YEAR 13 MOCK 2020 PAPER 1

1 (a)

$$A = 105 - 12e^{0.08t}$$

$t = 0$

$$A = 105 - 12 = 93m^2$$

(b)

$$40 = 105 - 12e^{0.08t}$$

$$12e^{0.08t} = 65$$

$$e^{0.08t} = \frac{65}{12}$$

$$0.08t = \ln\left(\frac{65}{12}\right)$$

$$t = 21.1 \text{ days}$$

(c) $t = 30$ $A = -27.3$

cannot use as it gives a negative area

(2)

a) $(1 + 4x)^{\frac{1}{2}}$

$n = \frac{1}{2}$ $\quad x = 4x$

$$1 + \frac{1}{2}(4x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(4x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{2}{3})}{6}(4x)^3$$

$$= 1 + 2x - 2x^2 + 4x^3$$

b) Expansion valid if $|4x| < 1$

$|x| < \frac{1}{4}$

$\frac{25}{4}$ is greater than $\frac{1}{4}$

so should not use

c) when $x = \frac{1}{100}$ $\quad (1+4x)^{\frac{1}{2}} = \frac{\sqrt{26}}{5}$

$x = \frac{1}{100}$ sub into the expansion

then multiplied with give approx for $\sqrt{26}$

(3)

(a) $u_1 = 1$

$u_2 = 4$

$u_3 = -2$

$u_1 = u_3$

period 3

$u_4 = 1$

$u_5 = 4$

$u_3 = -2$

(b) $\sum_{1}^{48} \underbrace{16}_{48 \div 3} \times \underbrace{(3)}_{\text{sum of } u_1 + u_2 + u_3} = 48$

$\sum_{1}^{50} 48 + \underbrace{(1+4)}_{\text{terms } u_{49} + u_{50}} = 53$

④

(a)

$$\begin{array}{r|l} & 4x - 3 \\ \hline x^2-4x+4 & 4x^3 - 19x^2 + 28x - 4 \\ - & 4x^3 - 16x^2 + 16x \\ \hline & \quad -3x^2 + 12x - 4 \\ - & \quad -3x^2 + 12x - 12 \\ \hline & \qquad\qquad 8 \end{array}$$

$$4x - 3 + \frac{8}{(x-2)^2}$$

(b)

$$\int 4x - 3 + \frac{8}{(x-2)^2}$$

$$= \int 4x - 3 + 8(x-2)^{-2}$$

$$= \frac{4x^2}{2} - 3x + \frac{8(x-2)^{-1}}{-1} + C$$

$$= 2x^2 - 3x - 8(x-2)^{-1} + C$$

$$= 2x^2 - 3x - \frac{8}{(x-2)} + C$$

(5)

(a) $AB = 3i + pj - (-2i + 3j)$

$AB = 5i + (p-3)j$

$|AB| = 5^2 + (p-3)^2$

$(5\sqrt{2})^2 = 25 + (p-3)^2$

$50 = 25 + (p-3)^2$

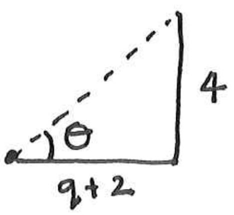
$25 = (p-3)^2$

$\pm 5 = p - 3$

$\therefore \quad p = 8, -2$

(b) $AC = qi + 7j - (-2i + 3j)$

$AC = (q+2)i + 4j$



$\tan\theta = \dfrac{4}{q+2}$

$\tan\left(\dfrac{\pi}{3}\right) = \dfrac{4}{q+2}$

$\sqrt{3} = \dfrac{4}{q+2}$

$q = \dfrac{4}{\sqrt{3}} - 2$

(6)

Addition Formula

(a) $\tan(2\theta+\theta) = \dfrac{\tan 2\theta + \tan\theta}{1-\tan 2\theta \tan\theta}$

double angle

$$= \frac{\frac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\tan\theta}$$

$$= \frac{\frac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1-\frac{2\tan^2\theta}{1-\tan^2\theta}}$$

$$= \frac{\frac{2\tan\theta}{1-\tan^2\theta} + \frac{\tan\theta(1-\tan^2\theta)}{1-\tan^2\theta}}{\frac{1-\tan^2\theta}{1-\tan^2\theta}-\frac{2\tan^2\theta}{1-\tan^2\theta}}$$

$$= \frac{\frac{2\tan\theta + \tan\theta - \tan^3\theta}{1-\tan^2\theta}}{\frac{1-\tan^2\theta-2\tan^2\theta}{1-\tan^2\theta}}$$

$$= \frac{3\tan\theta - \tan^3\theta}{1-3\tan^3\theta}$$

⑥

(b) $$\tan 3\beta = \frac{3\tan\beta - \tan^3\beta}{1 - 3\tan^2\beta}$$

$\tan\beta = \sqrt{6}$

$$\tan 3\beta = \frac{3\sqrt{6} - (\sqrt{6})^3}{1 - 3(\sqrt{6})^2}$$

$$= \frac{3\sqrt{6} - 6\sqrt{6}}{1 - 18}$$

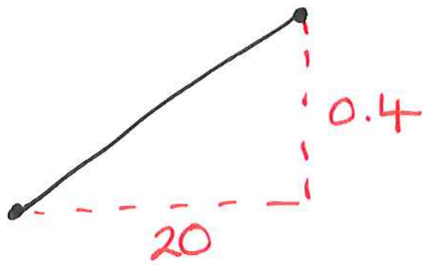
$$= \frac{-3\sqrt{6}}{-17}$$

$$= \frac{3\sqrt{6}}{17}$$

$$k = \frac{3}{17}$$

7

(a)



gradient $= \frac{0.4}{50} = 0.02$

y intercept $= 2$

$y = mx + c$

$\log_{10} P = 0.02t + 2$

(b) $\log_{10} P = 0.02t + 2$

$t = 0$

$\log_{10} P = 2$

$10^2 = P$

$P = 100$

(c) $\log_{10} P = 0.02t + 2$

$10^{0.02t+2} = P$

$(10^{0.02t})(10^2) = P$

$P = 100(10^{0.02})^t$

$\therefore$ $a = 100$

$b = 10^{0.02} = 1.047$

7

(d)(i) $P = 100(1.047)^t$

1996 to 2019 $t = 23$

$P = 100(1.047)^{23}$

$P = 288$

$288 > 198$

significantly more squirrels using the model. model not valid in 2019

(ii) - wood not big enough to sustain growth

- predators may effect numbers

- disease may effect numbers

(8)

(a) $180 + 30 = 210^\circ$ $(210, -5)$

(b) $5\cos(x - 30) = 4\sin x$

Additional Formula

$$5(\cos x \cos 30 + \sin x \sin 30) = 4\sin x$$

$$5 \cos x \frac{\sqrt{3}}{2} + 5\sin x \frac{1}{2} = 4\sin x$$

$$\frac{5\sqrt{3}}{2} \cos x + \frac{5}{2}\sin x = 4\sin x$$

$$\frac{5\sqrt{3}}{2} \cos x = \frac{3}{2}\sin x$$

$$\frac{10\sqrt{3}}{6} = \frac{\sin x}{\cos x}$$

$$\tan x = \frac{5\sqrt{3}}{6}$$

$$x = 70.9, 250.9$$

(c) 1 cycle $\tan x$ 2 roots

1 cycle $\tan 2x$ 4 roots

10 cycles $\tan 2x$ 40 roots

(9)

(a) $2 \div 4 = 0.5$ $h = 0.5$

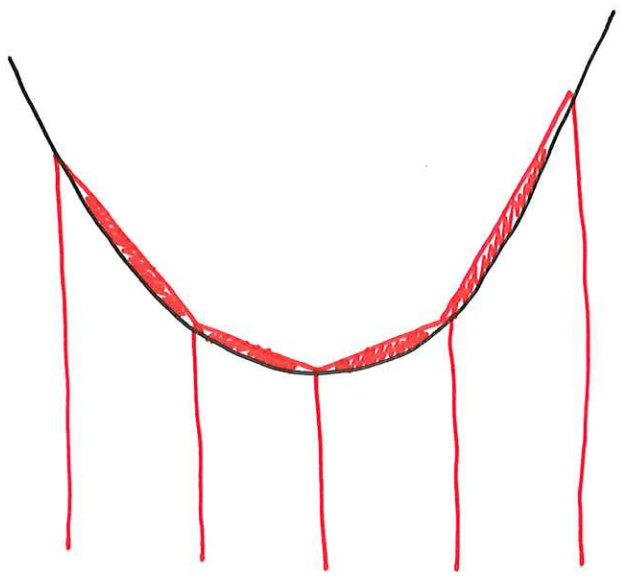
x	-0.5	0	0.5	1	1.5
$y = 2^{x^2} - x$	1.689	1	0.689	1	3.257

$$\text{Area} = \frac{1}{2}(0.5)\left[1.689 + 3.257 + 2\left(1 + 0.689 + 1\right)\right]$$

$$= 2.581$$

$$= 2.58 \text{ (2dp)}$$

(b)



overestimate

(9)

(c) $y_a = 2^{x^2} - x$

$\times 2 \quad +4x$

$y_b = 2\left(2^{x^2} - x\right) + 4x$

$y_b = 2\left(2^{x^2}\right) - 2x + 4x$

$y_b = 2^{x^2+1} + 2x$

$$\int 2^{x^2+1} + 2x = \int 2\left(2^{x^2} - x\right) + 4x$$

$$= 2\int 2^{x^2} - x + \int 4x$$

$$= 2\,(2.58) + \left[2x\right]_{-0.5}^{1.5}$$

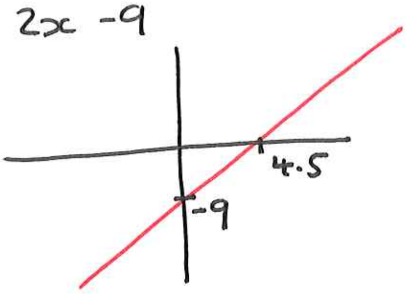
$$= 5.16 + 4$$

$$= 9.16$$

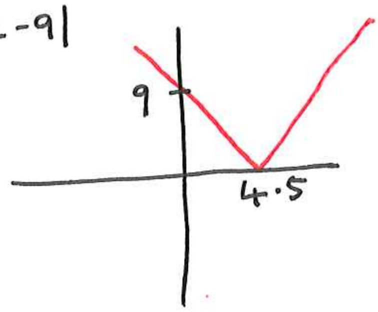
10

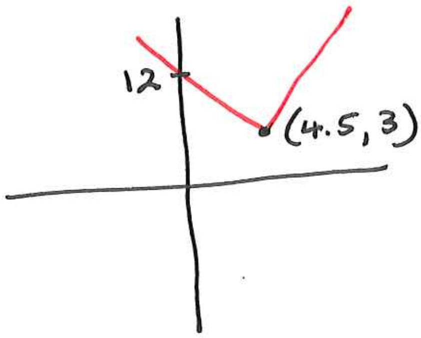
(a)

$y = 2x - 9$

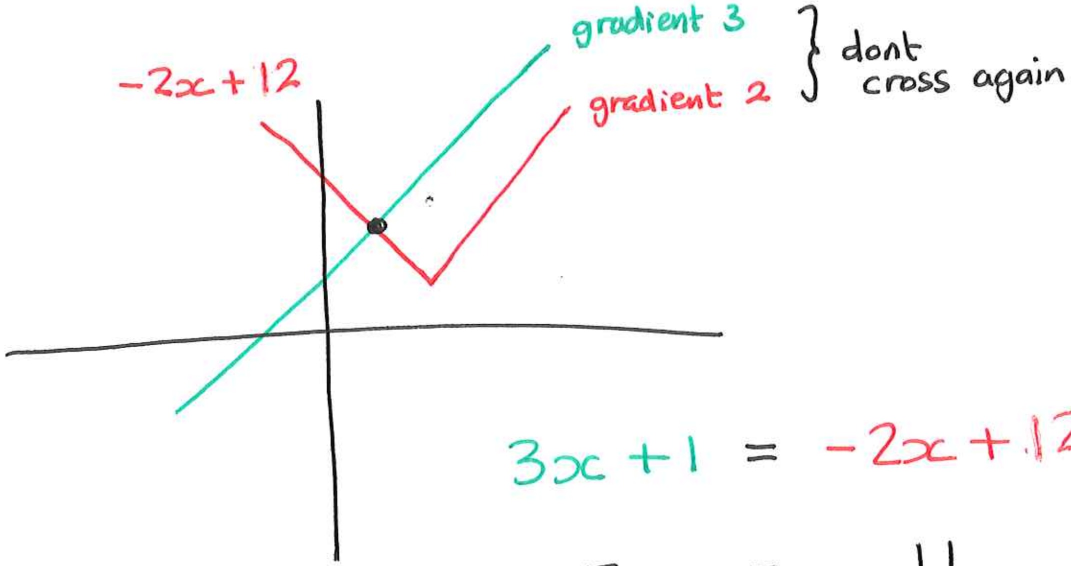


$y = |2x - 9|$





(b)

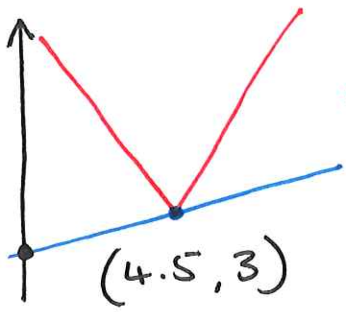


$$3x + 1 = -2x + 12$$

$$5x = 11$$

$$x = \frac{11}{5}$$

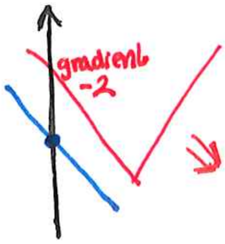
(c)



$\Rightarrow$ $y = kx + 1$

$$3 = k(4.5) + 1$$

$$k = \frac{4}{9}$$



$\Rightarrow$ $k \geqslant -2$

$$\therefore \quad k < \frac{4}{9} \text{ and } k \geqslant -2$$

(11)

(a)

$$y = \frac{2+3x}{x}$$

$$yx = 2+3x$$

$$yx - 3x = 2$$

$$x(y-3) = 2$$

$$x = \frac{2}{y-3}$$

$$f^{-1}(x) = \frac{2}{x-3}$$

$$\frac{5}{3} = \frac{2}{x-3}$$

$$5x - 15 = 6$$

$$5x = 21$$

$$x = \frac{21}{5}$$

(11)

(b)
$$\frac{2+3x}{x} < \frac{3}{2}$$

$$4 + 6x < 3x$$

$$4 < -3x$$

$$-4 > 3x$$

$$x < \frac{-4}{3}$$

function defined for $x > 0$

(c)
$$f(x) = \frac{2+3x}{x}$$

$$f(x) = \frac{2}{x} + 3$$

$$f(x) = 2x^{-1} + 3$$

$$f'(x) = -2x^{-2}$$

$$f'(x) = \frac{-2}{x^2}$$

$\frac{-2}{x^2} < 0$ for all values of x

$\therefore$ decreasing function

(12) $$\int_1^5 \frac{4x+9}{x+3}\,dx$$

let $u = x+3$
$x = u-3$

$\frac{dx}{du} = 1 \quad \therefore dx = du$

limits $x=5 \Rightarrow u=8$
$x=1 \Rightarrow u=4$

$$\int_4^8 \frac{4(u-3)+9}{u}\,du$$

$$= \int_4^8 \frac{4u-12+9}{u}\,du$$

$$= \int_4^8 \frac{4u-3}{u}\,du$$

$$= \int_4^8 4 - \frac{3}{u}\,du$$

$$= \left[4u - 3\ln u\right]_4^8$$

$$= [32 - 3\ln 8] - [16 - 3\ln 4]$$

$$= [32 - 3\ln 2^3] - [16 - 3\ln 2^2]$$

$$= [32 - 9\ln 2] - [16 - 6\ln 2]$$

$$= 16 - 3\ln 2$$

13

(a) Need to find $\frac{dh}{dt}$

Given $\frac{dV}{dt} = -\frac{\pi}{512}\sqrt{h}$

$$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$$

need V in terms of h

$$V = \frac{1}{3}\pi r^2 h$$

$r : h$

$2.5 : 4$

$0.625 : 1$

$r = 0.625h$

$$V = \frac{1}{3}\pi (0.625h)^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{25}{64}\right) h^3$$

$$V = \frac{25\pi}{192} h^3$$

$$\frac{dV}{dh} = \frac{75\pi}{192} h^2$$

(13)

(b) $$h^{3/2}\frac{dh}{dt} = -\frac{1}{200}$$

diff equation

$$\int h^{3/2}\,dh = \int -\frac{1}{200}\,dt$$

$$\frac{2h^{5/2}}{5} = \frac{-t}{200} + c$$

when full $t=0$, $h=4$ $\quad \therefore \quad c = \frac{64}{5}$

$$\frac{2h^{5/2}}{5} = \frac{-t}{200} + \frac{64}{5}$$

(c) empty $h=0$

$$0 = \frac{-t}{200} + \frac{64}{5}$$

$t = 2560$ sec

$t = 42$min 40sec

real time and predicted time are very close
model seems suitable

(14)

(a)

double angle

$x = 2\sin t \qquad y = 3(2\sin t\cos t)$

$\frac{x}{2} = \sin t \qquad y = 6\sin t\cos t$

$$y = 6\left(\frac{x}{2}\right)\cos t$$

$\cos^2 t = 1 - \sin^2 t$

$\cos^2 t = 1 - \frac{x^2}{4}$

$\cos t = \sqrt{1 - \frac{x^2}{4}}$

$$y = 6\left(\frac{x}{2}\right)\left(1 - \frac{x^2}{4}\right)^{\frac{1}{2}}$$

$$y = 6\left(\frac{x}{2}\right)\left(\frac{4 - x^2}{4}\right)^{\frac{1}{2}}$$

$$y = 6\left(\frac{x}{2}\right)\frac{\sqrt{4 - x^2}}{2}$$

$$y = \frac{3}{2}x\sqrt{4 - x^2}$$

$$y^2 = \frac{9}{4}x^2(4 - x^2)$$

(14)

(b) $y^2 = \frac{9}{4}x^2(4-x^2)$

Circle $x^2 + y^2 = r^2$

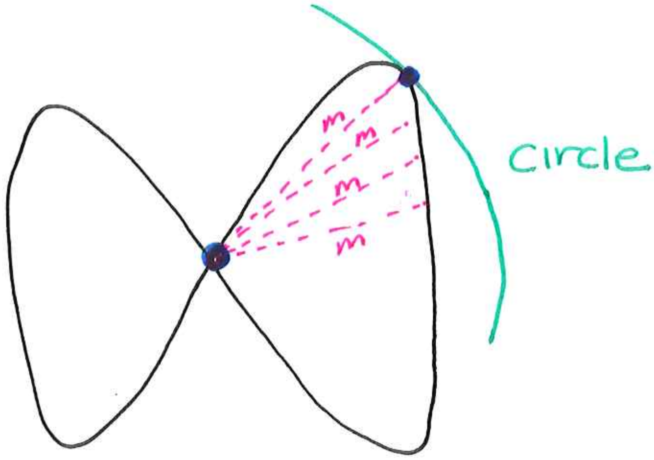
$$x^2 + \frac{9}{4}x^2(4-x^2) = r^2$$

$$x^2 + 9x^2 - \frac{9}{4}x^4 = r^2$$

$$10x^2 - \frac{9}{4}x^4 = r^2$$

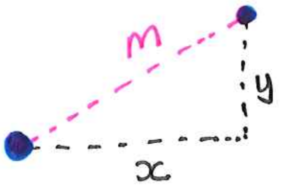
14

c)



$x = 2\sin t$

$y = 3\sin 2t$



$m^2 = x^2 + y^2$

$m^2 = (2\sin t)^2 + (3\sin 2t)^2$

$m^2 = 4\sin^2 t + 9\sin^2 2t$

implicit diff to find $\frac{dm}{dt}$

$4\sin^2 t$

$4u^2$	$\sin t$
$8u$	$\cos t$

chain rule

$9\sin^2 2t$

$9u^2$	$\sin 2t$
$18u^2$	$2\cos 2t$

chain rule

$2m\frac{dm}{dt} = 8\sin \cos t + 36\sin 2t \cos 2t$

m = max value $\therefore \frac{dm}{dt} = 0$

$0 = 8\sin \cos t + 36\sin 2t \cos 2t$

$0 = 8\sin \cos t + 36(2\sin t \cos t)\cos 2t$

$0 = 8\sin \cos t + 72\sin t \cos t \cos 2t$

$0 = 8\sin \cos t\,(1 + 9\cos 2t)$

$\therefore \quad 0 = 1 + 9\cos 2t \quad \Rightarrow \quad \cos t\ 2t = \frac{1}{-9}$

(c) $\cos t \; 2t = -\frac{1}{9}$

double angle

$1 - \sin^2 t = -\frac{1}{9} \Rightarrow \sin^2 t = \frac{5}{9}$

$m^2 = 4\sin^2 t + 9\sin^2 2t$

$\sin^2 2t + \cos^2 2t = 1$

$m^2 = 4\sin^2 t + 9(1 - \cos^2 2t)$

$m^2 = 4\left(-\frac{1}{9}\right) + 9\left(1 - \frac{1}{81}\right)$

$m^2 = \frac{100}{9}$

$m = \frac{10}{3}$

when $m = \max$ $m = r$

$r = \frac{10}{3}$