} \leq x < \frac{\pi}{2}.$$

Give your answer to 4 decimal places.

(5)

- (ii) Given $x = \sin^2 2y$, $0 < y < \frac{\pi}{4}$, find $\frac{dy}{dx}$ as a function of y .

Write your answer in the form

$$\frac{dy}{dx} = p \operatorname{cosec}(qy), \quad 0 < y < \frac{\pi}{4},$$

where p and q are constants to be determined.

(5)

(Total 10 marks)

7. (i) Given $y = 2x(x^2 - 1)^5$, show that

(a) $\frac{dy}{dx} = g(x)(x^2 - 1)^4$ where $g(x)$ is a function to be determined. (4)

(b) Hence find the set of values of x for which $\frac{dy}{dx} \geq 0$ (2)

(ii) Given

$$x = \ln(\sec 2y), \quad 0 < y < \frac{\pi}{4}$$

find $\frac{dy}{dx}$ as a function of x in its simplest form. (4)

(Total 10 marks)

8. $f(x) = \frac{4x - 5}{(2x + 1)(x - 3)} - \frac{2x}{x^2 - 9}, \quad x \neq \pm 3, \quad x \neq -\frac{1}{2}.$

(a) Show that

$$f(x) = \frac{5}{(2x + 1)(x + 3)}.$$

(5)

The curve C has equation $y = f(x)$. The point $P \left(-1, -\frac{5}{2} \right)$ lies on C .

(b) Find an equation of the normal to C at P . (8)

(Total 13 marks)

9.

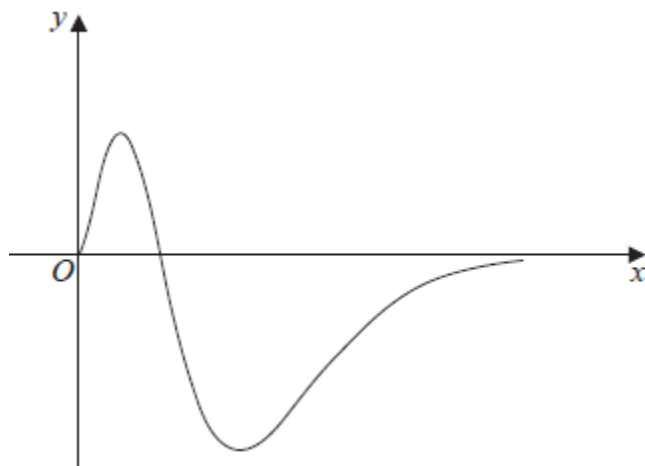


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \quad x \geq 0.$$

(a) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found.

(3)

(b) Hence find the range of g .

(6)

(c) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

(Total 10 marks)

10. Given that k is a **negative** constant and that the function $f(x)$ is defined by

$$f(x) = 2 - \frac{(x-5k)(x-k)}{x^2 - 3kx + 2k^2}, \quad x \geq 0,$$

- (a) show that $f(x) = \frac{x+k}{x-2k}$. (3)

- (b) Hence find $f'(x)$, giving your answer in its simplest form. (3)

- (c) State, with a reason, whether $f(x)$ is an increasing or a decreasing function.
Justify your answer.

(2)

(Total 8 marks)

11. $f(x) = \frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6}, \quad x > 2, \quad x \in \mathbb{R}.$

- (a) Given that

$$\frac{x^4 + x^3 - 3x^2 + 7x - 6}{x^2 + x - 6} \equiv x^2 + A + \frac{B}{x-2},$$

find the values of the constants A and B .

(4)

- (b) Hence or otherwise, using calculus, find an equation of the normal to the curve with equation $y = f(x)$ at the point where $x = 3$. (5)

(Total 9 marks)

TOTAL FOR PAPER: 100 MARKS